

# **JEE - 2025**

## **MATHS - MODULES**



**THEORY  
EXERCISE  
SOLUTIONS**

- ✓ Useful for JEE MAINS and ADVANCED Exams
- ✓ Each topic contains Detailed Theory with images
- ✓ Every topic contains Exercises and Detailed solutions

HINTS & SOLUTIONS

EXERCISE - 1

Single Choice

$$7. D = \begin{vmatrix} a_1 & a_2 & a_3 \\ 5 & 4 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$$

Since  $a_n = \frac{20}{n}, d = \frac{1}{20}$

Hence,  $D = \begin{vmatrix} 20 & \frac{20}{2} & \frac{20}{3} \\ 20 & 20 & 20 \\ 4 & 5 & 6 \end{vmatrix} = \frac{(20)^3}{4 \times 7} \begin{vmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ 1 & 4 & 2 \\ 1 & \frac{7}{5} & \frac{7}{3} \end{vmatrix}$

$R_1 \rightarrow R_1 - R_2$  and  $R_2 \rightarrow R_2 - R_3$

$$= \frac{(20)^3}{4 \times 7} \begin{vmatrix} 0 & -\frac{3}{10} & -\frac{1}{3} \\ 1 & -\frac{3}{40} & -\frac{1}{9} \\ 1 & \frac{7}{8} & \frac{7}{9} \end{vmatrix} = \frac{50}{21} \Rightarrow [D] = 2 \text{ (Sol)}$$

$$8. D = \begin{vmatrix} \sin \theta \cos \phi & \sin \theta \sin \phi & \cos \theta \\ \cos \theta \cos \phi & \cos \theta \sin \phi & -\sin \theta \\ -\sin \theta \sin \phi & \sin \theta \cos \phi & 0 \end{vmatrix}$$

$$\Delta = \sin^2 \theta \cos \theta \begin{vmatrix} \cos \phi & \sin \phi & \cos \theta \\ \cos \phi & \sin \phi & -\tan \theta \\ -\sin \phi & \cos \phi & 0 \end{vmatrix}$$

Applying  $R_1 \rightarrow R_1 - R_2$

$$\Delta = \sin^2 \theta \cos \theta \begin{vmatrix} 0 & 0 & \cot \theta + \tan \theta \\ \cos \phi & \sin \phi & -\tan \theta \\ -\sin \phi & \cos \phi & 0 \end{vmatrix}$$

$\Delta = \sin \theta$

10. Applying  $R_3 \rightarrow R_3 - 3R_1 - 2R_2$  we get  $\Delta = 0$   
 $\Rightarrow$  infinite solution.

11. Directly open by  $R_1$  to get a form of  $\sin(A - B)$  etc

$$12. \begin{vmatrix} 3u^2 & 2u^3 & 1 \\ 3v^2 & 2v^3 & 1 \\ 3w^2 & 2w^3 & 1 \end{vmatrix} = 0$$

$R_1 \rightarrow R_1 - R_2$  and  $R_2 \rightarrow R_2 - R_3$

$$\Rightarrow \begin{vmatrix} u^2 - v^2 & u^3 - v^3 & 0 \\ v^2 - w^2 & v^3 - w^3 & 0 \\ w^2 & w^3 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} u + v & u^2 + v^2 + uv & 0 \\ v + w & v^2 + w^2 + vw & 0 \\ w^2 & w^3 & 1 \end{vmatrix} = 0$$

$R_1 \rightarrow R_1 - R_2$

$$\Rightarrow \begin{vmatrix} u - w & (u^2 - w^2) + v(u - w) & 0 \\ v + w & v^2 + w^2 + vw & 0 \\ w^2 & w^3 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & u + w + v & 0 \\ v + w & v^2 + w^2 + vw & 0 \\ w^2 & w^3 & 1 \end{vmatrix} = 0$$

$\Rightarrow (v^2 + w^2 + vw) - (v + w)[(v + w) + u] = 0$

$\Rightarrow v^2 + w^2 + vw = (v + w)^2 + u(v + w)$

$\Rightarrow uv + vw + wu = 0$

$$13. \Delta = \begin{vmatrix} a^2(1+x) & ab & ac \\ ab & b^2(1+x) & bc \\ ac & bc & c^2(1+x) \end{vmatrix}$$

$$= a^2 b^2 c^2 \begin{vmatrix} (1+x) & 1 & 1 \\ 1 & (1+x) & 1 \\ 1 & 1 & (1+x) \end{vmatrix}$$

Applying  $C_1 \rightarrow C_2 + C_3$

$$a^2 b^2 c^2 (3+x) \begin{vmatrix} 1 & 1 & 1 \\ 1 & (1+x) & 1 \\ 1 & 1 & (1+x) \end{vmatrix}$$

Applying  $R_1 \rightarrow R_1 - R_2, R_2 \rightarrow R_2 - R_3$   
 $a^2 b^2 c^2 (3+x) x^2$

Which is divisible by  $x^2$

$$16. f(x) = \begin{vmatrix} 1 + \sin^2 x & \cos^2 x & 4 \sin 2x \\ \sin^2 x & 1 + \cos^2 x & 4 \sin 2x \\ \sin^2 x & \cos^2 x & 1 + 4 \sin 2x \end{vmatrix}$$

Applying  $C_1 \rightarrow C_1 + C_2 + C_3$

$$= (2 + 4 \sin 2x) \begin{vmatrix} 1 & \cos^2 x & 4 \sin 2x \\ 1 & 1 + \cos^2 x & 4 \sin 2x \\ 1 & \cos^2 x & 1 + 4 \sin 2x \end{vmatrix} = (2 + 4 \sin 2x)$$

$$\begin{vmatrix} 1 & \cos^2 x & 4 \sin 2x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$f(x) = (2 + 4 \sin 2x)$$

$$f(x)_{\max} = 6$$

$$18. a = a_0 r_1^{p-1} \Rightarrow \log a = (p-1) \log r_1 + \log a_0$$

$$b = a_0 r_1^{q-1} \Rightarrow \log b = (q-1) \log r_1 + \log a_0$$

$$c = a_0 r_1^{r-1} \Rightarrow \log c = (r-1) \log r_1 + \log a_0$$

$$\begin{vmatrix} \log a_0 + (p-1) \log r_1 & p & 1 \\ \log a_0 + (q-1) \log r_1 & q & 1 \\ \log a_0 + (r-1) \log r_1 & r & 1 \end{vmatrix}$$

$$= \begin{vmatrix} \log a_0 & p & 1 \\ \log a_0 & q & 1 \\ \log a_0 & r & 1 \end{vmatrix} + \log r_1 \begin{vmatrix} p-1 & p & 1 \\ q-1 & q & 1 \\ r-1 & r & 1 \end{vmatrix} = 0$$

21. A is non singular  $\det A \neq 0$

Given  $AB - BA = A$  hence  $AB = A + BA = A(I + B)$

$$\det. A \cdot \det. B = \det. A \cdot \det. (I + B)$$

$$\det. B = \det. (I + B) \quad \dots(1)$$

(as A is non singular)

$$\text{again } AB - A = BA$$

$$A(B - I) = BA$$

$$(\det. A) \cdot \det.(B - I) = \det. B \cdot \det. A$$

$$\Rightarrow \det. (B - I) = \det. (B) \quad \dots(2)$$

from (1) and (2)

$$\det. (B - I) = \det. (B + I)$$

22. Consider the det. B, using  $R_1 \rightarrow R_1 + R_2 + R_3$

$$B = 2 \begin{vmatrix} a + p + x & b + q + y & c + r + z \\ a + x & b + y & c + z \\ a + p & b + q & c + r \end{vmatrix}$$

using  $R_2 \rightarrow R_2 - R_1$  and  $R_3 \rightarrow R_3 - R_1$

$$= 2 \begin{vmatrix} a + p + x & b + q + y & c + r + z \\ -p & -q & -r \\ -x & -y & -z \end{vmatrix}$$

using  $R_1 \rightarrow R_1 + R_2 + R_3$

$$B = 2 \det. A = 2 \cdot 6 = 12$$

$$23. \text{ Using: } C_3 \rightarrow C_3 - (C_1 + C_2), D_1 = \begin{vmatrix} a & b & a + b \\ c & d & c + d \\ a & b & a - b \end{vmatrix}$$

$$\text{and } D_2 = \begin{vmatrix} a & c & a + c \\ b & d & b + d \\ a & c & a + b + c \end{vmatrix}$$

$$\therefore \frac{D_1}{D_2} = \frac{-2b(ad - bc)}{b(ad - bc)} = -2$$

$$27. \begin{vmatrix} (1+x)^2 & (1-x)^2 & -(2+x^2) \\ 2x+1 & 3x & 1-5x \\ x+1 & 2x & 2-3x \end{vmatrix} + \begin{vmatrix} (1+x)^2 & (1-x)^2 & 1-2x \\ 2x+1 & 3x & 3x-2 \\ x+1 & 2x & 2x-3 \end{vmatrix}$$

Since two columns are same in above determinants therefore we can add them along  $C_3$ .

$$\Rightarrow \begin{vmatrix} (1+x)^2 & (1-x)^2 & -(x+1)^2 \\ 2x+1 & 3x & -(1+2x) \\ x+1 & 2x & -(1+x) \end{vmatrix} = 0$$

$$\Rightarrow 0 = 0 \Rightarrow \text{infinite solution}$$

$$30. U_n = \begin{vmatrix} n & 1 & 5 \\ n^2 & 2N+1 & 2N+1 \\ n^3 & 3N^2 & 3N+1 \end{vmatrix}$$

$$\Rightarrow \sum_{n=1}^N U_n = \begin{vmatrix} \frac{N(N+1)}{2} & 1 & 5 \\ \frac{N(N+1)(2N+1)}{6} & (2N+1) & (2N+1) \\ \left[\frac{N(N+1)}{2}\right]^2 & 3N^2 & (3N+1) \end{vmatrix}$$

$$= \frac{N(N+1)(2N+1)}{2} \begin{vmatrix} 1 & 1 & 5 \\ \frac{1}{3} & 1 & 1 \\ \frac{N(N+1)}{2} & 3N^2 & 3N+1 \end{vmatrix}$$

Applying  $R_1 \rightarrow R_1 - R_2$

$$\frac{N(N+1)(2N+1)}{2} \begin{vmatrix} \frac{2}{3} & 0 & 4 \\ \frac{1}{3} & 1 & 1 \\ \frac{N(N+1)}{2} & 3N^2 & 3N+1 \end{vmatrix} = 2 \sum_{n=1}^N n^2$$

$$31. \Delta = \begin{vmatrix} 1 & 2 & 3 \\ 1 & p & 2 \\ 1 & 4 & \mu \end{vmatrix} \quad \Delta_x = \begin{vmatrix} 4 & 2 & 3 \\ 3 & p & 2 \\ 3 & 4 & \mu \end{vmatrix}$$

$$\Delta_y = \begin{vmatrix} 1 & 4 & 3 \\ 1 & 3 & 2 \\ 1 & 3 & \mu \end{vmatrix} \quad \Delta_z = \begin{vmatrix} 1 & 2 & 4 \\ 1 & p & 3 \\ 1 & 4 & 3 \end{vmatrix}$$

For infinite no. of solution  $\Delta = \Delta_x = \Delta_y = \Delta_z = 0$

$$\Rightarrow \mu = 2, p = 4$$

32.  $R_3 \rightarrow R_1 + R_3$

$$33. D = \frac{1}{abc} \begin{vmatrix} ab^2c^2 & abc & a(b+c) \\ bc^2a^2 & bca & b(c+a) \\ ca^2b^2 & cab & c(a+b) \end{vmatrix}$$

$(R_1 \rightarrow aR_1, R_2 \rightarrow bR_2, R_3 \rightarrow cR_3)$

$$= abc \begin{vmatrix} bc & 1 & ab+ac \\ ca & 1 & bc+ab \\ ab & 1 & ca+cb \end{vmatrix}$$

$$= abc(ab+bc+ca) \begin{vmatrix} bc & 1 & 1 \\ ca & 1 & 1 \\ ab & 1 & 1 \end{vmatrix} \quad (C_3 \rightarrow C_3 + C_1)$$

$$= 0$$

35.  $\Delta = 0$

$$36. \text{ for non trivial solution } \begin{vmatrix} \sin \theta & -\cos \theta & \lambda + 1 \\ \cos \theta & \sin \theta & -\lambda \\ \lambda & \lambda + 1 & \cos \theta \end{vmatrix} = 0; \text{ this}$$

$$\text{gives } 2 \cos \theta (\lambda^2 + \lambda + 1) = 0$$

37.  $D = \cos \theta - \cos^2 \theta + 6 \neq 0$  since  $D \neq 0$

$$\Rightarrow \text{only trivial solution is possible} \Rightarrow (A)$$

38.  $|A| = 2; |B| = 3; |C| = 5$

$$\det(A^2BC^{-1}) = |A^2BC^{-1}| = \frac{|A|^2|B|}{|C|} = \frac{4 \cdot 3}{5} = \frac{12}{5}$$

39. Multiply  $R_1$  by  $a$ ,  $R_2$  by  $b$  &  $R_3$  by  $c$  & divide the determinant by  $abc$ . Now take  $a, b$  &  $c$  common from  $c_1, c_2$  &  $c_3$ . Now use  $C_1 \rightarrow C_1 + C_2 + C_3$  to get

$$(a^2 + b^2 + c^2 + 1) \begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2 + 1 & b^2 \\ c^2 & c^2 & c^2 + 1 \end{vmatrix} = 1.$$

Now use  $c_1 \rightarrow c_1 - c_2$  &  $c_2 \rightarrow c_2 - c_3$

we get  $1 + a^2 + b^2 + c^2 = 1$

$$\Rightarrow a = b = c = 0 \Rightarrow (D)$$

$$40. D = \begin{vmatrix} \sin \theta & 0 & 2 \\ \cos \theta & \sin \theta & 0 \\ 0 & \cos \theta & 2 \end{vmatrix}$$

$$\sin \theta (2 \sin \theta) + 2 \cos^2 \theta = 2$$

$$41. D = \begin{vmatrix} 1 & -2 & 1 \\ 2 & 1 & -2 \\ 1 & 3 & -3 \end{vmatrix} \text{ which vanishes ;}$$

hencefor atleast one solution  $D_1 = D_2 = D_3 = 0$

$$\therefore D_1 = \begin{vmatrix} a & -2 & 1 \\ b & 1 & -2 \\ c & 3 & -3 \end{vmatrix} = 0 \Rightarrow a - b + c = 0$$

**Note :** Same condition is obtained by putting

$$D_2 = 0 \text{ or } D_3 = 0$$

42. 1<sup>st</sup> two columns of 1<sup>st</sup> determinant are same as 1<sup>st</sup> two rows of 2<sup>nd</sup>. Hence transpose the 2<sup>nd</sup>. Add the two determinants and use  $C_1 \rightarrow C_1 + C_3 \Rightarrow D = 0$

43.  $x17 = k1 = 100x + 10 + 7$

$$3y6 = km = 300 + 10y + 6$$

$$12z = kn = 100 + 20 + z$$

use  $R_2 \rightarrow R_2 + 100R_1 + 10R_3$  to get the result

EXERCISE - 2

Part # I : Multiple Choice

5. Get Result  $R_1 \rightarrow 2R_1 - R_2$  and  $R_2 \rightarrow R_2 - R_3$

$$6. \Delta = \begin{vmatrix} -x & a & b \\ b & -x & a \\ a & b & -x \end{vmatrix} = (a+b-x) \begin{vmatrix} 1 & a & b \\ 1 & -x & a \\ 1 & b & -x \end{vmatrix}$$

Applying  $R_2 \rightarrow R_2 - R_1$ ,  $R_3 \rightarrow R_3 - R_1$

$$= (a+b-x) \begin{vmatrix} 1 & a & b \\ 0 & -(x+a) & a-b \\ 0 & b-a & -(x+b) \end{vmatrix} = (a+b-x) \{ (x+a)$$

$$(x+b) + (a-b)^2 \}$$

If  $a = b$  then  $x = a, -b, (a+b)$

$$8. \Delta = \begin{vmatrix} a & a^2 & 0 \\ 1 & 2a+b & (a+b)^2 \\ 0 & 1 & 2a+2b \end{vmatrix} = a \begin{vmatrix} 1 & a & 0 \\ 1 & (2a+b) & (a+b)^2 \\ 0 & 1 & 2a+3b \end{vmatrix}$$

$R_2 \rightarrow R_2 - R_1$

$$= a(a+b) \begin{vmatrix} 1 & a & 0 \\ 0 & ab & (a+b)^2 \\ 0 & 1 & 2a+3b \end{vmatrix}$$

$$= a(a+b) \begin{vmatrix} 1 & a & 0 \\ 0 & 1 & (a+b) \\ 0 & 1 & 2a+3b \end{vmatrix}$$

$$= a(a+b) \begin{vmatrix} 1 & a & 0 \\ 0 & 1 & (a+b) \\ 0 & 1 & 2a+3b \end{vmatrix} = a(a+b)(2a+3b-a-b)$$

$$= a(a+b)(a+2b)$$

$$9. \begin{vmatrix} 1 + \sin^2 A & \cos^2 A & 2\sin 4\theta \\ \sin^2 A & 1 + \cos^2 A & 2\sin 4\theta \\ \sin^2 A & \cos^2 A & 1 + 2\sin 4\theta \end{vmatrix} = 0$$

$C_1 \rightarrow C_1 + C_2$

$$\Rightarrow \begin{vmatrix} 2 & \cos^2 A & 2\sin 4\theta \\ 2 & 1 + \cos^2 A & 2\sin 4\theta \\ 1 & \cos^2 A & 1 + 2\sin 4\theta \end{vmatrix} = 0$$

$R_1 \rightarrow R_1 - R_2$

$$\Rightarrow \begin{vmatrix} 0 & -1 & 0 \\ 2 & 1 + \cos^2 A & 2\sin 4\theta \\ 1 & \cos^2 A & 1 + 2\sin 4\theta \end{vmatrix} = 0$$

$$\Rightarrow 2(1+2\sin 4\theta) - 2\sin 4\theta = 0$$

$$\Rightarrow 1 + \sin 4\theta = 0$$

$$\sin 4\theta = -1$$

$$4\theta = -\frac{\pi}{2} \text{ or } 4\theta = \frac{3\pi}{2}$$

$$\theta = -\frac{\pi}{8} \text{ or } \theta = \frac{3\pi}{8} \text{ \& } A \in \mathbb{R}$$

$$13. \text{ Let } A = \begin{bmatrix} a_1 & b_1 \\ c_1 & -a_1 \end{bmatrix} \text{ and } B = \begin{bmatrix} a_2 & b_2 \\ c_2 & -a_2 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} a_1 a_2 + b_2 c_2 & a_1 b_2 + b_1 a_2 \\ c_1 a_2 - a_1 c_2 & c_1 b_2 + a_1 a_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow a_1 a_2 + b_1 c_2 = a_1 b_2 - b_1 a_2 = c_1 a_2 - a_1 c_2 = c_1 b_2 + a_1 a_2 = 0$$

$$BA = \begin{bmatrix} a_2 a_1 + b_2 c_1 & a_2 b_1 - b_2 a_1 \\ c_2 a_1 - a_2 c_1 & c_2 b_1 + a_2 a_1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$15. x + y = 3 \quad \dots\dots(i)$$

$$(1+K)x + (K+2)y = 8 \quad \dots\dots(ii)$$

$$x - (1+K)y = -K - 2 \quad \dots\dots(iii)$$

If system is consistent then  $\Delta = 0$

on solving we get

$$K = 1, \frac{-5}{3}$$

$$19. \frac{a^2}{b} = \frac{b}{b^2} = \frac{a^2 - b}{2 + 4b}$$

$$\Rightarrow a^2 b^2 = b^2 \Rightarrow b^2(a^2 - 1) = 0$$

$$\Rightarrow a = 1 \text{ or } -1 \text{ \& } b = 0,$$

$$\text{if } a = 1 \text{ then } 2 + 4b = b - b^2 \Rightarrow b^2 + 3b + 2 = 0$$

$$\Rightarrow b = -2 \text{ or } -1 ;$$

$$\text{if } a = -1 \text{ then } b = -2 \text{ or } -1$$

$$20. \text{ Start: } p = a ; q = a + d ; r = a + 2d ; s = a + 3d$$

$$\Rightarrow f(x) = -2d^2$$

Also use  $R_1 \rightarrow R_1 - R_2$  and  $R_2 \rightarrow R_2 - R_3$

Part # II : Assertion & Reason

2. Statement-I :  $B_1 = c_2a_3 - a_2c_3$

$$B_2 = a_1c_3 - c_1a_3, \quad B_3 = a_2c_1 - a_1c_2$$

$$a_1B_1 = a_1a_3c_2 - a_1a_2c_3$$

$$a_2B_2 = a_1a_2c_3 - a_2a_3c_1$$

$$a_3B_3 = a_1a_2c_3 - a_1a_3c_2$$

Statement-II is obviously true.

EXERCISE - 3

Part # II : Comprehension

Comprehension # 1

$$\begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = (\alpha - 1)^2 (\alpha + 2)$$

1.  $\Delta \neq 0 \Rightarrow$  unique solution

$$2. \quad \alpha = -2 \Rightarrow \Delta = 0, \quad \Delta_1 = \begin{vmatrix} m & -2 & 1 \\ n & 1 & -2 \\ p & 1 & 1 \end{vmatrix}$$

$$\Delta_1 = 3(m + n + p) \neq 0$$

$$\Rightarrow \Delta_1 \neq 0$$

Hence no solution

3.  $x + y + z = m$

$$x + y + z = p$$

$$\therefore m \neq p \Rightarrow \text{no solution}$$

Comprehension # 3

Hint :

$$\Delta = (x - y)(y - z)(z - x)[xyz(xy + yz + zx) - (x + y + z)]$$

**EXERCISE - 4**  
**Subjective Type**

3.  $a^2 + b^2 + c^2 = 1$

$$\Delta = \begin{vmatrix} a^2 + (b^2 + c^2)\cos\phi & ab(1 - \cos\phi) & ac(1 - \cos\phi) \\ ba(1 - \cos\phi) & b^2 + (c^2 + a^2)\cos\phi & bc(1 - \cos\phi) \\ ca(1 - \cos\phi) & cb(1 - \cos\phi) & c^2 + (a^2 + b^2)\cos\phi \end{vmatrix}$$

$$\Delta = \frac{1}{abc}$$

$$\begin{vmatrix} a^3 + a(b^2 + c^2)\cos\phi & b^2a(1 - \cos\phi) & ac^2(1 - \cos\phi) \\ ba^2(1 - \cos\phi) & b^3 + b(c^2 + a^2)\cos\phi & bc^2(1 - \cos\phi) \\ ca^2(1 - \cos\phi) & cb(1 - \cos\phi) & c^3 + c(a^2 + b^2)\cos\phi \end{vmatrix}$$

Applying  $C_1 \rightarrow C_1 + C_2 + C_3$

$$\frac{abc}{abc} \begin{vmatrix} a^2 + (b^2 + c^2)\cos\phi & b^2(1 - \cos\phi) & c^2(1 - \cos\phi) \\ a^2(1 - \cos\phi) & b^2 + (c^2 + a^2)\cos\phi & c^2(1 - \cos\phi) \\ a^2(1 - \cos\phi) & b^2(1 - \cos\phi) & c^2 + (a^2 + b^2)\cos\phi \end{vmatrix}$$

$$= (a^2 + b^2 + c^2)$$

$$\begin{vmatrix} 1 & b^2(1 - \cos\phi) & c^2(1 - \cos\phi) \\ 1 & b^2 + (c^2 + a^2)\cos\phi & c^2(1 - \cos\phi) \\ 1 & b^2(1 - \cos\phi) & c^2 + (a^2 + b^2)\cos\phi \end{vmatrix}$$

Applying  $R_2 \rightarrow R_2 - R_1$  and  $R_3 \rightarrow R_3 - R_1$   
( $a^2 + b^2 + c^2$ )

$$\begin{vmatrix} 1 & b^2(1 - \cos\phi) & c^2(1 - \cos\phi) \\ 0 & (a^2 + b^2 + c^2)\cos\phi & 0 \\ 0 & 0 & (a^2 + b^2 + c^2)\cos\phi \end{vmatrix}$$

$$= (a^2 + b^2 + c^2)\cos^2\phi = \cos^2\phi$$

This is independent from a, b, c.

6. Let  $\Delta = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$

Operate :  $C_1 \rightarrow C_1 + C_2 + C_3$

$$\Rightarrow \Delta = \begin{vmatrix} 2(a+b+c) & c+a & a+b \\ 2(a+b+c) & a+b & b+c \\ 2(a+b+c) & b+c & c+a \end{vmatrix} = 2(a+b+c)$$

$$\begin{vmatrix} 1 & c+a & a+b \\ 1 & a+b & b+c \\ 1 & b+c & c+a \end{vmatrix}$$

Operate :  $R_2 \rightarrow R_2 - R_1$  ;  $R_3 \rightarrow R_3 - R_1$

$$\Rightarrow D = 2(a+b+c) \begin{vmatrix} 1 & c+a & a+b \\ 0 & b-c & c-a \\ 0 & b-a & c-b \end{vmatrix}$$

$$= 2(a+b+c) \cdot 1 \cdot \begin{vmatrix} b-c & c-a \\ b-a & c-b \end{vmatrix} \text{ open w.r.t. } R_1$$

$$= 2(a+b+c) [(b-c)(c-b) - (c-a)(b-a)]$$

$$= 2(a+b+c) [bc - b^2 - c^2 + cb - (cb - ac - ab + a^2)]$$

$$= 2(a+b+c) (ab + bc + ca - a^2 - b^2 - c^2)$$

10.  $\sum_{r=1}^n D_r = \begin{vmatrix} \sum_{r=1}^n 2^{r-1} & \sum_{r=1}^n 2(3^{r-1}) & \sum_{r=1}^n 4(5^{r-1}) \\ x & y & z \\ 2^n - 1 & 3^n - 1 & 5^n - 1 \end{vmatrix}$

$$= \begin{vmatrix} 1+2+\dots+2^{n-1} & 2\{1+3+\dots+3^{n-1}\} & 4\{1+5+\dots+5^{n-1}\} \\ x & y & z \\ 2^n - 1 & 3^n - 1 & 5^n - 1 \end{vmatrix}$$

$$= \begin{vmatrix} \frac{2^n - 1}{2 - 1} & \frac{2(3^n - 1)}{3 - 1} & \frac{4(5^n - 1)}{5 - 1} \\ x & y & z \\ 2^n - 1 & 3^n - 1 & 5^n - 1 \end{vmatrix} = 0$$

11. Let  $\Delta = \begin{vmatrix} 0 & ab^2 & ac^2 \\ a^2b & 0 & bc^2 \\ a^2c & cb^2 & 0 \end{vmatrix}$

Take :  $a^2, b^2$  and  $c^2$  respectively common from  $C_1, C_2$  and  $C_3$ .

$$\Delta = a^2 b^2 c^2 \begin{vmatrix} 0 & a & a \\ b & 0 & b \\ c & c & 0 \end{vmatrix}$$

operate :  $C_2 \rightarrow C_2 - C_3$

$$\Delta = a^2 b^2 c^2 \begin{vmatrix} 0 & 0 & a \\ b & -b & b \\ c & c & 0 \end{vmatrix}$$

Expand by  $R_1$

$$\Delta = a^2 b^2 c^2 \cdot a = \begin{vmatrix} b & -b \\ c & c \end{vmatrix} = 2a^3 b^3 c^3.$$

12. Let  $\Delta = \begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix}$

Operate :  $R_2 \rightarrow R_2 - 2R_1$ ;  $R_3 \rightarrow R_3 - 3R_1$

$$\Delta = \begin{vmatrix} x+y & x & x \\ 3x+2y & 2x & 0 \\ 7x+5y & 5x & 0 \end{vmatrix}$$

Expand by  $C_3$

$$\begin{aligned} \Delta &= x \begin{vmatrix} 3x+2y & 2x \\ 7x+5y & 5x \end{vmatrix} \\ &= x[5x(3x+2y) - 2x(7x+5y)] \\ &= x[15x^2 + 10xy - (14x^2 + 10xy)] = x^3 \end{aligned}$$

13.  $\therefore$  a, b, c are in A.P.

$$\therefore b - a = c - b \quad \dots(i)$$

Now L.H.S. =  $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$

Operate :  $R_1 \rightarrow R_2 - R_1$ ;  $R_3 \rightarrow R_3 - R_2$

$$= \begin{vmatrix} x+1 & x+2 & x+a \\ 1 & 1 & b-a \\ 1 & 1 & c-b \end{vmatrix}$$

$$= \begin{vmatrix} x+1 & x+2 & x+a \\ 1 & 1 & b-a \\ 1 & 1 & b-a \end{vmatrix} \quad [\text{Using (1)}]$$

$$= 0 \quad [\because R_2 \text{ and } R_3 \text{ are identical}]$$

$$= \text{R.H.S.}$$

14. (a)  $D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 1 & 1 & -2 \end{vmatrix} = 3$

$$\Rightarrow D_1 = \begin{vmatrix} 6 & 1 & 1 \\ 1 & 1 & -1 \\ -3 & 1 & -2 \end{vmatrix} = 3$$

$$D_2 = \begin{vmatrix} 1 & 6 & 1 \\ 2 & 1 & -1 \\ 1 & -3 & -2 \end{vmatrix} = 6$$

Similarly  $D_3 = 9$

so consistent having  $x = 1, y = 2, z = 3$

(d)  $D = \begin{vmatrix} 7 & -7 & 5 \\ 3 & 1 & 5 \\ 2 & 3 & 5 \end{vmatrix} = 0$

$$D_1 = \begin{vmatrix} 3 & -7 & 1 \\ 7 & 1 & 1 \\ 5 & 3 & 1 \end{vmatrix} = 24$$

Inconsistent system

15.  $\frac{\alpha_1}{\alpha_2} = \frac{\beta_1}{\beta_2} \Rightarrow \frac{(\alpha_1 + \alpha_2)^2}{(\alpha_1 + \alpha_2)^2 - 4\alpha_1\alpha_2} = \frac{(\beta_1 + \beta_2)^2}{(\beta_1 + \beta_2)^2 - 4\beta_1\beta_2}$

$$\Rightarrow \frac{b^2/a^2}{b^2/a^2 - 4c/a} = \frac{q^2/p^2}{q^2/p^2 - 4r/p}$$

$$\Rightarrow \frac{b^2}{q^2} = \frac{b^2 - 4ac}{q^2 - 4rp} \Rightarrow \frac{b^2}{q^2} = \frac{4ac}{4rp}$$

EXERCISE - 5

Part # I : AIEEE/JEE-MAIN

3. 
$$\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} + abc \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} (1 + abc) = 0$$

$$(a - b)(b - c)(c - a)(1 + abc) = 0$$

but  $a \neq b \neq c$  so  $abc = -1$

4. If  $a_1, a_2, \dots, a_n, \dots$  are in G.P.  
then  $\log a_1, \log a_2, \dots, \log a_n, \dots$  are in A.P.  
 $A, A+D, \dots$

Let common difference of A.P. is D

so 
$$\begin{vmatrix} \log a_n & \log a_{n+2} & \log a_{n+4} \\ \log a_{n+6} & \log a_{n+8} & \log a_{n+10} \\ \log a_{n+12} & \log a_{n+14} & \log a_{n+16} \end{vmatrix}$$

$$\begin{vmatrix} A & A+2D & A+4D \\ A+6D & A+8D & A+10D \\ A+12D & A+14D & A+16D \end{vmatrix}$$

$$C_2 \rightarrow 2C_2 - (C_1 + C_3)$$

$$\begin{vmatrix} A & 0 & A+4D \\ A+6D & 0 & A+10D \\ A+12D & 0 & A+16D \end{vmatrix} = 0$$

5.  $a^2 + b^2 + c^2 = -2$

applying  $C_1 \rightarrow C_1 + C_2 + C_3$

$$= \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 1 & 1+b^2x & (1+c^2)x \\ 1 & (1+b^2)x & 1+c^2x \end{vmatrix}$$

$$R_2 \rightarrow R_2 - R_1$$

$$R_3 \rightarrow R_3 - R_1$$

$$\Rightarrow \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 0 & 1-x & 0 \\ 0 & 0 & 1-x \end{vmatrix}$$

$$= (1-x)^2 \text{ so degree is } \rightarrow 2$$

6. For no solution

$\Delta = 0$  and  $\Delta_x$  or  $\Delta_y$  or  $\Delta_z$  at least one is not zero.

$$\Delta = \begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = 0$$

at  $\alpha = 1$  their are 3 row are identical so factor of determinant  $(\alpha - 1)^2$

and other factor will be find out by  $R_1 \rightarrow R_1 + R_3 + R_2$

$$\lambda(\alpha + 2)(\alpha - 1)^2 = 0$$

$$\alpha = -2, 1$$

but at  $\alpha = 1$

all equation are same so at  $\alpha = 1$  system of equation infinite solution and

at  $\alpha = -2$

$$\Delta x = \begin{vmatrix} -3 & 1 & 1 \\ -3 & -2 & 1 \\ -3 & 1 & -2 \end{vmatrix}$$

$$-3(4-1) - 1(6+3) + 1(-3+6)$$

$$-9 - 9 + 3 = -15 \neq 0$$

so at  $\alpha = -2$  system have no solution.

9.  $x - cy - bz = 0$

$$cx - y + az = 0 \quad x \neq 0 ; y \neq 0, z \neq 0$$

$$bx + ay - z = 0$$

these system is homogeneous

$$\text{so } \Delta_x = \Delta_y = \Delta_z = 0$$

and at  $\Delta = 0 \rightarrow$  system have non zero solution.

$$\Delta = \begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$$

$$1 - a^2 + c(-c - ab) - b(ac + b) = 0$$

$$1 - a^2 - b^2 - c^2 - abc - abc = 0$$

$$a^2 + b^2 + c^2 + 2abc = 1$$

12.  $\Delta = 0$  (For Non zero solution)

$$\begin{vmatrix} 4 & K & 2 \\ K & 4 & 1 \\ 2 & 2 & 1 \end{vmatrix} = 0$$

$$8 - K(K - 2) + 2(2K - 8) = 0$$

$$8 - K^2 + 2K + 4K - 16 = 0$$

$$-K^2 + 6K - 8 = 0$$

$$K^2 - 6K + 8 = 0$$

$$(K - 4)(K - 2) = 0$$

$$K = 2, 4 \quad \text{Two solution}$$

13. For Trivial sol<sup>n</sup>  $\Delta \neq 0$

$$\begin{vmatrix} 1 & -K & 1 \\ K & 3 & -K \\ 3 & 1 & -1 \end{vmatrix} \neq 0$$

$$(-3 + K) + K(-K + 3K) + (K - 9) \neq 0$$

$$2K^2 + 2K - 12 \neq 0$$

$$K^2 + K - 6 \neq 0$$

$$(K + 3)(K - 2) \neq 0$$

$$K \neq -3$$

$$K \neq 2$$

$$\text{So Ans. } R = \{2, -3\}$$

Part # II : IIT-JEE ADVANCED

6. (a) Method : 1

$$P \equiv (-\sin(\beta - \alpha), -\cos\beta) \equiv (x_1, y_1),$$

$$Q \equiv (\cos(\beta - \alpha), \sin\beta) \equiv (x_2, y_2)$$

$$\text{and } R \equiv (x_2\cos\theta + x_1\sin\theta, y_2\cos\theta + y_1\sin\theta)$$

We see that

$$T \equiv \left( \frac{x_2 \cos\theta + x_1 \sin\theta}{\cos\theta + \sin\theta}, \frac{y_2 \cos\theta + y_1 \sin\theta}{\cos\theta + \sin\theta} \right) \text{ and}$$

$$P, Q, T \text{ are collinear} \Rightarrow P, Q, R \text{ are non-collinear}$$

Method : 2

$$\begin{vmatrix} -\sin(\beta - \alpha) & -\cos\beta & 1 \\ \cos(\beta - \alpha) & \sin\beta & 1 \\ \cos(\beta - \alpha + \theta) & \sin(\beta - \theta) & 1 \end{vmatrix}$$

$$\text{Applying } R_3 \rightarrow R_3 + R_1\sin\theta - R_2\cos\theta$$

$$\begin{vmatrix} -\sin(\beta - \alpha) & -\cos\beta & 1 \\ \cos(\beta - \alpha) & \sin\beta & 1 \\ 0 & 0 & 1 + \sin\theta - \cos\theta \end{vmatrix}$$

$$= (1 + \sin\theta - \cos\theta) [-\sin\beta \sin(\beta - \alpha) + \cos\beta \cos(\beta - \alpha)]$$

$$= (1 + \sin\theta - \cos\theta) \cos(2\beta - \alpha) \neq 0$$

Hence P, Q, R are non collinear.

(b)  $x - 2y + 3z = -1, -x + y - 2z = k$

$$\& x - 3y + 4z = 1$$

$$\Delta = \begin{vmatrix} 1 & -2 & 3 \\ -1 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix} = 0 \quad \& \quad \begin{vmatrix} 1 & 3 & -1 \\ -1 & -2 & k \\ 1 & 4 & 1 \end{vmatrix} = 3 - k$$

Hence if  $k = 3$  then system will have infinite solutions and  $k \neq 3$  then system will have no solution. so S(I) & S(II) both are true & (II) is correct explanation for (I).

7.  $(y + z) \cos 3\theta = (xyz) \sin 3\theta \quad \dots(i)$

$$x \sin 3\theta = \frac{2 \cos 3\theta}{y} + \frac{2 \sin 3\theta}{z} \quad \dots(ii)$$

$$(xyz)\sin 3\theta = (y + 2z)\cos 3\theta + y\sin 3\theta \quad \dots(iii)$$

$$\text{where } yz \neq 0 \text{ and } 0 < \theta < \pi$$

from (i) & (iii)

$$(y + z) \cos 3\theta = (y + 2z) \cos 3\theta + y \sin 3\theta$$

$$\Rightarrow z \cos 3\theta + y \sin 3\theta = 0 \quad \dots(iv)$$

from eq<sup>n</sup> (ii)

$$2z \cos 3\theta + 2y \sin 3\theta = xyz \sin 3\theta \quad \dots(v)$$

from equation (iv) & (v)

$$\Rightarrow xyz \sin 3\theta = 0$$

$$\Rightarrow x \sin 3\theta = 0 \text{ as } yz \neq 0$$

Possible cases are either  $x = 0$  or  $\sin 3\theta = 0$

Case (1) : if  $x = 0$

$$\Rightarrow y + z = 0 \Rightarrow y = -z$$

from eq<sup>n</sup> (iv)  $\cos 3\theta = \sin 3\theta$

$$\Rightarrow 3\theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4} \Rightarrow \theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{9\pi}{12}$$

Case (2) : if  $\sin 3\theta = 0$

$$\Rightarrow \theta = \frac{\pi}{3}, \frac{2\pi}{3}$$

But these values does not satisfy given equations.

Hence, total number of possible values of  $\theta$  are 3.

$$10. \quad x^3 \begin{vmatrix} 1 & 1 & 1+x^3 \\ 2 & 4 & 1+8x^3 \\ 3 & 9 & 1+27x^3 \end{vmatrix} = 10$$

$$x^3 \begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & 1 \\ 3 & 9 & 1 \end{vmatrix} + x^6 \begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & 8 \\ 3 & 9 & 27 \end{vmatrix} = 10$$

$$x^3 \begin{vmatrix} 1 & 0 & 0 \\ 2 & 2 & -1 \\ 3 & 6 & -2 \end{vmatrix} + x^6 \begin{vmatrix} 1 & 0 & 0 \\ 2 & 2 & 6 \\ 3 & 6 & 24 \end{vmatrix} = 10$$

$$6x^3 + x^3 - 5 = 0$$

$$\Rightarrow 6x^6 + 6x^3 - 5x^3 - 5 = 0$$

$$(6x^3 - 5)(x^3 + 1) = 0$$

$$x^3 = \frac{5}{6} \quad \text{or} \quad x^3 = -1. \quad \text{Two real distinct values of } x.$$

$$11. \quad ax + 2y = \lambda$$

$$3x - 2y = \mu$$

$$\Delta = \begin{vmatrix} \alpha & 2 \\ 3 & -2 \end{vmatrix} = -2\alpha - 6$$

$$\Delta = 0 \quad \Rightarrow \quad \alpha = -3$$

$$\Delta_1 = \begin{vmatrix} \lambda & 2 \\ \mu & -2 \end{vmatrix} = -2\lambda - 2\mu = -(\lambda + \mu)$$

$$\Delta_2 = \begin{vmatrix} -3 & \lambda \\ 3 & \mu \end{vmatrix} = -3\mu - 3\lambda = -(\lambda + \mu)$$

MOCK TEST

1. (C)

$$|\text{adj } A^{-1}| = |A^{-1}|^2 = \frac{1}{|A|^2}$$

$$|(\text{adj } A^{-1})^{-1}| = \frac{1}{|\text{adj } A^{-1}|} = |A|^2 = 2^2 = 4$$

$$2. \quad \begin{vmatrix} x^k & x^{k+2} & x^{k+3} \\ y^k & y^{k+2} & y^{k+3} \\ z^k & z^{k+2} & z^{k+3} \end{vmatrix} = x^k y^k z^k \begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix}$$

$$= (x - y)(y - z)(z - x)(xy + zy + zx) x^k y^k z^k \quad \text{at } k = -1$$

$$= (x - y)(y - z)(z - x) \left( \frac{1}{x} + \frac{1}{y} + \frac{1}{z} \right)$$

3. (A)

$$\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix} = pqr(a^3 + b^3 + c^3) - abc(p^3 + q^3 + r^3) = pqr$$

$$(3abc) - abc(3pqr) = 0$$

$$4. \quad \begin{vmatrix} a + p & \ell + x & u + f \\ b + q & m + y & v + g \\ c + r & n + z & w + h \end{vmatrix} = \begin{vmatrix} a + p & \ell + x & u \\ b + q & m + y & v \\ c + r & n + z & w \end{vmatrix}$$

$$+ \begin{vmatrix} a + p & \ell + x & f \\ b + q & m + y & g \\ c + r & n + z & h \end{vmatrix}$$

$$= \begin{vmatrix} a + p & \ell & u \\ b + q & m & v \\ c + r & n & w \end{vmatrix} + \begin{vmatrix} a + p & x & u \\ b + q & y & v \\ c + r & z & w \end{vmatrix}$$

$$+ \begin{vmatrix} a + p & \ell & f \\ b + q & m & g \\ c + r & n & h \end{vmatrix} + \begin{vmatrix} a + p & x & f \\ b + q & y & g \\ c + r & z & h \end{vmatrix}$$

Now each det. may splits in 2 det.

So, total determinants are 8.

5. (D)

2ABC is not defined

∴ there is no solution

$$6. \begin{vmatrix} a & a^3 & a^4 - 1 \\ b & b^3 & b^4 - 1 \\ c & c^3 & c^4 - 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} a & a^3 & a^4 \\ b & b^3 & b^4 \\ c & c^3 & c^4 \end{vmatrix} = \begin{vmatrix} a & a^3 & 1 \\ b & b^3 & 1 \\ c & c^3 & 1 \end{vmatrix}$$

$$abc(a-b)(b-c)(c-a)(ab+bc+ca) \\ = (a-b)(b-c)(c-a)(a+b+c)$$

$$\therefore abc(ab+bc+ca) = (a+b+c)$$

7. (C)

$$A^2 = I \Rightarrow A^2 - I = O$$

$$\Rightarrow (A+I)(A-I) = O$$

$$\therefore \text{either } |A+I| = 0 \text{ or } |A-I| = 0$$

$$\text{If } |A-I| \neq 0, \text{ then } (A+I)(A-I) = O$$

$$\Rightarrow A+I = O \text{ which is not so}$$

$$\therefore |A-I| = 0 \text{ and } A-I \neq O.$$

8.  $f(x) = \log_{10} x, g(x) = e^{i\pi x} = \cos \pi x + i \sin \pi x$

$$f(10) = 1, g(10) = 1$$

$$h(10) = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 0 \\ 3 & 3 & 1 \end{vmatrix} = 0$$

$$9. D = \begin{vmatrix} a & 1+2i & 3-5i \\ 1-2i & b & -7-3i \\ 3+5i & -7+3i & c \end{vmatrix}$$

$$= abc - a(58) - (1+2i)\{c(1-2i) - (-21-35i-9i+15)\} \\ + (3-5i)\{-7+14i+3i+6\} - 6(3+5i)$$

$$= abc - 58a - (1+2i)(c-2ic+6+44i) - (3-5i)(36+56i) + (3-5i)(-1+17i)$$

$$= abc - 58a - c - 2ic + 2ic - 6 - 44i - 4c - 12i + 88 - 3 \\ - 5i - 51i + 85 - 96 - 25b + 15ib - 15ib$$

$$= abc - 58a - 34b - c + 85 \text{ Purely real.}$$

Aliter : observe that  $D = D^T$

and  $\bar{D} = D^T = D$  i.e.  $\bar{D} = D$

i.e. D is real

$\bar{D}$  is a conjugate of D

10. (A)

$$S_1 : D = \begin{vmatrix} \ln x & \ln y & \ln z \\ \ln 2x & \ln 2y & \ln 2z \\ \ln 3x & \ln 3y & \ln 3z \end{vmatrix}$$

$$= \begin{vmatrix} \ln x & \ln y & \ln z \\ \ln 2 + \ln x & \ln 2 + \ln y & \ln 2 + \ln z \\ \ln 3 + \ln x & \ln 3 + \ln y & \ln 3 + \ln z \end{vmatrix}$$

$$= \begin{vmatrix} \ln x & \ln y & \ln z \\ \ln 2 & \ln 2 & \ln 2 \\ \ln 3 & \ln 3 & \ln 3 \end{vmatrix} = 0$$

$$S_2 : 0 = \begin{vmatrix} x & a & b & 1 \\ \lambda & x & b & 1 \\ \lambda & \mu & x & 1 \\ \lambda & \mu & v & 1 \end{vmatrix} = \begin{vmatrix} x & a & b & 1 \\ \lambda-x & x-a & 0 & 0 \\ \lambda-x & \mu-a & x-b & 0 \\ \lambda-x & \mu-a & v-b & 0 \end{vmatrix}$$

$$\begin{cases} R_2 \rightarrow R_2 - R_1 \\ R_3 \rightarrow R_3 - R_1 \\ R_4 \rightarrow R_4 - R_1 \end{cases}$$

$$= (\lambda-x) \begin{vmatrix} 1 & x-a & 0 \\ 1 & \mu-a & x-b \\ 1 & \mu-a & v-b \end{vmatrix} \begin{cases} R_2 \rightarrow R_2 - R_1 \\ R_3 \rightarrow R_3 - R_1 \end{cases}$$

$$= (\lambda-x) \begin{vmatrix} 1 & x-a & 0 \\ 0 & \mu-x & x-b \\ 0 & \mu-x & v-b \end{vmatrix} = (\lambda-x)(\mu-x)(v-x)$$

$$S_3 : \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$= -\frac{a+b+c}{2} ((a-b)^2 + (b-c)^2 + (c-a)^2) < 0$$

$$S_4 : f(x) = \begin{vmatrix} \frac{1}{x} & \ell n x & x^n \\ 1 & -\frac{1}{n} & (-1)^n \\ 1 & a & a^2 \end{vmatrix} = f'(x)$$

$$= \begin{vmatrix} -\frac{1}{x^2} & \frac{1}{x} & nx^{n-1} \\ 1 & -\frac{1}{n} & (-1)^n \\ 1 & a & a^2 \end{vmatrix} f^{(n)}(x)$$

$$= \begin{vmatrix} (-1)^n \frac{n!}{x^{n+1}} & (-1)^{n-1} \frac{(n-1)!}{x^n} & n! \\ 1 & -\frac{1}{n} & (-1)^n \\ 1 & a & a^2 \end{vmatrix}$$

$$f^{(n)}(1) = \begin{vmatrix} (-1)^n n! & (-1)^{n-1} (n-1)! & n! \\ 1 & -\frac{1}{n} & (-1)^n \\ 1 & a & a^2 \end{vmatrix}$$

$$= (-1)^n n! \begin{vmatrix} 1 & -\frac{1}{n} & (-1)^n \\ 1 & -\frac{1}{n} & (-1)^n \\ 1 & a & a^2 \end{vmatrix} = 0$$

11. (A, B, C, D)

Here, (a), (b), (c), (d) are the properties of adjoint.

$$12. f(x) = \begin{vmatrix} 2 \sin x & \sin^2 x & 0 \\ 1 & 2 \sin x & \sin^2 x \\ 0 & 1 & 2 \sin x \end{vmatrix} = 2 \sin x (4 \sin^2 x - \sin^2 x)$$

$$- \sin^2 x (2 \sin x) = 6 \sin^3 x - 2 \sin^3 x$$

$$f(x) = 4 \sin^3 x$$

$$f(\pi) = 12 \sin^2 x \cos x$$

(B)  $f'(\pi/2) = 12 \sin^2(\pi/2) \cos(\pi/2) = 0$

(C)  $f(-x) = -f(x)$  odd function

$$\therefore \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x) dx = 0$$

(D) at  $x = 0, y = 0$

$$\left(\frac{dy}{dx}\right)_{(0,0)} = 0 \text{ tangent at } (0, 0)$$

$$y - 0 = \left(\frac{dy}{dx}\right)_{(0,0)} (x - 0)$$

$$y = 0$$

13. (B, C, D)

$$x + 3y + 2z = 6 \quad \dots\dots(i)$$

$$x + \lambda y + 2z = 7 \quad \dots\dots(ii)$$

$$x + 3y + 2z = \mu \quad \dots\dots(iii)$$

(A) If  $\lambda = 2$ , then  $D = 0$ , therefore unique solution is not possible

(B) If  $\lambda = 4, \mu = 6$

$$x + 3y = 6 - 2z$$

$$x + 4y = 7 - 2z$$

$$\therefore y = 1 \text{ and } x = 3 - 2z$$

substituting in equation (iii)

$$3 - 2z + 3 + 2z = 6 \text{ is satisfied}$$

$\therefore$  infinite solutions

(C)  $\lambda = 5, \mu = 7$

consider equation (ii) and (iii)

$$x + 5y = 7 - 2z$$

$$x + 3y = 7 - 2z$$

$$\therefore y = 0 \quad x = 7 - 2z \text{ are solution}$$

sub. in (i)

$$7 - 2z + 2z = 6 \quad \text{does not satisfy}$$

$\therefore$  no solution

(D) if  $\lambda = 3, \mu = 5$

then equation (i) and (ii) have no solution

$\therefore$  no solution

$$14. f(x) = \begin{vmatrix} 1/x & \log x & x^n \\ 1 & -1/n & (-1)^n \\ 1 & a & a^2 \end{vmatrix}$$

$$f'(x) = \begin{vmatrix} -1/x^2 & 1/x & nx^{n-1} \\ 1 & -1/n & (-1)^n \\ 1 & a & a^2 \end{vmatrix}$$

$$f''(x) = \begin{vmatrix} 2/x^3 & -1/x^2 & n(n-1)x^{n-2} \\ 1 & -1/n & (-1)^n \\ 1 & a & a^2 \end{vmatrix}$$

$$f^n(x) = \begin{vmatrix} (-1)^n \frac{n!}{x^{n+1}} & \frac{(-1)^{n-1}(n-1)!}{x^n} & n! \\ 1 & -1/n & (-1)^n \\ 1 & a & a^2 \end{vmatrix}$$

$$f^n(1) = \begin{vmatrix} (-1)^n n! & (-1)^{n-1}(n-1)! & n! \\ 1 & -1/n & (-1)^n \\ 1 & a & a^2 \end{vmatrix}$$

$$= (-1)^n n! \begin{vmatrix} 1 & -1/n & (-1)^n \\ 1 & -1/n & (-1)^n \\ 1 & a & a^2 \end{vmatrix} = 0$$

and  $y = a(x - f^n(1))$

$$y = ax$$

15. (A, B, C, D)

Obvious ( using properties)

$$16. \Delta = \begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 3 & -4 \end{vmatrix}$$

Applying  $R_2 \rightarrow R_2 - 3R_1$

and  $R_3 \rightarrow R_3 - 2R_1$  then,

$$\Delta = \begin{vmatrix} 1 & \dots & k & \dots & 3 \\ \vdots & & & & \\ 0 & & -2k & & -11 \\ \vdots & & & & \\ 0 & & 3-2k & & -10 \end{vmatrix}$$

$$= 20k + 33 - 22k = 0 \quad \therefore k = 33/2$$

17. (A)

$$A = -A^T \Rightarrow |A| = -|A^T| = -|A|$$

$$\Rightarrow 2|A| = 0$$

$$\Rightarrow |A| = 0$$

18. (D)

$A^{-1}$  exists only for non-singular matrix

$$AB = AC$$

$$\Rightarrow B = C \text{ if } A^{-1} \text{ exists}$$

If  $A^{-1}$  exists

19. (C)

$$\det(AB^{-1}) = \det A \cdot \det B^{-1} = \frac{\det A}{\det B} = \frac{-6}{2} = -3$$

$$20. \Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -(a^3 + b^3 + c^3 - 3abc)$$

$$= -(a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca) = 0$$

( $\because a+b+c=0$ ) (non trivial)

and  $a^2 + b^2 + c^2 - ab - bc - ca$

$$= \frac{1}{2} \{(a-b)^2 + (b-c)^2 + (c-a)^2\} > 0$$

22. (A)  $\rightarrow$  (p, t), (B)  $\rightarrow$  (q), (C)  $\rightarrow$  (r), (D)  $\rightarrow$  (p, t)

$$(A) |A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$|B| = \begin{vmatrix} a_{11} & k^{-1}a_{12} & k^{-2}a_{13} \\ ka_{21} & a_{22} & k^{-1}a_{23} \\ k^2a_{31} & ka_{32} & a_{33} \end{vmatrix} = \frac{1}{k^3} \begin{vmatrix} k^2a_{11} & ka_{12} & a_{13} \\ k^2a_{21} & ka_{22} & a_{23} \\ k^2a_{31} & ka_{32} & a_{33} \end{vmatrix}$$

$$= |A|$$

$$k_1 |A| + k_2 |B| = 0$$

$$k_1 + k_2 = 0$$

$$(B) \begin{vmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ 1 & 1 & 1 \end{vmatrix} = 4$$

$$(C) \begin{vmatrix} 1 & \cos\alpha & \cos\beta \\ \cos\alpha & 1 & \cos\gamma \\ \cos\beta & \cos\gamma & 1 \end{vmatrix} = \begin{vmatrix} 0 & \cos\alpha & \cos\beta \\ \cos\alpha & 0 & \cos\gamma \\ \cos\beta & \cos\gamma & 0 \end{vmatrix}$$

$$\Rightarrow \sin^2 \gamma - \cos\alpha (\cos\alpha - \cos\beta \cos\gamma) + \cos\beta (\cos\alpha \cos\gamma - \cos\beta)$$

$$= -\cos\alpha (-\cos\beta \cos\gamma) + \cos\beta (\cos\alpha \cos\gamma)$$

$$\Rightarrow \sin^2\gamma - \cos^2\alpha + 2 \cos\alpha \cos\beta \cos\gamma - \cos^2\beta = 2\cos\alpha \cos\beta \cos\gamma$$

$$\Rightarrow \sin^2\gamma = \cos^2\alpha + \cos^2\beta$$

$$\Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$(D) \begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix}$$

$$R_2 \rightarrow R_2 - (R_1 + R_3)$$

$$= \begin{vmatrix} x^2+x & x+1 & x-2 \\ -4 & 0 & 0 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = 4 \begin{vmatrix} x+1 & x-2 \\ 2x-1 & 2x-1 \end{vmatrix}$$

$$= 4 \begin{vmatrix} x+1 & -3 \\ 2x-1 & 0 \end{vmatrix} = (24x-12)$$

$$\therefore A = 24, B = -12$$

$$\therefore A + 2B = 0$$

23.

1. (A)

$$b_1 \cdot C_{31} + b_2 \cdot C_{32} + b_3 \cdot C_{33} =$$

$$b_1 \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - b_2 \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + b_3 \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = 0$$

2. (B)

$$\text{Value of new determinant} = 2^3\Delta = 8\Delta$$

3. (D)

$$a_3 M_{13} - b_3 \cdot M_{23} + d_3 \cdot M_{33} = a_3 C_{13} + b_3 \cdot C_{23} + d_3 \cdot C_{33} = \Delta$$

by definition

24.

1.  $a + b + c = p, ab + bc + ca = 0$

$$\therefore a^2 + b^2 + c^2 = (a + b + c)^2 - 2(ab + bc + ca) = p^2 - 0$$

$$= p^2$$

$$\text{If } \Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$\therefore \Delta^c = \begin{vmatrix} bc - a^2 & ca - b^2 & ab - c^2 \\ ca - b^2 & ab - c^2 & bc - a^2 \\ ab - c^2 & bc - a^2 & ca - b^2 \end{vmatrix} = \Delta^{3-1}$$

$$= \Delta^2 = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \times \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$= \begin{vmatrix} a^2 + b^2 + c^2 & ab + bc + ca & ab + bc + ca \\ ab + bc + ca & a^2 + b^2 + c^2 & ab + bc + ca \\ ab + bc + ca & ab + bc + ca & a^2 + b^2 + c^2 \end{vmatrix}$$

$$= \begin{vmatrix} p^2 & 0 & 0 \\ 0 & p^2 & 0 \\ 0 & 0 & p^2 \end{vmatrix} = p^6$$

2.

$$\begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix} \times \begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix}$$

$$= \begin{vmatrix} l_1^2 + m_1^2 + n_1^2 & l_1 l_2 + m_1 m_2 + n_1 n_2 \\ l_1 l_2 + m_1 m_2 + n_1 n_2 & l_2^2 + m_2^2 + n_2^2 \\ l_3 l_1 + m_3 m_1 + n_3 n_1 & l_2 l_3 + m_2 m_3 + n_2 n_3 \end{vmatrix}$$

$$\begin{vmatrix} l_1 l_3 + m_1 m_3 + n_1 n_3 \\ l_2 l_3 + m_2 m_3 + n_2 n_3 \\ l_3^2 + m_3^2 + n_3^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1$$

$$\therefore \begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix}^2 = 1$$

$$\therefore \begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix} = \pm 1$$

3.  $x^3 - 3x^2 + 3x + 7 = 0$

$$\Rightarrow (x-1)^3 + 8 = 0$$

$$\Rightarrow (x-1)^3 = (-2)^3$$

$$\Rightarrow \left(\frac{x-1}{-2}\right)^3 = 1$$

$$\Rightarrow \frac{x-1}{-2} = (1)^{1/3} = 1, \omega, \omega^2$$

$$\Rightarrow x-1 = -2, -2\omega, -2\omega^2$$

or  $x = -1, 1-2\omega, 1-2\omega^2$

$$\therefore a = -1, b = 1-2\omega, c = 1-2\omega^2$$

$$\therefore \begin{vmatrix} a & b & c^2 \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$= \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \times \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$= \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \times \begin{vmatrix} -a & c & b \\ -b & a & c \\ -c & b & a \end{vmatrix} \quad (\text{row by row})$$

$$= \begin{vmatrix} 2bc - a^2 & c^2 & b^2 \\ c^2 & 2ac - b^2 & a^2 \\ b^2 & a^2 & c^2 \end{vmatrix}$$

$$= \begin{vmatrix} a & b & c^2 \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$= (a^3 + b^3 + c^3 - 3abc)^2$$

$$= \{(a+b+c)(a^2+b^2+c^2-ab-bc-ca)\}^2$$

$$= \frac{1}{4} (a+b+c)^2 \{(a-b)^2 + (b-c)^2 + (c-a)^2\}^2$$

$$= \frac{9}{4} \{-12(1+\omega+\omega^2)\} = 0$$

25.

1. (C)

As 2<sup>nd</sup> row of all the options is same, we are to look at the elements of the first row.

Let left inverse be  $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$ , then

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore a + b + 2c = 1$$

$$-a + b + 3c = 0 \quad \text{i.e. } b = \frac{1-5c}{2}, a = \frac{1+c}{2}$$

Thus matrices in the options A, B and D are the inverses and matrix in option C is not the left inverse.

2. (D)

Let right inverse is

$$\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$$

Now  $a - c + 2e = 1$

$$b - d + 2f = 0$$

$$2a - c + e = 0$$

$$2b - d + f = 1$$

infinite solution

so answer is (D)

3. (C)

By observation there can't be any left inverse for (B) & (D) so we will check for (A) & (C) only.

For (A) let left inverse be  $\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$ , then

$$\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix} \begin{bmatrix} 1 & 2 & 4 \\ -3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now  $a - 3b = 1, 2a + 2b = 0$

and  $4a + b = 0$  which is not possible.

For (C)  $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 2 & -3 \\ 5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow a + 2b + 5c = 1, 4a - 3b + 4c = 0,$   
 $d + 2e + 5f = 0, 4d - 3e + 4f = 1$

$\therefore$  there are infinite number of left inverses.

$$\begin{bmatrix} 1 & 4 \\ 2 & -3 \\ 5 & 4 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow a + 4d = 1, 2a - 3d = 0$  and  $5a + 4d = 0$

which is not possible

$\therefore$  There is no right inverse.

**26. (4)**

On solving we get

$(2\lambda + 15)f(x+1) - (\lambda + 10)f(x+8) - f(x+1) = 0$

$(2\lambda + 14)f(x+1) = (\lambda + 10)f(x+8)$

Since  $f$  is periodic with period 7

$\therefore f(x+1) = f(x+8)$

$\Rightarrow 2\lambda + 14 = \lambda + 10$

$\Rightarrow |\lambda| = 4$

**27. (1)**  $f(-x) = -f(x) = g(x)$

$\therefore f(x) \cdot g(x) = -(f(x))^2$

or  $f(1)g(1) = -(f(1))^2 = - \begin{vmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 1 & 3 & 1 \end{vmatrix}^2 = -4$

$\Rightarrow \lambda f(1)g(1) = 4 \Rightarrow \lambda(-4) = 4 \Rightarrow \lambda = -1$

**28.** Clearly  $f'(x) = 0$

$\therefore f(x) = c = 6$

$\therefore \sum_{r=1}^{25} f(r) = \sum_{r=1}^{25} 6 = 150$

**29. (0)** Let

$$\begin{vmatrix} (1+x)^{a_1b_1} & (1+x)^{a_1b_2} & (1+x)^{a_1b_3} \\ (1+x)^{a_2b_1} & (1+x)^{a_2b_2} & (1+x)^{a_2b_3} \\ (1+x)^{a_3b_1} & (1+x)^{a_3b_2} & (1+x)^{a_3b_3} \end{vmatrix}$$

$= \lambda_0 + \lambda_1x + \lambda_2x^2 + \lambda_3x^3 + \dots$

for  $\lambda_1$  differentiate w.r.t.  $x$  and put  $x = 0$

so  $\lambda_1 = 0$

**30.**  $p^2 + q^2 - pq - p - q + 1 \leq 0$

or  $2p^2 + 2q^2 - 2pq - 2p - 2q + 2 \leq 0$

or  $(p^2 + q^2 - 2pq) + (p^2 - 2p + 1) + (q^2 - 2q + 1) \leq 0$

$\Rightarrow (p-q)^2 + (p-1)^2 + (q-1)^2 \leq 0$

which is possible

$(p-q)^2 + (p-1)^2 + (q-1)^2 = 0$

or  $p - q = 0, p - 1 = 0, q - 1 = 0$

$\therefore p = 1, q = 1$

Then,  $\begin{vmatrix} 1 & \cos\gamma & \cos\beta \\ \cos\gamma & p & \cos\alpha \\ \cos\beta & \cos\alpha & q \end{vmatrix} = \begin{vmatrix} 1 & \cos\gamma & \cos\beta \\ \cos\gamma & 1 & \cos\alpha \\ \cos\beta & \cos\alpha & 1 \end{vmatrix}$

$= 1 + 2 \cos\alpha \cos\beta \cos\gamma - \cos^2\alpha - \cos^2\beta - \cos^2\gamma$

$= 2 \cos\alpha \cos\beta \cos\gamma - (\cos^2\alpha - \sin^2\beta) - \cos^2\gamma$

$= 2 \cos\alpha \cos\beta \cos\gamma - \cos(\alpha + \beta)\cos(\alpha - \beta) - \cos^2\gamma$

$= 2 \cos\alpha \cos\beta \cos\gamma - \cos(-\gamma)\cos(\alpha - \beta) - \cos^2\gamma$

$(\because \alpha + \beta + \gamma = 0)$

$= \cos\gamma \{ 2 \cos\alpha \cos\beta - \cos(\alpha - \beta) - \cos\gamma \}$

$= \cos\gamma \{ 2 \cos\alpha \cos\beta - \cos(\alpha - \beta) - \cos(\alpha + \beta) \}$

$= \cos\gamma (2 \cos\alpha \cos\beta - 2 \cos\alpha \cos\beta)$

$= 0$