Solution

APPLICATION OF DERIVATIVES WS 1

Class 12 - Mathematics

1.

(d) $\tan^{-1} \frac{3}{5}$

Explanation: $\tan^{-1} \frac{3}{5}$

2.

(b) 15

Explanation: $y = -x^3 + 3x^2 + 12x - 5$

$\frac{dy}{dx} = -3x^3 + 6x + 12 = f(x)$

$\frac{d^2y}{dx^2} = -6x + 6$

$\frac{d^2y}{dx^2} = 0$

$-6x + 6 = 0$

$x = 1$

$\frac{d^3y}{dx^2} = -6 < 0$,maximum

i.e, $\frac{dy}{dx}$ or slope of y is maximum at x = 1

Slope = $f(x) = -3(1)^2 + 6(1) + 12$

$= -3 + 6 + 12$

$= 15$

3.

(c) Increasing $(-\infty, -1)$

Decreasing $(-1, \infty)$

Explanation: $f(x) = -x^2 - 2x + 15$

$f'(x) = -2x - 2 = -2(x + 1) > 0$

if $x < -1$ i.e., in $(-\infty, -1)$

$f'(x) < 0$ if $x > -1$ i.e., in $(-1, \infty)$

Hence f(x) is increasing in $(-\infty, -1)$ and decreasing in $(-1, \infty)$.

4.

(c) $\lambda > 1/2$

Explanation: $\lambda > 1/2$

5.

(b) $0 < X < 2$

Explanation: $0 < X < 2$

$f(x) = x^2e^{-x}$

$f'(x) = 2xe^{-x} - x^2e^{-x}$

$= e^{-x}(2 - x)$

For f(x) to be monotonic increasing, we must have

$f'(x) > 0$

$\Rightarrow e^{-x}x(2 - x) > 0$ $[\because e^{-x} > 0]$

$\Rightarrow x(2 - x) > 0$

$\Rightarrow x(x - 2) < 0$

$\Rightarrow 0 < x < 2$

6. **(a)** $k \geq 1$

Explanation: Given,$f(x) = \sin x - kx$

$f'(x) = \cos x - k$

$\therefore$ f decreases, if f'$(x) \leq 0$
$\Rightarrow \cos x - k \leq 0$
$\Rightarrow \cos x \leq k$
Therefore, for decreasing $k \geq 1$

7.

(c) 2abc

Explanation: S = b^2x + c^2y and xy = a^2

$\Rightarrow$ S = b^2x + $\frac{a^2c^2}{x}$

$\therefore \frac{dS}{dx} = b^2 - \frac{c^2a^2}{x^2}$ = 0

$\Rightarrow b^2 - \frac{c^2a^2}{x^2}$ = 0 $\Rightarrow x^2 = \frac{c^2a^2}{b^2} \Rightarrow x = \pm\frac{ca}{b}$

$\frac{d^2S}{dx^2} = \frac{2c^2a^2}{x^3}$

$\therefore \frac{d^2S}{dx^2}\Big|_{x=\frac{ca}{b}} = \frac{2c^2a^2}{c^3a^3} = \frac{2b^3}{ca}$ > 0

$\therefore$ Minimum value of S = $b^2 \times \frac{ca}{b} + \frac{c^2a^2}{\frac{ca}{b}}$ = abc + abc = 2abc

8.

(b) $\lambda > 2$

Explanation: $\lambda > 2$

9.

(c) a > 1

Explanation: a > 1

10. **(a)** -128

Explanation: f(x) = $2x^3$ - $21x^2$ + 36x - 20

$\Rightarrow$ f'(x) = $6x^2$ - 42x + 36

For local maxima or minima

$6x^2$ - 42x + 36 = 0

x^2 - 7x + 6 = 0

$\Rightarrow$ x = 1 or x = 6

f''(x) = 12x - 42

$\Rightarrow$ f''(1) = -30 < 0

also, f''(6) = 30 > 0

function has minima at x = 6

$\Rightarrow$ f(6) = -128

11.

(d) 1

Explanation: Given f(x) = x^2 - 8x + 17

$\Rightarrow$ f'(x) = 2x - 8

For minimum valye of f(x) we have f'(x) = 0

$\Rightarrow$ 2x - 8 = 0 $\Rightarrow$ x = $\frac{8}{2}$ = 4

Now, f''(x) = 2 > 0, hence the minimum of f(x) exist at x = 4 and minimum value = f(4) = 1

12.

(d) $1, \frac{1}{3}$

Explanation: Given, f(x) = x^3 - $2x^2$ + x + 1

$\therefore$ f'(x) = $3x^2$ - 4x + 1

For critical points, put f'(x) = 0

$\therefore$ $3x^2$ - 4x + 1 = 0

$\Rightarrow$ $3x^2$ - 3x - x + 1 = 0

$\Rightarrow$ 3x(x - 1) - 1(x - 1) = 0

$\Rightarrow$ (3x - 1)(x - 1) = 0 $\Rightarrow$ x = 1, $\frac{1}{3}$

13. **(a)** $(1, \infty)$

Explanation: Given, function

$\Rightarrow$ f(x) = $(x + 1)^3 . (x - 3)^3$

$\Rightarrow$ f'(x) = $3(x + 1)^2 (x - 3)^3 + 3(x - 3)^3 (x + 1)^3$

Put f'(x) = 0

$\Rightarrow$ $3(x + 1)^2 (x - 3)^3 = -3(x - 3)^2 (x + 1)^3$

$\Rightarrow$ x - 3 = -(x + 1)

$\Rightarrow$ 2x = 2

$\Rightarrow$ x = 1

When x > 1 the function is increasing

x < 1 function is decreasing

Therefore f(x) is increasing in $(1, \infty)$.

14. **(a)** strictly increasing

Explanation: strictly increasing

15.

(c) $\frac{4}{27}$

Explanation: Here,it is given function ff(x) = $(x - 2) (x - 3)^2$

f(x) = $(x - 2) (x^2 - 6x + 9)$

f(x) = $x^3 - 8x^2 + 21x + 18$

f'(x) = $3x^2 - 16x + 21$

f''(x) = 6x - 16

For maximum or minimum value f'(x) = 0

$\therefore$ $3x^2 - 9x - 7x + 21 = 0$

$\Rightarrow$ 3x(x - 3) - 7(x - 3) = 0

$\Rightarrow$ x = 3 or x = $\frac{7}{3}$

f''(c) at x = 3

$\therefore$ f''(x) = 2

f''(x) > 0 it is decreasing and has minimum value at x = 3

at x = $\frac{7}{3}$

f'' (x) = -2

f''(x) < 0 it is increasing and has maximum value at x = $\frac{7}{3}$

Putting, x = $\frac{7}{3}$ in f(x) we obtain,

$\Rightarrow \left(\frac{7}{3} - 2\right)\left(\frac{7}{3} - 3\right)^2$

$= \left(\frac{1}{3}\right)\left(\frac{-2}{3}\right)^2$

$= \frac{4}{27}$

16.

(d) $k \in (-\infty, 4)$

Explanation: $\because$ f(x) = X^2 - kx + 5 is increasing in x $\in$ [2, 4]

f'(x) = 2x - k

f'(x) > 0, for increasing function

2x - k > 0

k < 2x (k should be less than minimum value of 2x)

k < 4

$k \in (-\infty, 4)$

17.

(c) $\frac{3}{4}$

Explanation: Given, f(x) = $x^2 + x + 1$

$\Rightarrow$ f'(x) = 2x + 1

For minimum value of f(x) we have f'(x) = 0

$\Rightarrow$ 2x + 1 = 0 $\Rightarrow$ x = $\frac{-1}{2}$

Now, f''(x) = 2 >0, hence the minimum of f(x) exist at x = $\frac{-1}{2}$ and minimum value = f($\frac{-1}{2}$) = $\frac{3}{4}$

18.

(b) $\frac{1}{2}$

Explanation: Let f(x) = sinx . cosx

$\Rightarrow f(x) = \frac{1}{2}(sin2x)$

Now, f'(x)=$\frac{1}{2}(cos2x).2 = cos2x$

For maximum and minimum values of x, we have f'(x) = 0

f'(x) = 0$\Rightarrow cos2x = 0$

$\Rightarrow x = \frac{\pi}{4}$

Now, f''(x) = -2sin2x

i.e, f''($\frac{\pi}{4}$) = -2sin$\frac{\pi}{2}$ = -2 < 0

Hence, f(x) has a maximum value at x = $\frac{\pi}{4}$ and the max value of f($\frac{\pi}{4}$) = sin$\frac{\pi}{4}$ cos$\frac{\pi}{4}$ = $\frac{1}{2}$.

19.

(c) $2\sqrt{ab}$

Explanation: Given f(x) = ax + $\frac{b}{x}$ (a > 0, b > 0, x > 0)

f'(x) = a $\frac{b}{x^2}$

for critical points

f'(x) = 0

a - $\frac{b}{x^2}$ = 0

x^2 = $\frac{b}{a}$

x = $\pm\sqrt{\frac{b}{a}}$

but x > 0

$\Rightarrow x = \sqrt{\frac{b}{a}}$

f''(x) = -b$\left(\frac{-2}{x^3}\right)$

f''(x) = $\frac{2b}{x^3}$

f''$\left(\sqrt{\frac{b}{a}}\right) = \frac{2b}{\left(\sqrt{\frac{b}{a}}\right)^3}$ > 0 as a > 0, b > 0

$\Rightarrow$ at x = $\sqrt{\frac{b}{a}}$ we will get minimum value

$\Rightarrow$ f $\left(\sqrt{\frac{b}{a}}\right) = a \cdot \sqrt{\frac{b}{a}} + b \cdot \sqrt{\frac{a}{b}}$

= $\sqrt{ab} + \sqrt{ab}$

= $2\sqrt{ab}$

20.

(c) $\frac{1}{4}$

Explanation: Given f(x) = $x^{25}(1 - x)^{75}$

f'(x) = x^{25}. $75(1 - x)^{74}(-1)+(1 - x)^{75}$. $25x^{24}$

= $25x^{24}(1 - x)^{74}$\{-3x + (1-x)\}

= $25x^{24}(1 - x)^{74}(1 - 4x)$

For maximum value of f(x) we have f'(x) = 0

$\Rightarrow$ $25x^{24}(1 - x)^{74}(1 - 4x) = 0$

$\Rightarrow$ x = 0, x = 1, x= $\frac{1}{4}$

All the values of x $\in$ [0,1]

Note that f(0) = f(1) = 0 and f($\frac{1}{4}$) = $\frac{3^{75}}{4^{100}}$

So, f(x) is maximum at x = $\frac{1}{4}$

21.

(c) Decreasing on R

Explanation: Given, f'(x) = $-x^3 + 3x^2 - 3x + 4$

f'(x) = $-3x^2 + 6x - 3$

f'(x) = $-3(x^2 - 2x + 1)$

f'(x) = $-3(x - 1)^2$

As f'(x) has -ve sign before 3

$\Rightarrow$ f'(x) is decreasing over R.

22. **(a)** 2

Explanation: Here ,it is given the function f(x) = $e^x + e^{-x}$

$\Rightarrow f(x) = e^x + \frac{1}{e^x}$

$\Rightarrow f(x) = \frac{e^{2x}+1}{e^x}$

f(x) is always increasing at x = 0 it has the least value

$\Rightarrow f(x) = \frac{1+1}{1}$ = 2

$\therefore$ The least value is 2

23. **(a)** point of inflexion at x = 0

Explanation: Given f(x) = x^3

f'(x) = $3x^2$

For point of inflexion, we have f'(x) = 0

$f'(x) = 0 \Rightarrow 3x^2 = 0 \Rightarrow x = 0$

Hence, f(x) has a point of inflexion at x = 0.

But x = 0 is not a local extremum as we cannot find an interval I around x = 0 such that

$f(0) \geq f(x) \quad or \quad f(0) \leq f(x) \qquad for \quad all \quad x \epsilon I$

24.

(b) e

Explanation: $f(x) = \frac{x}{\log x}$

$\Rightarrow f'(x) = \frac{log x.1 - x.\frac{1}{x}}{(\log x)^2}$

For maximum or minimum values of x we have f'(x) = 0

$f'(x) = 0 \Rightarrow \frac{\log x - 1}{(\log x)^2} = 0 \Rightarrow (\log x - 1) = 0$

$\Rightarrow \log x = 1 \Rightarrow x = e$

Now, $f''(x) = (\log x - 1)\frac{-2}{(\log x)^3} \quad + (\log x)^{-2}. \frac{1}{x}$

$f''(e) = \frac{1}{e} > 0$

Hence, f(x) has a minimum value f(e) = e.

25.

(c) $e^{\pi} > \pi^e$

Explanation: Let y = f(x) = $x^{\frac{1}{x}}$

Then, log y = log $x^{\frac{1}{x}}$ = $\frac{1}{x}$. logx

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{x}.\frac{1}{x} + log x.\frac{-1}{x^2} = \frac{1 - \log x}{x^2}$

$\Rightarrow f'(x) = x^{\frac{1}{x}}(\frac{1 - \log x}{x^2})$

$f'(x) = 0 \Rightarrow (1 - \log x) = 0 \ldots\ldots (\because x^{\frac{1}{x}} \neq 0)$

$\Rightarrow \log x = 1 \Rightarrow x = e$

$\therefore f \ (e) > f\ (\ \pi\ )$

$\Rightarrow e^{\frac{1}{e}} > \pi^{\frac{1}{\pi}}$

$\Rightarrow \left(e^{\frac{1}{e}}\right)^{\pi e} > \left(\pi^{\frac{1}{\pi}}\right)^{\pi e}$
$\Rightarrow e^{\pi} > \pi^{e}$

26.
(d) odd and increasing
Explanation: odd and increasing

27.
(c) decreasing in $\left(\frac{\pi}{2}, \pi\right)$

Explanation: We have, f(x) = 4 $\sin^3$x - 6 $\sin^2$x + 12 sin x + 100

$\therefore$ f'(x) = 12 $\sin^2$ x $\cdot$ cos x - 12 sin x $\cdot$ cos x + 12 cos x

= 12 cos x [$\sin^2$x - sin x + 1]

= 12 cos x [$\sin^2$ x + (1 - sinx)]

Now 1 - sin x $\geq$ 0 and $\sin^2$ x $\geq$ 0

$\therefore$ $\sin^2$ x + 1 - sin x > 0
Hence f'(x) > 0, when cos x > 0 i.e., $x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$
So, f(x) is increasing when $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
and f'(x) < 0, when cos x < 0 i.e., $x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$
Hence, f(x) is decreasing when $x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$
Hence, f(x) is decreasing in $\left(\frac{\pi}{2}, \pi\right)$

28.
(c) 0 < x < 1 and x > 2
Explanation: Given function is

y = $[x(x - 2)]^2 = [x^2 - 2x]^2$
On differentiating w.r.t. x, we get
$\frac{dy}{dx}$ = 2(x^2 - 2x) $\frac{d}{dx}$(x^2 - 2x)
= 2(x^2 - 2x(2x - 2)
= 4x(x - 2)(x - 1)
On putting $\frac{dy}{dx}$ = 0, we get
x = 0, 1 and 2
which divides real line in disjoint intervals (-∞, 0), (0, 1), (1, 2) and (2, ∞).

Intervals	Sign of f'(x)	Nature of f(x)
(-∞, 0)	(-)(-)(-) = -ve	Strictly decreasing
(0, 1)	(+)(-)(-) = +ve	Strictly increasing
(1, 2)	(+)(-)(+) = -ve	Strictly decreasing
(2, ∞)	(+)(+)(+) = +ve	Strictly increasing

Therefor, y in increasing in (0, 1) and (2, ∞).

29. **(a)** Minimum at x = $\frac{\pi}{2}$

Explanation: f(x) = 1 + 2 sinx + 3$\cos^2$ x
$\Rightarrow$ f'(x) = 2 cosx - 6 cosx sinx
$\Rightarrow$ f'(x) = 2 cosx - 3 sin2x
to find minima or maxima of the function
2 cosx - 6 cosx sinx = 0
2 cos x (1 - 3sinx) = 0

$\Rightarrow$ cos x = 0 or sinx = $\frac{1}{3}$

x = $\frac{\pi}{2}$ or x = sinx$^{-1}\left(\frac{1}{3}\right)$

f''(x) = - 2 sinx - 6 cos2x

f''$\left(\frac{\pi}{2}\right)$ = 4 > 0

$\Rightarrow$ x =$\frac{\pi}{2}$ is a local minima.

f''$\left(\sin^{-1}\left(\frac{1}{3}\right)\right)$ = - $\left(\frac{2}{3}+4\sqrt{2}\right)$ < 0

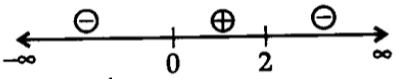
function has maxima at x = sin$^{-1}\left(\frac{1}{3}\right)$

30. **(a)** $(-\infty, 0) \cup (2, \infty)$

Explanation: We have,

f(x) = y = x^2e^{-x}

$\therefore \frac{dy}{dx}$ = 2x e^{-x} + $x^2(-1)e^{-x}$ = $xe^{-x}(2 - x)$



Now, put $\frac{dy}{dx}=0$

$\Rightarrow$ x = 2 and x = 2

The points x = 0 and x = divide the real time into three disjoint intervals i.e., $(-\infty, 0)$, (0, 2) and $(2, \infty)$.

In intervals, $(-\infty, 0)$ and $(2, \infty)$, f'(x) < 0 as e^{-x} is always positive.

$\therefore$ f(x) or y is decreasing in $(-\infty, 0)$ and $(2, \infty)$.

31.

(d) $\left(\frac{1}{e}\right)$

Explanation: $\Rightarrow f(x)=\frac{\log x}{x}$

$\therefore f'(x)=\frac{\log x - x\frac{1}{x}}{x^2}$

$\Rightarrow$ f'(x) = log x - 1

$\Rightarrow$ substitute f'(x) = 0

We get x = e

F''(x) = $\frac{1}{x}$

Substitute x = e in f''(x)

$\frac{1}{e}$ is point of maxima

$\therefore$ The max value is $\frac{1}{e}$

32. **(a)** decreases on [0, a]

Explanation: decreases on [0, a]

33. **(a)** $\mathbb{R}$

Explanation: $\mathbb{R}$

34. **(a)** f(x) is an increasing function

Explanation: f'(x) = $\frac{\sin x - x\cos x}{\sin^2 x}$, g'(x) = $\frac{\tan x - x\sec^2 x}{\tan^2 x}$

Now $\frac{d}{dx}$(sin x - x cos x) = cos x + x sin x - cos x

= x sin x > 0 for 0 $\leq$ x < 1

$\therefore$ sin x - x cos x is an increasing function.

But at x = 0, x sin x is 0

$\therefore$ In 0 < x $\leq$ 1, sin x - x cos x > 0

$\therefore$ f'(x) > 0 for 0 < x $\leq$ 1

So f(x) is increasing in the interval 0 < x $\leq$ 1

Again $\frac{d}{dx}$(tan x - x $\sec^2$x) = $\sec^2$x - 2x $\sec^2$x tan x - $\sec^2$x

= -2x $\sec^2$x tan x < 0 for $0 \leq x \leq 1$

$\therefore$ g(x) is decreasing in 0 < x $\leq$ 1

35. **(a)** cos x

Explanation: $f_1(x)$ = $\sin^2$x, increases from '0' to '1' in $\left(0, \frac{\pi}{2}\right)$

$f_2(x)$ = tan x is increasing function in each quadrant

$f_3(x)$ = cos x, decreases from '1' to '0' in $\left(0, \frac{\pi}{2}\right)$

$f_4(x)$ = cos 3x, decreases if $3x \in \left(0, \frac{\pi}{2}\right)$ or $x \in \left(0, \frac{\pi}{6}\right)$

36.

(c) -39

Explanation: Given function,

$f(x) = 3x^4 - 8x^3 - 48x + 25$

$F'(x) = 12x^3 - 24x^2 - 48 = 0$

$F'(x) = 12(x^3 - 2x^2 - 4) = 0$

Differentiating again, we obtain

$F''(x) = 3x^2 - 4x = 0$

x(3x - 4) = 0

x = 0 or x = $\frac{4}{3}$

Putting the value in equation, we obtain

f(x) = -39

37.

(b) 1 < x < 2

Explanation: 1 < x < 2

38. **(a)** f(x) is invertible

Explanation: f(x) is invertible

39.

(c) $(-\infty, -1)$

Explanation: f(x) = sin x - ax + b

$\Rightarrow$ f'(x) = cos x - a

For increasing function

$f'(x) \geq 0$

cos x - a $\geq 0 \Rightarrow$ cos x $\geq$ a

i.e. a $\leq$ cos x a $\leq$ min (cos x) = -1

$\therefore$ a $\in (-\infty, -1)$

40. **(a)** strictly decreasing

Explanation: Given function is f(x) = $\frac{5}{x}$ - 9, x $\neq 0$

$\Rightarrow f(x) = 5x^{-1} - 9$

On differentiating both sides w.r.t. x, we get

$f'(x) = 5(-1)\, x^{-2} - 0 = \frac{-5}{x^2} < 0, \forall\, x \in R - \{0\}$

$\therefore$ f(x) is strictly decreasing for x $\in$ R, (x $\neq$ 0).

41. **(a)** $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

Explanation: $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$Given function,f(x) is sin x

$\therefore$ f'(x) = cos x

$\Rightarrow$ f'(x) = cos x

= 0

$\Rightarrow$ for $x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

f'(x) is increasing,we

$\therefore$ f(x) is increasing in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$.

Which is the required solution.

42.

(c) $\frac{1}{6}$

Explanation: f(x) = $\frac{x}{4+x+x^2}$

$\Rightarrow$ f'(x) = $\frac{4-x^2}{\left(4+x+x^2\right)^2}$

For a local maxima or minima,
f'(x) = 0
$\frac{4-x^2}{(4+x+x^2)^2}$ = 0
$\Rightarrow$ x = $\pm$2 $\in$ [-1, 1]
f(1) = $\frac{1}{6}$ > 0
f(-1) = $\frac{-1}{4}$ < 0
$\Rightarrow$ $\frac{1}{6}$ is the maximum value.

43.

(c) 36π cm^2/s
Explanation: The rate of change of radius of circle is $\frac{dr}{dt} = 6 \text{ cm/s}$
A = πr^2
Differentiating w.r.t. 't', we get
$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$
$\Rightarrow \frac{dA}{dt} = 2\pi r(6)$
$\Rightarrow \frac{dA}{dt} = 12\pi r$
$\therefore \left(\frac{dA}{dt}\right)_{\text{at } r=3} = 12\pi \times 3 = 36\pi \text{cm}^2/\text{s}$

44.

(c) Increasing on R
Explanation: Given,f'(x) = 3x + cos 3x
f'(x) = 3 - 3sin 3x
f'(x) = 3(t - sin 3x)
Sin 3x varies from [-1, 1]
When sin 3x is 1 f'(x) = 0 and sin 3x is -1 f'(x) = 6
As the function is increasing in 0 to 6.
$\therefore$ The function is increasing of R.

45.

(c) always increases
Explanation: We have, f(x) = tan x - x
$\therefore$ f'(x) = sec^2x - 1
$\Rightarrow$ f'(x) $\geq$ 0, $\forall$ x $\in$ R
So, f(x) always increases.

46.

(d) (-∞, 4)
Explanation: $f(x) = 2x^2 - kx + 5$
$f'(x) = 4x - k$
for f(x) to be increasing, we must have
f(x) > 0
4x - k > 0
K < 4 x
since $x \in [1, 2], 4x \in [4, 8]$
so, the minimum value of 4 x is 4.
since K < 4x, K < 4.
k $\in (-\infty, 4)$

47.

(b) [1, ∞)
Explanation: f(x) = $\frac{2x^2-1}{x^4}$
f'(x) = $\frac{x^4(4x)-(2x^2-1)(4x^3)}{(x^4)^2} = \frac{4x^5-8x^5+4x^3}{x^8}$
= $\frac{-4x^5+4x^3}{x^8} = \frac{-4x^2+4}{x^5}$

f is decreasing if f'(x) ≤ 0

$\Rightarrow \frac{-4x^2+4}{x^5} \leq 0 \Rightarrow \frac{4x^2-4}{x^5} \geq 0$

$\Rightarrow$ x^2 - 1 $\geq 0 \Rightarrow$ $x^2 \geq 1 \Rightarrow |x| \geq$ 1 and x > 0

$\Rightarrow$ x $\in [1, \infty)$

48. **(a)** increasing in $[0, \frac{\pi}{2}]$

Explanation: f(x) = 2 $\sin^3$x - 3 $\sin^2$x + 12 sin x + 5

f'(x) = 6 $\sin^2$x cos x - 6 sin x cos x + 12 cos x

= 6 cos x ($\sin^2$x - sin x + 2}

= 6 cos x {$\sin^2$x - 2 sin x $\times \frac{1}{2} + \frac{1}{4} - \frac{1}{4}$ + 2}

= 6 cos x $\left\{\left(\sin x - \frac{1}{2}\right)^2 + \frac{7}{4}\right\} \geq 0 \forall x \in \left\lfloor 0, \frac{\pi}{2}\right\rfloor$

$\therefore$ f(x) is increasing in $[0, \frac{\pi}{2}]$

49.

(b) 0 < x < 1

Explanation: 0 < x < 1

50.

(d) Maximum value = 2π

Minimum value = 0

Explanation: f(x) = x + sin 2x

f(0) = 0 and f(2π) = 2π

Hence f(x) has maximum value 2π and minimum value is 0.

51.

(c) increasing

Explanation: increasing

52.

(d) $\frac{1}{a_1} - \frac{1}{a} = \frac{1}{b_1} - \frac{1}{b}$

Explanation: $\frac{1}{a_1} - \frac{1}{a} = \frac{1}{b_1} - \frac{1}{b}$

53. **(a)** k > 3

Explanation: f(x) = kx^3 - $9x^2$ + 9x + 3

f'(x) = $3kx^2$ - 18x + 9

= 3(kx^2 - 6x + 3)

Given: f(x) is monotonically increasing in every interval.

$\Rightarrow$ f'(x) > 0

$\Rightarrow$ 3(kx^2 - 6x + 3) > 0

$\Rightarrow$ (kx^2 - 6x = 3) > 0

$\Rightarrow$ K > 0 and $(-6)^2$ - 4(k)(3) < 0 [$\because$ ax^2 + bx + c > 0 and D is c < 0]

$\Rightarrow$ k > 0 and $(-6)^2$ - 4(k) (3) < 0

$\Rightarrow$ k > 0 and 36 - 12k < 0

$\Rightarrow$ k > 0 and 12k > 36

$\Rightarrow$ k > 0 and k > 3

$\Rightarrow$ k > 3

54. **(a)** none of these

Explanation: Given: f(x) = x^3 - $6x^2$ + 9x

$\Rightarrow$ f'(x) = $3x^2$ - 12x + 9

For a local maxima or a local minima, we must have

f'(x) = 0

$\Rightarrow$ $3x^2$ - 12x + 9 = 0

$\Rightarrow$ x^2 - 4x + 3 = 0
$\Rightarrow$ (x - 1)(x - 3) = 0
$\Rightarrow$ x = 1, 3
Now,
f(0) = 0^3 - $6(1)^2$ + 9(1) + 9(1) = 1 - 6 + 9 = 4
f(1) = 1^3 - 6(1)2 + 9(1) = 1 - 6 + 9 = 4
f(3) = 3^3 - $6(3)^3$ + 9(3) = 27 - 54 + 27 = 0
f(6) = 6^3 - $6(6)^2$ + 9(6) = 216 - 216 + 54 = 54
The least and greatest values of f(x) = x^3 - $6x^2$ + 9x in [0, 6] are 0 and 54, respectively.

55.

(b) ab $\geq \frac{c^2}{4}$
Explanation: f(x) = ax + $\frac{b}{x}$
$\Rightarrow$ f'(x) = a - $\frac{b}{x^2}$
f'(x) = 0
a - $\frac{b}{x^2}$ = 0
$\Rightarrow$ x = $\pm\sqrt{\frac{b}{a}}$
f''(x) = $\frac{2b}{x^3}$
f''$\left(\sqrt{\frac{b}{a}}\right)$ = $\frac{2b}{(\sqrt{\frac{b}{a}})^3} > 0$
$\Rightarrow$ x = $\sqrt{\frac{b}{a}}$ has a minima.
$f\left(\sqrt{\frac{b}{a}}\right)$ = $2\sqrt{ab} \geq c$
$\frac{c}{2} \leq \sqrt{ab}$
$\Rightarrow \frac{c^2}{4} \leq ab$

56. **(a)** 4 + $\sqrt{2}$
Explanation: Maximum value of 4 $\sin^2$x + 3 $\cos^2$x = 4 sin 2x + 3(1 − $\sin^2$x) = $\sin^2$x + 3 is 4 as 0 $\leq$ $\sin^2$x $\leq$ 1 and that of sin $\frac{\pi}{2}$ + cos$\frac{\pi}{2}$ is $\frac{\pi}{\sqrt{2}} + \frac{\pi}{\sqrt{2}} = \sqrt{2}$ (as −a^2 + b^2 $\leq$ a sin x + b cos x $\leq$ a^2 + b^2)
both attained at x = $\frac{\pi}{2}$
Hence, the given function has maximum value 4 + $\sqrt{2}$.

57.

(b) -1 < k < 1
Explanation: -1 < k < 1

58.

(b) neither maximum value nor minimum value
Explanation: Given, f(x) = x^3 + 1
$\therefore$ f'(x) = $3x^2$ and f''(x) = 6x
Put f'(x) = 0
$\Rightarrow$ $3x^2$ = 0 $\Rightarrow$ x = 0
At x = 0, f''(x) = 0
Thus, f(x) has neither maximum value nor minimum value.

59.

(c) $\sqrt{a^2 - b^2}$
Explanation: Let y = a sec θ - b tan θ
$\frac{dy}{d\theta}$ = a sec θ tan θ - b $\sec^2 \theta$
For minimum value of y
$\frac{dy}{d\theta}$ = 0 = a sec θ tan θ - b $\sec^2 \theta$

$\Rightarrow \sin\theta = \frac{b}{a}$

Hence, y = $\frac{a - b\sin\theta}{\cos\theta} = \frac{a^2 - b^2}{\sqrt{a^2 - b^2}}$ = $\sqrt{a^2 - b^2}$

60.

(c) strictly increasing

Explanation: Let x_1 and x_2 be any two numbers in R,

where $x_1 < x_2$.

Given, f(x) = e^{2x}

Then, we have $x_1 < x_2 \Rightarrow 2x_1 < 2x_2$

$\Rightarrow e^{2x_1} < e^{2x_2}$

[$\because x_1 < x_2$ then $a^{x_1} < a^{x_2}$, when a > 1]

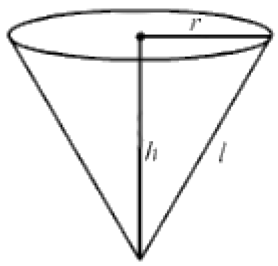
$f(x_1) < f(x_2)$.

Hence, f is strictly increasing on R.

61.

(b) $54\pi \mathrm{cm}^2/\mathrm{min}$

Explanation: Let r be the radius, h be the height and S be the lateral surface area of the cone at any time t.



Given: $\frac{dr}{dt}$ =3cm/min and $\frac{dh}{dt} = -4cm/\min$

Here,

$l^2 = h^2 + r^2$

$\Rightarrow l = \sqrt{(24)^2 + (7)^2}$

$\Rightarrow l = \sqrt{625}$

$\Rightarrow$ l = 25

$S = \pi r l$

$\Rightarrow S^2 = (\pi r l)^2$

$\Rightarrow S^2 = \pi^2 r^2 \left(h^2 + r^2\right)$

$\Rightarrow S^2 = \pi^2 r^4 + \pi^2 h^2 r^2$

$\Rightarrow 2S\frac{dS}{dt} = 4\pi^2 r^3 \frac{dr}{dt} + 2\pi^2 r^2 h \frac{dh}{dt} + 2\pi^2 h^2 r \frac{dr}{dt}$

$\Rightarrow 2\pi r l \frac{dS}{dt} = 2\pi^2 r h \left[\frac{2r^2}{h}\frac{dr}{dt} + r\frac{dh}{dt} + h\frac{dr}{dt}\right]$

$\Rightarrow 25\frac{dS}{dt} = 24\pi \left[\frac{2(7)^2}{24} \times 3 - 7 \times 4 + 24 \times 3\right]$ [Given: r = 7, h = 24]

$\Rightarrow 25\frac{dS}{dt} = 24\pi \left[\frac{49}{4} - 28 + 72\right]$

$\Rightarrow 25\frac{dS}{dt} = 24\pi \left[\frac{49 + 288 - 112}{4}\right]$

$\Rightarrow \frac{dS}{dt} = 24\pi \left[\frac{225}{100}\right]$

$\Rightarrow \frac{dS}{dt} = 24\pi(2.25)$

$\Rightarrow \frac{dS}{dt} = 54\pi \mathrm{cm}^2/\mathrm{sec}$

62. **(a)** $\left(0, \frac{\pi}{4}\right)$

Explanation: Given function is f(x) = sin x + cos x

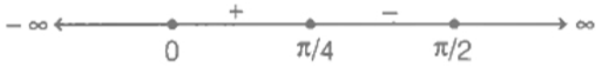
On differentiating both sides w.r.t. x, we get

f'(x) = cos x - sin x

Put f'(x) = 0 $\Rightarrow$ cos x - sin x = 0

$\Rightarrow$ tan x = 1 $\Rightarrow$ x = $\frac{\pi}{4} \in (0, \frac{\pi}{2})$

Now, $\frac{\pi}{4}$ divides the interval (0, $\frac{\pi}{2}$) into two sub intervals (0, $\frac{\pi}{4}$) and ($\frac{\pi}{4}$, $\frac{\pi}{2}$).



In interval (0, $\frac{\pi}{4}$), cos x > sin x

$\therefore$ cos x - sin x > 0 $\Rightarrow$ f'(x) > 0

So, f(x) is strictly increasing function in (0, $\frac{\pi}{4}$).

In interval ($\frac{\pi}{4}$, $\frac{\pi}{2}$), cos x < sin x

$\therefore$ cos x - sin x < 0 $\Rightarrow$ f'(x) < 0

So, f(x) is strictly decreasing function in ($\frac{\pi}{4}$, $\frac{\pi}{2}$).

63.

(c) $(-\infty, 1) \cup (2, 3)$

Explanation: Given that;

f(x) = 2 log(x - 2) - x^2 + 4x +1

f'(x) = $\frac{2}{(x-2)}$ - 2x + 4

= $\frac{2}{(x-2)}$ - 2(x - 2)

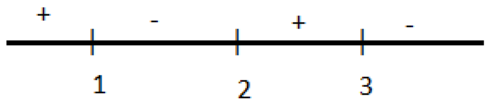
= $\frac{2(1-(x-2)^2)}{(x-2)}$

= $\frac{2(1-x+2)(1+x-2)}{(x-2)}$

= $\frac{2(3-x)(x-1)}{(x-2)}$

Critical points are;

1, 2 and 3



F(x) is increasing in $(-\infty$, 1) $\cup$ (2, 3)

64.

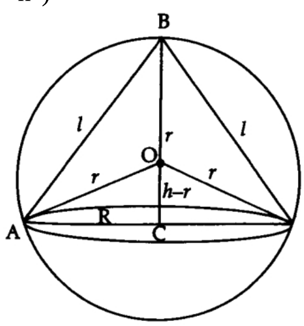
(c) a = 11, b = -6

Explanation: a = 11, b = -6

65.

(d) $2\pi^2$r(2rh^2 - h^3)

Explanation:



Here, CSA of cone = πRl

Radius of sphere = r

height of cone = h

In $\triangle$AOC,

$AO^2 = AC^2 + OC^2$

$\Rightarrow r^2 = R^2 + (h - r)^2$

$\Rightarrow R^2 = 2hr - h^2$

$\therefore$ Radius of cone, $R = \sqrt{2hr - h^2}$ ...(i)

In $\triangle$ABC,

$AB^2 = AC^2 + BC^2$

$\Rightarrow l^2 = R^2 + h^2$

$\Rightarrow l^2 = 2hr - h^2 + h^2$
$\therefore$ slant height $= \sqrt{2hr}$ ...(ii)
CSA of cone = πRl
$= \pi\sqrt{2hr - h^2}\sqrt{2hr}$
$(\text{CSA of cone})^2 = \pi^2(2hr - h^2)(2hr)$
$= 2\pi^2 hr(2hr - h^2)$
$= 2\pi^2 r(2rh^2 - h^3)$

66.

(c) $e^{\frac{1}{e}}$

Explanation: $f(x) = x^{\frac{1}{x}}$
taking log on both sides
$\log f(x) = \log x^{\frac{1}{x}}$
$\frac{1}{f(x)} f'(x) = \frac{1}{x} \times \frac{1}{x} + \log x \left(\frac{-1}{x^2}\right)$
$f'(x) = f(x) \left(\frac{1}{x^2} - \frac{\log x}{x^2}\right)$
consider,
$f'(x) = 0$
$\Rightarrow 1 - \log x = 0$
$\Rightarrow x = e$
Hence the max point is $x = e$
and Maximum value = $e^{\frac{1}{e}}$

67.

(d) (1, 2)

Explanation: $y^2 = 4x \Rightarrow x = \frac{y^2}{4}$
$\Rightarrow d = \sqrt{(x-2)^2 + (y-1)^2}$
$\Rightarrow d^2 = (x - 2)^2 + (y - 1)^2$
$\Rightarrow d^2 = \left(\frac{y^2}{4} - 2\right)^2 + (y - 1)^2$
Let $u = \left(\frac{y^2}{4} - 2\right)^2 + (y - 1)^2$
$\Rightarrow \frac{du}{dy} = 2\left(\frac{y^2}{4} - 2\right) \frac{y}{2} + 2(y - 1)$
To find minima
$\frac{du}{dy} = 0$
$2\left(\frac{y^2}{4} - 2\right) \frac{y}{2} + 2(y - 1) = 0$
$\Rightarrow y = 2 \Rightarrow x = 1 \left(x = \frac{y^2}{4}\right)$
$\frac{d^2u}{dy^2} = \frac{3y^2}{4}$
$\Rightarrow \left(\frac{d^2u}{dy^2}\right)_{(1,2)} = 3 > 0$
Hence, nearest point is (1, 2).

68.

(b) $a = -\frac{3}{4}, b = -\frac{1}{8}$
Explanation: $a = -\frac{3}{4}, b = -\frac{1}{8}$

69. **(a)** $(-\infty, \infty) - \{0\}$

Explanation: Given, $f(x) = \frac{3}{x} + 7 = 3x^{-1} + 7$
$\therefore f'(x) = \frac{-3}{x^2} < 0 \ \forall\ x \in R - \{0\}$

70.

(c) 120

Explanation: $f'(x) = 4x^3 - 62 \times 2x + a$

f'(x) = $4x^3$ - 124x + a
As function attains maximum at x = 1 $\in$ [0, 2]
f' (1) = 0
$\Rightarrow$ 4 - 124 + a = 0 $\Rightarrow$ a = 120

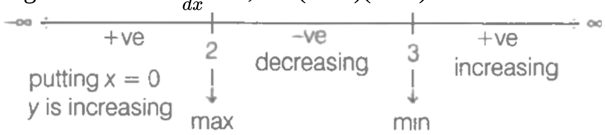
71.

(d) (0 , $\frac{1}{e}$)
Explanation: (0 , $\frac{1}{e}$)
Let y = x^x
$\Rightarrow \log(y)$ = xlogx
$\Rightarrow \frac{1}{y} \times \frac{dy}{dx}$ = 1 + logx
$\Rightarrow \frac{dy}{dx} = x^x(1 + \log x)$
Since the function is decreasing,
$\Rightarrow x^x x(1 + |\log x) < 0$
$\Rightarrow 1 + \log x < 0$
$\Rightarrow \log x < -1$
$\Rightarrow x < \frac{1}{e}$
Therefore, function is decreasing on (0,$\frac{1}{e}$)

72.

(d) h is increasing whenever f(x) is increasing
Explanation: h(x) = f(x) - $\{f(x))^2 + f(x)\}^3$
h'(x) = f'(x) - 2 (f(x) f'(x) + 3$\{(f(x)\}^2$ f'(x)
= {1 - 2(f(x) + 3$\{(f(x)\}^2\}$f'(x)
= 3{$\frac{1}{3} - \frac{2}{3}$f(x) + $(f(x))^2$} f'(x)
= $\left(3\left|(f(x))^2 - \frac{2}{3}f(x) + \frac{1}{9}\right| + \frac{2}{3}\right)$ f'(x)
= $\left|3\left\{f(x) - \frac{1}{3}\right\}^2 + \frac{2}{3}\right|$ f'(x)
If f(x) is increasing, f'(x) > 0 and therefore h'(x) > 0 i.e., h(x) is increasing.
If f(x) is decreasing, f'(x) < 0 and h' (x) < 0 i.e., h(x) is decreasing.

73.

(d) 39, 38
Explanation: Let y = $2x^3 - 15x^2 + 36x + 11$ ...(i)
$\therefore \frac{dy}{dx}$ = $6x^2 - 30x + 36 = 6(x^2 - 5x + 6)$
= 6(x - 2)(x - 3) ...(ii)
Sign scheme for $\frac{dy}{dx}$ i.e., for (x - 2)(x - 3) is


y has maximum value at x = 2
From Eq. (i), the corresponding maximum value of
y = $2(2)^3 - 15(2)^2 + 36(2) + 11 = 39$
y has minimum value at x = 3
From Eq. (i), minimum value of
y = $2(3)^3 - 15(3)^2 + 36(3) + 11 = 38$

74.

(c) 1
Explanation: f(x) = cos x + cos($\sqrt{2}$x)
$\therefore$ f(x) = 2 cos $\frac{\sqrt{2}+1}{2}$x cos $\frac{\sqrt{2}-1}{2}$ x $\leq$ 2
and it is 2 when cos $\frac{\sqrt{2}+1}{2}$x and cos $\frac{\sqrt{2}-1}{2}$ are both equal to 1 for a value of x. This is possible only when x = 0.

75. **(a)** $\frac{\pi}{6}$

Explanation: f(x) = sin x + $\sqrt{3}$ cos x

$\Rightarrow$ f'(x) = cos x - $\sqrt{3}$ sin x

for maxima or minima

f'(x) = 0

cos x - $\sqrt{3}$ sin x = 0

$\Rightarrow$ tan x = $\frac{1}{\sqrt{3}} \Rightarrow x = \frac{\pi}{6}$

f''(x) = - sin x - $\sqrt{3}$ cos x

$\Rightarrow$ f''$\left(\frac{\pi}{6}\right)$ = - sin$\frac{\pi}{6} - \sqrt{3}$ cos $\frac{\pi}{6} = \frac{-1-\sqrt{3}}{2} < 0$

function has local maxima at x = $\frac{\pi}{6}$