Solution

CONTINUITY AND DIFFERENTIABILITY WS 1

Class 12 - Mathematics

1.

(d) $\frac{1}{2}$

Explanation: $\frac{1}{2}$

2.

(c) $\frac{1}{\sqrt{x^2+a^2}}$

Explanation: Given that y $= \log_e(x+\sqrt{x^2+a^2})$

Differentiating with respect to x, we obtain

$\frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+a^2}}\left(1+\frac{1}{2\sqrt{x^2+a^2}}\times 2x\right)$

$\frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+a^2}} \times \frac{x+\sqrt{x^2+a^2}}{\sqrt{x^2+a^2}} = \frac{1}{\sqrt{x^2+a^2}}$

3.

(b) is equal to 0

Explanation: $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-} e^{\frac{1}{x}} = 0, \quad \lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} x = 0 \therefore \lim\limits_{x\to 0} f(x) = 0$

4.

(d) log (ex)

Explanation: Given, y = x log x

On differentiating both sides w.r.t. x, we get

$\frac{dy}{dx}$ = $\frac{x}{x}$ + log x

$\Rightarrow$ $\frac{dy}{dx}$ = log e + log x $\Rightarrow$ $\frac{dy}{dx}$ = log (ex)

5.

(d) 1

Explanation: Here, given

$\Rightarrow f(x) = \frac{1-\cos 4x}{8x^2}$ is continuous at x = 0

$\Rightarrow f(x) = \lim_{x\to 0} \frac{2\sin^2 2x}{8x^2}$

$\Rightarrow f(x) = \lim_{x\to 0} \frac{2\sin^2 2x}{2\times 4x^2}$

$\Rightarrow f(x) = \lim_{x\to 0} \left(\frac{\sin 2x}{2x}\right)^2$

$\Rightarrow$ f(x) = 1

$\therefore$ k = 1

6.

(b) 3

Explanation: f(x) = $\frac{1}{\log|x|}$

f(x) is not defined for x = 0, -1 ,1

$\therefore$ f(x) is not continuous at x = 0, -1, 1

7. **(a)** has oscillating discontinuity

Explanation: f(x) = $\begin{cases} \sin\frac{1}{x'} & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$

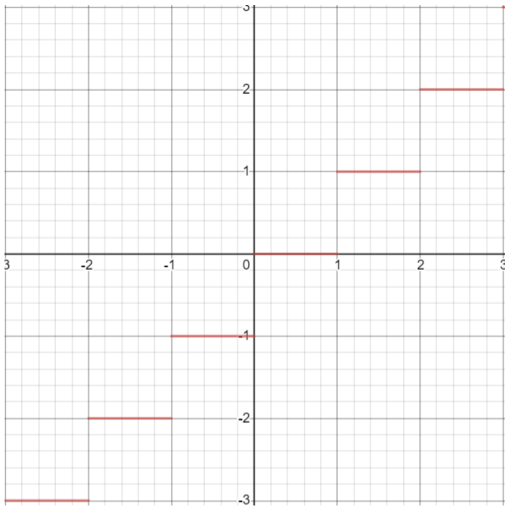
$\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0} \sin\left(\frac{1}{x}\right)$ does not exist.

But value of sin $\left(\frac{1}{x}\right)$ oscillates between -1 and 1.

8.

(b) None of these

Explanation: Let us see that graph of the floor function, we get



We can see that f(x) = [x] is neither continuous and non differentiable at x = 2.

9.

(d) discontinuous at exactly three points

Explanation: We have, $f(x)=\frac{4-x^2}{4x-x^3}=\frac{\left(4-x^2\right)}{x\left(4-x^2\right)}$

$=\frac{\left(4-x^2\right)}{x\left(2^2-x^2\right)}=\frac{4-x^2}{x(2+x)(2-x)}$

Clearly, f(x) is discontinuous at exactly three points x = 0, x = -2 and x = 2.

10.

(c) $\frac{\log x}{(1+\log x)^2}$

Explanation: $x^y = e^{x-y}$

Taking log on both sides,

$\log x^y = \log e^{x-y}$

y log x = x - y

y log x + y = x

$y=\frac{x}{\log x+1}$

Differentiate with respect to x,

$\frac{dy}{dx}=\frac{(\log x+1)-x\times\frac{1}{x}}{(\log x+1)^2}$

$\frac{dy}{dx}=\frac{(\log x+1)-1}{(\log x+1)^2}$

$\frac{dy}{dx}=\frac{\log x}{(\log x+1)^2}$

11.

(b) f(x) is continuous at x = 0 and at x = 2

Explanation: f(x) is continuous at x = 0 and at x = 2

12.

(d) f is everywhere continuous but not differentiable at $x=n\pi, n\in\mathbf{Z}$

Explanation: We have, $f(x)=|\sin x|$

Let f(x) = vou(x) = v[u(x)] [where, u(x) = sin x and v(x) = |x|]

= v(sin x) = |sin x|

Where, u(x) and v(x) are both continuous.

Hence, f(x) = vou(x) is also a continuous function but v(x) is not differentiable at x = 0

So, f(x) is not differentiable where sin x = 0 $\Rightarrow$ x = nπ, n $\in$ Z

Hence, f(x) is continuous everywhere but not differentiable at x = nπ, n $\in$ Z

13.

(d) continuous at x = 0

Explanation: Given f(x) = $\sin^{-1}$(cos x),

Checking differentiability and continuity,

LHL at x = 0,

$\lim\limits_{\mathbf{x}\to 0^-}\mathbf{f}(\mathbf{x})=\lim\limits_{\mathbf{h}\to 0}\mathbf{f}(0-\mathbf{h})=\lim\limits_{\mathbf{h}\to 0}\sin^{-1}(\cos(0-\mathbf{h}))=\lim\limits_{\mathbf{h}\to 0}\sin^{-1}(\cos(-\mathbf{h}))=\sin^{-1}1=\frac{\pi}{2}$

RHL at x =0,

$\lim\limits_{\mathbf{x}\to 0^+}\mathbf{f}(\mathbf{x})=\lim\limits_{\mathbf{h}\to 0}\mathbf{f}(0+\mathbf{h})=\lim\limits_{\mathbf{h}\to 0}\sin^{-1}(\cos(0+\mathbf{h}))=\lim\limits_{\mathbf{h}\to 0}\sin^{-1}(\cos(\mathbf{h}))=\sin^{-1}1=\frac{\pi}{2}$

And $f(0)=\frac{\pi}{2}$

Hence, f(x) is continuous at x =0.

LHD at x = 0,

$\lim\limits_{x\to 0^-}\frac{f(x)-f(0)}{x-0}=\lim\limits_{h\to 0}\frac{f(0-h)-f(0)}{0-h-0}$

$=\lim\limits_{h\to 0}\frac{\sin^{-1}(\cos(0-h))-\left(\frac{\pi}{2}\right)}{-h}=1$

RHD at x =0,

$\lim\limits_{x\to 0^+}\frac{f(x)-f(0)}{x-0}=\lim\limits_{h\to 0}\frac{f(0+h)-f(0)}{0+h-0}$

$=\lim\limits_{h\to 0}\frac{\sin^{-1}(\cos(0+h))-\left(\frac{\pi}{2}\right)}{h}=-1$

$\because$ LHD $\neq$ RHD

$\therefore$ f(x) is not differentiable at x =0.

14.

(b) 7

Explanation: $\Rightarrow f(x)=\frac{3x+4\tan x}{x}$ is continuous at x = 0

$\Rightarrow f(x)=\lim\limits_{x\to 0}\frac{3x+4\tan x}{x}$

$\Rightarrow f(x)=\lim\limits_{x\to 0}\frac{3x}{x}+\frac{4\tan x}{x}$

$\Rightarrow f(x)=3+4\lim\limits_{x\to 0}\frac{\tan x}{x}$

$\Rightarrow$ f(x) = 3 + 4

$\therefore$ k = 7

15.

(b) 6

Explanation: Since, f(x) is continuous at x = $\frac{\pi}{2}$.

Therefore, $\lim\limits_{x\to\frac{\pi}{2}}f(x)=f\left(\frac{\pi}{2}\right)$

$=\lim\limits_{x\to\frac{\pi}{2}}\frac{k\cos x}{\pi-2x}$

$\Rightarrow k\lim\limits_{x\to\frac{\pi}{2}}\frac{\sin\left(\frac{\pi}{2}-x\right)}{2\left(\frac{\pi}{2}-x\right)}=3$

$\Rightarrow \frac{k}{2}\lim\limits_{x\to\frac{\pi}{2}}\frac{\sin\left(\frac{\pi}{2}-x\right)}{\left(\frac{\pi}{2}-x\right)}=3$

$\Rightarrow \frac{k}{2}\times 1=3\Rightarrow k=6$

16.

(d) f(x) is not continuous at x = 1 and x = 2

Explanation: Since, $0\leq x<1\Rightarrow -1\leq x-1<0\Rightarrow [x-1]=-1$

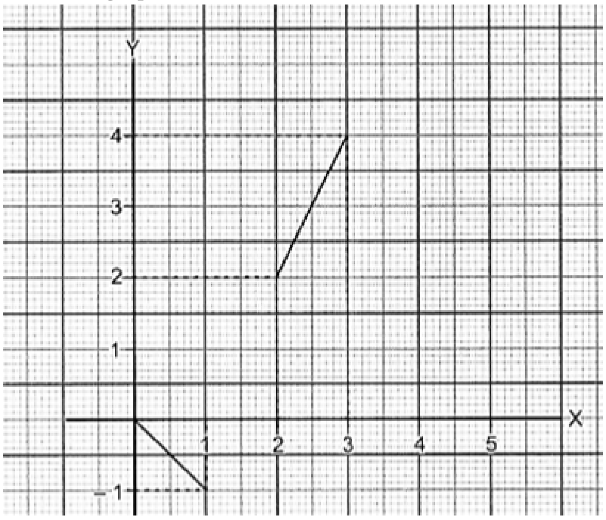
$1\leq x<2\Rightarrow 0\leq x-1<1\Rightarrow [x-1]=0$

And $2\leq x<3\Rightarrow [x]=2$

Therefore, given function may be written as

f(x) = $\begin{cases}-x, & 0\leq x<1\\ 0, & 1\leq x<2\\ 2(x-1), & 2\leq x<3\end{cases}$

From the graph it is obvious that f(x) is not continuous at x = 1 and 2, and thus not differentiable.



17.

(c) a = - 1, b = -1

Explanation: f(x) is continuous at x = 1

$\Rightarrow \lim_{x \to 1} f(x) = f(1)$

$\Rightarrow \lim_{h \to 0} f(1+h) = a$

$\Rightarrow \lim_{h \to 0} \frac{(1+h)^2}{a} = a$

$\Rightarrow \frac{1}{a} = a$

$\Rightarrow$ a^2 = 1

$\Rightarrow a = \pm 1$

Consider,

$\lim_{x \to \sqrt{2}} \frac{2b^2 - 4b}{x^2} = a$

b^2 - 2b = ± 1

for a = 1

using formulas for quadratic equation

$b^2 - 2b - 1 = 0 \Rightarrow b = 1 \pm \sqrt{2}$

for a = -1

b^2 - 2b = -1

b^2 - 2b + 1 = 0

$(b - 1)^2$ = 0

b = 1

a = -1, b = 1

18.

(d) $\frac{2}{\sqrt{1+x^2}}$

Explanation: Given that $y = \log_e \left(\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}-x} \right)$

Differentiating with respect to x, we obtain

$\Rightarrow \frac{dy}{dx} = \frac{\sqrt{1+x^2}-x}{\sqrt{1+x^2}+x} \times \frac{(\sqrt{1+x^2}-x) \times \left(\frac{1}{2\sqrt{1+x^2}} \times 2x + 1 \right) - (\sqrt{1+x^2}+x) \times \left(\frac{1}{2\sqrt{1+x^2}} \times 2x - 1 \right)}{(\sqrt{1+x^2}-x)^2}$

Hence, $\frac{dy}{dx} = \frac{2}{\sqrt{1+x^2}}$

19. **(a)** (1 + sin 2x) y_1

Explanation: y = $e^{\tan x}$

$y_1 = \sec^2 x\, e^{\tan x}$

$\Rightarrow \cos^2 x y_1 = e^{\tan x}$

Again differentiating w.r.t. x we get

$\cos^2(x).y_2 - 2\cos x \sin x y_1 = \sec^2 x e^{\tan x}$

$\Rightarrow \cos^2(x).y_2 = y_1 \sin 2x + y_1.$

20. **(a)** $\frac{2}{1+x^2}$

Explanation: Given, y = $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

On differentiating both sides w.r.t. x, we get

$\frac{dy}{dx} = \frac{1}{\sqrt{1-\left(\frac{2x}{1+x^2}\right)^2}}\frac{d}{dx}\left(\frac{2x}{1+x^2}\right)$

$= \frac{1+x^2}{\sqrt{(1+x^2)^2-4x^2}}\left[\frac{(1+x^2)(2)-2x(2x)}{(1+x^2)^2}\right]$

$= \frac{1+x^2}{\sqrt{1+x^4+2x^2-4x^2}}\left[\frac{2-2x^2}{(1+x^2)^2}\right]$

$= \frac{2(1+x^2)(1-x^2)}{(1-x^2)(1+x^2)^2} = \frac{2}{1+x^2}$

21.

(b) f(x) is continuous for all x in its domain but not differentiable at x = ± 1

Explanation: Here,the given function is f(x) = |log|x|| where

$$|\mathrm{x}| = \begin{cases} -\mathrm{x}, -\infty < \mathrm{x} < -1 \\ -\mathrm{x}, -1 < \mathrm{x} < 0 \\ \mathrm{x}, 0 < \mathrm{x} < 1 \\ \mathrm{x}, 1 < \mathrm{x} < \infty \end{cases}$$

$$\log|\mathrm{x}| = \begin{cases} \log(-\mathrm{x}), -\infty < \mathrm{x} < -1 \\ \log(-\mathrm{x}), -1 < \mathrm{x} < 0 \\ \log \mathrm{x}, 0 < \mathrm{x} < 1 \\ \log \mathrm{x}, 1 < \mathrm{x} < \infty \end{cases}$$

$$|\log|\mathrm{x}|| = \begin{cases} \log(-\mathrm{x}), -\infty < \mathrm{x} < -1 \\ -\log(-\mathrm{x}), -1 < \mathrm{x} < 0 \\ -\log \mathrm{x}, 0 < \mathrm{x} < 1 \\ \log \mathrm{x}, 1 < \mathrm{x} < \infty \end{cases}$$

We can see that function is continuous for all x. Now, checking the points of non differentiability.

Now,L.H.D at x =1,we get

$\lim_{x\to 1^-}\frac{f(x)-f(1)}{x-1} = \lim_{h\to 0}\frac{f(1-h)-f(1)}{1-h-1}$

$= \lim_{\mathrm{h}\to 0}\frac{\log(1-\mathrm{h})-\log 1}{-\mathrm{h}} = -1$

RHD at x =1,

$\lim_{x\to 1^+}\frac{f(x)-f(1)}{x-1} = \lim_{h\to 0}\frac{f(1+h)-f(1)}{1+h-1}$

$= \lim_{h\to 0}\frac{\log(1+h)-\log 1}{h} = 1$

$\because \mathrm{L.H.D} \neq \mathrm{R.H.D}$

Thus, function is not differentiable at x =1.

L.H.D at x =-1,

$\lim_{x\to -1^-}\frac{f(x)-f(-1)}{x-(-1)} = \lim_{h\to 0}\frac{f(-1-h)-f(-1)}{-1-h-(-1)}$

$= \lim_{h\to 0}\frac{\log(-1-h)-\log(-1)}{-h} = -1$

R.H.D at x =-1,

$\lim_{x\to -1^+}\frac{f(x)-f(-1)}{x-1} = \lim_{h\to 0}\frac{f(-1+h)-f(-1)}{(-1)+h-(-1)}$

$= \lim_{h\to 0}\frac{\log(-1+h)-\log(-1)}{h} = 1$

$\because \mathrm{L.H.D} \neq \mathrm{R.H.D}$

So, function is not differentiable at x =-1.

At x =0 function is not defined.

$\therefore$ Function is not differential at x = 0 and ±1.

22.

(d) $a=\log_e\left(\frac{2}{3}\right), b=\frac{2}{3}, c=1$

Explanation: $f(0)=\lim\limits_{x\to 0}(1+ax)^{\frac{1}{x}}$

$b=\lim\limits_{x\to 0}(1+ax)^{\frac{1}{ax}}$

b = e^a

a = $\log_e$ b

$f(0)=\lim\limits_{x\to 0^+}\frac{(x+c)^{1/3}-1}{(x+1)^{1/2}-1}$

Here, c = 1

x + 1 = y

$x\to 0\Rightarrow y\to 1$

$f(0)=\lim\limits_{y\to 1}\frac{y^{1/3}-1}{y^{1/2}-1}$

$b=\lim\limits_{y\to 1}\frac{\frac{y^{1/3}-1}{y-1}}{\frac{y^{1/2}-1}{y-1}}=\frac{\frac{1}{3}}{\frac{1}{2}}=\frac{2}{3}$

a = log b = log $\frac{2}{3}$

23.

(d) $(\sin x)^{\log x}\cdot\left\{\frac{x\cot x\log x+\log\sin x}{x}\right\}$

Explanation: Given that y = (sinx) $^{\log_e x}$

Taking log both sides, we obtain

$\log_e$ y = $\log_e$ x $\times$ $\log_e$ sinx (Since $\log_a b^c$ = c $\log_a$ b)

Differentiating with respect to x, we obtain

$\frac{1}{y}\frac{dy}{dx}=\log_e x\times\frac{1}{\sin x}\times\cos x+\log_e\sin x\times\frac{1}{x}$

$=\frac{x\cot x\log_e x+\log_e\sin x}{x}$

Therefore $\frac{dy}{dx}=\frac{x\cot x\log_e x+\log_e\sin x}{x}\times y$

$=\frac{x\cot x\log_e x+\log_e\sin x}{x}(\sin x)^{\log_e x}$

24. **(a)** continuous

Explanation: We have, $f(x)=\begin{cases}\frac{x^2-x-6}{x+2} & \text{if } x\neq -2\\ -5, & \text{if } x=-2\end{cases}$

At x = -2,,

$\lim\limits_{x\to -2}f(x)$ = $\lim\limits_{x\to -2}\frac{x^2-x-6}{x+2}$ ...(i)

Now, factorising the numerator, we get

$\lim\limits_{x\to -2}f(x)$ = $\lim\limits_{x\to -2}\frac{(x-3)(x+2)}{(x+2)}$= $\lim\limits_{x\to -2}(x-3)$= -2 - 3 = -5

Also, at x = -2,

f(x) =-5 i.e.f(-2)= -5

Thus, $\lim\limits_{x\to -2}$ f(x) = f(-2)

Hence, f(x) is continuous at x = -2.

25.

(b) 0

Explanation: 0

f(x) = $\begin{cases}x^2, & x\geq 0\\ kx, & x<0\end{cases}$

Given function is differentiable

LHD=RHD

$\lim\limits_{h\to 0}\frac{f(0-h)-f(0)}{-h}$ = \lim\limits_ {h\to 0}}\frac{f(0+h)-f(0)}{h}

$\lim_{h \to 0} \frac{k(0-h)-k(0)}{-h}$ = $\lim_{h \to 0} \frac{(0+h)^2-(0)^2}{h}$

$\lim_{h \to 0} \frac{k(-h)}{-h}$ = $\lim_{h \to 0} \frac{(h)^2-(0)^2}{h}$

k = 0

26. **(a)** continuous at R

Explanation: We have, f(x) = $\begin{cases} -2, & \text{if } x \leq -1 \\ 2x, & \text{if } -1 < x < \infty \end{cases}$

Since, f(x) is a polynomial function, so it is continuous everywhere except at x = -1. So, we have to check the continuity at x = -1 only.

At x = -1 , f(-1) = -2

LHL = $\lim_{x \to -1^-}$ f(x) = $\lim_{x \to -1^-}$ (-2) = -2

RHL = $\lim_{x \to -1^+}$ 2x = $\lim_{h \to 0}$ 2 (-1 + h)

[put x = -1 + h; when x $\rightarrow$ -1^+, then h $\rightarrow$ 0]

= $\lim_{h \to 0}$(-2 + 2h) = -2

Thus, f(-1) = LHL = RHL

$\therefore$ f(x) is continuous at x = -1.

Hence, f(x) is continuous for every value of x.

27.

(d) $x\frac{d^2y}{dx^2} = y_1$

Explanation: y = a + bx^2

$\frac{dy}{dx}$ = 2bx

$\frac{d^2y}{dx^2}$ = 2b

$\Rightarrow \frac{d^2y}{dx^2} = \frac{1}{x}\frac{dy}{dx}$

$\Rightarrow x\frac{d^2y}{dx^2} = \frac{dy}{dx}$

28. **(a)** $\frac{y}{(1-y)}$

Explanation: We can write it as

$\Rightarrow y = e^{x+y}$

log y = (x + y) log e

Differentiating with respect to x,we get

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = 1 + \frac{dy}{dx}$

$\Rightarrow \left(\frac{1}{y} - 1\right)\frac{dy}{dx} = 1$

$\Rightarrow \frac{dy}{dx} = 1\left(\frac{y}{1-y}\right)$

29.

(d) 1, 3

Explanation: Here, f(x) = $\begin{cases} 3, & \text{if } 0 \leq x \leq 1 \\ 4, & \text{if } 1 < x < 3 \\ 5, & \text{if } 3 \leq x \leq 10 \end{cases}$

For 0 $\leq$ x $\leq$ 1, f(x) = 3; 1 < x < 3 ; f(x) = 4 and 3 $\leq$ x $\leq$ 10, f(x) = 5 are constant functions, so it is continuous in the given interval, so we have to check the continuity at x = 1, 3.

At x = 1, LHL = $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-}(3)$ = 3,

RHL = $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+}(4)$ = 4

$\therefore$ LHL $\neq$ RHL

Thus, f(x) is discontinuous at x = 1.

At x = 3, LHL = $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-}(4)$ = 4,

RHL = $\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+}(5)$ = 5

$\therefore$ LHL $\neq$ RHL.

Thus, f(x) is discontinuous at x = 3.

Hence, f (x) is continuous everywhere except at x = 1, 3.

30.

(c) -4

Explanation: We have,

$\Rightarrow f(x)=\frac{x^2-2x-3}{x+1}$ is continuous at x = 0.

$\Rightarrow f(x)=\lim_{x\to-1}\frac{(x+1)(x-3)}{x+1}$

$\Rightarrow f(x)=\lim_{x\to-1}x-3$

$\Rightarrow$ f(x) = -4

$\therefore$ k = -4

31. **(a)** continuous at x = π and not differentiable at x = π

Explanation: continuous at x = π and not differentiable at x = π

32.

(d) 2

Explanation: Since the given function is continuous,

$\therefore$ k = $\lim\limits_{x\to0}\frac{Sinx}{x}+Cosx$

$\Rightarrow$ k = 1 + 1 = 2

33. **(a)** $-e^x \tan(e^x)$

Explanation: Let y = log (cos e^x)

On differentiating both sides w.r.t. x, we get

$\frac{dy}{dx}=\frac{d}{dx}[\log\{\cos(e^x)\}]$

$=\frac{1}{\cos(e^x)}\frac{d}{dx}\{\cos(e^x)\}$ [using chain rule]

$=\frac{1}{\cos(e^x)}\{-\sin(e^x)\}\frac{d}{dx}(e^x)$[using chain rule]

$= -\tan(e^x)\cdot e^x = -e^x \tan(e^x)$

34. **(a)** No value

Explanation: No value

35.

(c) $\frac{b}{a}cosec\theta$

Explanation: x = a secθ ,we get

$\therefore \frac{dx}{d\theta}=\text{a}\sec\theta\cdot\tan\theta$

$\therefore \frac{d\theta}{dx}=\frac{1}{\text{a}\sec\theta\cdot\tan\theta}$

y = b tan θ ,we get

$\therefore \frac{dy}{d\theta}=b\cdot\sec^2\theta$

$\Rightarrow \frac{dy}{dx}=\frac{dy}{d\theta}\times\frac{d\theta}{dx}$

$\Rightarrow \frac{dy}{dx}=b\cdot\sec^2\theta\times\frac{1}{\text{a}\sec\theta\cdot\tan\theta}$

$\Rightarrow \frac{dy}{dx}=\frac{b\sec\theta}{\text{a}\tan\theta}$

$\Rightarrow \frac{\text{dy}}{\text{dx}}=\frac{\text{b}\cdot\frac{1}{\cos\theta}}{\text{a}\cdot\frac{\sin\theta}{\cos\theta}}$

$\Rightarrow \frac{dy}{dx}=\frac{b}{a}cosec\theta$

36.

(c) { $\frac{n\pi}{4}$: n = 0, $\pm$ 1, $\pm$ 2, ...}

Explanation: f(x) = sec 2x + cosec 2x = $\frac{1}{\cos 2x}+\frac{1}{\sin 2x}$

cos 2x = 0 $\Rightarrow$ 2x = (2n + 1) $\frac{\pi}{2}$ $\Rightarrow$ x = (2n + 1) $\frac{\pi}{4}$ where n = 0, $\pm$ 1, $\pm$ 2, ...

And sin 2x = 0 $\Rightarrow$ 2x = n π $\Rightarrow x=\frac{n\pi}{2}$ where n = 0, $\pm$ 1, $\pm$ 2, $\pm$ 3, ...

None of the options (a), (b) and (c) are correct.

37.

(d) 1/8

Explanation: If f(x) is continuous at $x = \frac{\pi}{2}$ then

$\lim_{x \to \frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$

$\lim_{x \to \frac{\pi}{2}} \frac{1-\sin x}{(\pi-2x)^2} = f\left(\frac{\pi}{2}\right)$ ...(i)

Now lets suppose

$\left(\frac{\pi}{2} - x\right)$ = t, then limit becomes

$\lim_{t \to 0}\left[\frac{1-\sin\left(\frac{\pi}{2}-t\right)}{(2t)^2}\right] = f\left(\frac{\pi}{2}\right)$ [from equation (i)]

$\Rightarrow \lim_{t \to 0}\left[\frac{1-\cos t}{4t^2}\right] = f\left(\frac{\pi}{2}\right)$

$\Rightarrow \frac{1}{4}\lim_{t \to 0}\left[\frac{2\sin^2\left(\frac{t}{2}\right)}{t^2}\right] = f\left(\frac{\pi}{2}\right)$

$\Rightarrow \frac{1}{4}\lim_{t \to 0}\left[\frac{\frac{2}{4}\sin^2\left(\frac{t}{2}\right)}{\frac{t^2}{4}}\right] = f\left(\frac{\pi}{2}\right)$

$\Rightarrow \frac{1}{8}\lim_{t \to 0}\left[\frac{\sin^2\left(\frac{t}{2}\right)}{\frac{t^2}{4}}\right] = f\left(\frac{\pi}{2}\right)$

$\Rightarrow \frac{1}{8}\lim_{t \to 0}\left[\frac{\sin\left(\frac{t}{2}\right)}{\frac{t}{2}}\right]^2 = f\left(\frac{\pi}{2}\right)$

$\Rightarrow f\left(\frac{\pi}{2}\right) = \lambda = \frac{1}{8}$

38. **(a)** continuous everywhere but not differentiable at x = 0

Explanation: Given that f(x) = $e^{-|x|}$

$\Rightarrow \mathrm{f(x)} = \begin{cases} \mathrm{e^x, x < 0} \\ \mathrm{1, x = 0} \\ \mathrm{e^{-x}, x > 0} \end{cases}$

Checking continuity and differentiability at x = 0,

LHL:

$\lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \mathrm{lim} \mathrm{e}^{-h}_{h \to 0} = 1$

RHL:

$\lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} e^{-h} = 1$

And f(0)= 1

∴ LHL = RHL = f(0)

f(x) is continuous at x = 0.

LHD at x =0,

$\lim_{x \to 0^-} \frac{f(x)-f(0)}{x-0} = \lim_{h \to 0} \frac{f(0-h)-f(0)}{0-h-0}$

$= \lim_{h \to 0} \frac{e^{-h}-(0)}{-h} = \infty$

∴LHD does not exist, so f(x) is not differentiable at x = 0

39. **(a)** $\frac{1}{(1+x^2)}$

Explanation: Given that $y = \cot^{-1}\left(\frac{1-x}{1+x}\right)$

Let x = $\tan\theta \Rightarrow \theta = \tan^{-1} x$ and using $\cot^{-1} x = \frac{\pi}{2} - \tan^{-1} x$

Hence, $y = \frac{\pi}{2} - \tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)$

Using $\tan\left(\frac{\pi}{4} - x\right) = \frac{1-\tan x}{1+\tan x}$, we obtain

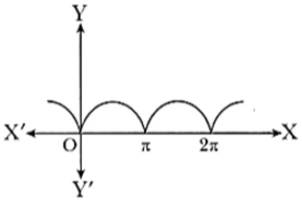
$y = \frac{\pi}{2} - \tan^{-1}\tan\left(\frac{\pi}{4} - \theta\right) = \frac{\pi}{2} - \left(\frac{\pi}{4} - \theta\right) = \frac{\pi}{4} + \theta = \frac{\pi}{4} + \tan^{-1} x$

Differentiating with respect to x, we obtain

$\frac{dy}{dx} = \frac{1}{1+x^2}$

40.

(c) f(x) is non-differentiable at 3 points and continuous everywhere

Explanation: 

It is clear from graph that f(x) is continuous everywhere in $0 \leq x \leq 2\pi$. And has sharp edge at x = 0, π, 2π so it is not differentiable at x = 0, π, 2π.

Because it has no unique tangents.

41.

(c) f(x) is right continuous

Explanation: LHL = $\lim_{x \to 0^-}$ f(x) = $\lim_{x \to 0^-}$ (1 - 3x) = 1

RHL = $\lim_{x \to 0^+}$ f(x) = $\lim_{x \to 0^+}$ (x^2 + 3) = 3

f(0) = 3, LHL $\neq$ RHL = f(0)

$\Rightarrow$ f(x) is right continuous but discontinuous from left.

42.

(d) $a = \frac{1}{3}, b = \frac{8}{3}$

Explanation: Let f(x) be continuous at x = $\frac{\pi}{2}$

Then

LHL = $\lim_{x \to (\frac{\pi}{2})^-} \frac{1-\sin^2 x}{3\cos^2 x}$

$\Rightarrow$ a = $\lim_{x \to (\frac{\pi}{2})^-} \frac{\cos^2 x}{3\cos^2 x}$ = $\frac{1}{3}$

$\Rightarrow$ a = $\frac{1}{3}$

Again,

RHL = $\lim_{x \to (\frac{\pi}{2})^+} \frac{b(1-\sin x)}{(\pi-2x)^2}$

$\Rightarrow$ a = $\lim_{h \to 0} \frac{b(1-\sin(\frac{\pi}{2}+h))}{[\pi-2(\frac{\pi}{2}+h)]^2}$

= b $\lim_{h \to 0} \frac{(1-\cos\ h)}{[\pi-\pi-2h]^2}$

= b $\lim_{h \to 0} \frac{(1-\cos\ h)}{4h^2}$

= b $\lim_{h \to 0} \frac{2\sin^2 \frac{h}{2}}{\frac{h^2}{4} \times 4^2}$

$\Rightarrow$ a = $\frac{2b}{16}$

$\Rightarrow$ $\frac{1}{3}$ = $\frac{b}{8}$

$\Rightarrow$ b = $\frac{8}{3}$

43.

(b) $\frac{1-x}{x}$

Explanation: Given expression is

x = $e^{y+e^{y}+\ldots to \infty}$ $\Rightarrow$ x = e^{y+x}

Taking log on both sides, we have

log x = log e^{y+x} $\Rightarrow$ log x = y + x

Differentiating, w.r.t. x, we get

$\frac{1}{x} = \frac{dy}{dx} + 1 \Rightarrow \frac{dy}{dx} = \frac{1}{x}$ - 1

$\Rightarrow \frac{dy}{dx} = \frac{1-x}{x}$

44.

(b) $n = \frac{m\pi}{2}$

Explanation: We have, $f(x) = \begin{cases} mx+1, & \text{if } x \leq \frac{\pi}{2} \\ \sin x + n, & \text{if } x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$

$\therefore LHL = \lim\limits_{x \to \frac{\pi}{2}} (mx + 1) = \lim\limits_{h \to 0} \left[m\left(\frac{\pi}{2} - h\right) + 1\right] = \frac{m\pi}{2} + 1$

and $RHL = \lim\limits_{x \to \frac{\pi'}{2}} (\sin x + n) = \lim\limits_{h \to \infty} \left[\sin\left(\frac{\pi}{2} + h\right) + n\right]$

$= \lim\limits_{n \to 0} \cos h + n = 1 + n$

Since the function is continuous, we have

$LHL = RHL$

$\Rightarrow m \cdot \frac{\pi}{2} + 1 = n + 1$

$\therefore\ n = m \cdot \frac{\pi}{2}$

45. **(a)** -sec$^2\left(\frac{\pi}{4} - \text{x}\right)$

Explanation: -sec$^2\left(\frac{\pi}{4} - \text{x}\right)$

y = $\frac{\cos \text{x} - \sin \text{x}}{\cos \text{x} + \sin \text{x}}$

y = $\frac{cosx(1 - \frac{sinx}{cosx})}{cosx(1 + \frac{sinx}{cosx})}$

y = $\frac{1 - tanx}{1 + tanx}$

y = $\frac{tan\frac{\pi}{4} - tanx}{1 + tan\frac{\pi}{4}tanx}$

y = tan ($\frac{\pi}{4}$-x)

$\frac{dy}{dx} = -sec^2(\frac{\pi}{4} - x)$

46. **(a)** $3x^2e^{x^3}$

Explanation: Let y = e^{x^3}

On differentiating both sides w.r.t. x, we get

$\frac{dy}{dx}$ = $e^{x^3}\frac{d}{dx}\left(x^3\right)$[by chain rule of derivative]

= $e^{x^3} \cdot$ 3x^2 = 3x^2 e^{x^3}

47.

(b) f ‘(x) exists for all x

Explanation: Since $[x - \pi]$ is an integer for all $x \in R$ & $\tan n\pi\ = 0\ \forall\ n\ \in I$ .Therefore, f(x)= 0 for all x in R. So, f(x) is a constant and hence derivatives of f(x) of all order exist.

48.

(b) 3 (xy_2 + y_1) y_2

Explanation: $y = \frac{ax+b}{x^2+c}$

$\Rightarrow$ y(x^2 + c) = ax + b

Differentiating both sides w.r.t. x we get

y_1 (x^2 + c) + 2xy = a

Again differentiating w.r.t to x we get

y_2 (x^2 +c) + 2xy_1 + 2y + 2xy_1 = 0

$\Rightarrow$ y_2 (x^2 + c) = -(4xy_1 + 2y) ...(i)

Again differentiating w.r.t to x we get

y_3 (x^2 + c) + 2xy_2 + 4xy_2 + 4y_1 + 2y_1 = 0

$\Rightarrow$ y_3 (x^2 + c) = -(6xy_2 + 6y_1) ...(ii)

$\frac{y_2}{y_3} = \frac{2xy_1+y}{3xy_2+3y_1}$

$\Rightarrow$ y_3 (2xy_1 + y) = 3y_2 (xy_2 + y_1)

49. **(a)** $\frac{11}{4}$

Explanation: $\frac{11}{4}$

50.

(c) -m^{2}y

Explanation: $y=a\sin mx+b\cos mx \Rightarrow y_1 \quad = am\cos mx - bm\sin mx$
$\Rightarrow y_2=-am^2\sin mx-bm^2\cos mx$
$\Rightarrow y_2=-m^2(a\sin mx+b\cos mx)=-m^2y$

51.

(b) $\frac{y}{x}$

Explanation: We have, $\sin^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right)=\log a$

$\Rightarrow \frac{x^2-y^2}{x^2+y^2}=\sin\log a$

$\Rightarrow \frac{(x^2+y^2)\left(2x-2y\frac{dy}{dx}\right)-(x^2-y^2)\left(2x+2y\frac{dy}{dx}\right)}{(x^2+y^2)^2}=0$

$\Rightarrow \frac{2x^3-2x^2y\frac{dy}{dx}+2xy^2-2y^3\frac{dy}{dx}-2x^3-2x^2y\frac{dy}{dx}+2xy^2+2y^3\frac{dy}{dx}}{(x^2+y^2)^2}=0$

$\Rightarrow -4x^2y\frac{dy}{dx}+4xy^2=0$

$\Rightarrow -4x^2y\frac{dy}{dx}=-4xy^2$

$\Rightarrow \frac{dy}{dx}=\frac{4xy^2}{4x^2y}$

$\therefore \frac{dy}{dx}=\frac{y}{x}$.

Which is the required solution.

52.

(b) 1.5

Explanation: [x] is always continuous at non-integer value of x. Hence, f(x) = [x] will be continuous at x = 1.5.

53.

(c) $\frac{y(1-x)}{x(y-1)}$

Explanation: Given that xy = e^{x+y}

Taking log both sides, we get

$\log_e xy = x + y$ (Since $\log_a b^c = c\log_a b$)

Since $\log_a bc = \log_a b + \log_a c$, we get

$\log_e x + \log_e y = x + y$

Differentiating with respect to x, we obtain

$\frac{1}{x}+\frac{1}{y}\frac{dy}{dx}=1+\frac{dy}{dx}$

Or

$\frac{dy}{dx}\left(\frac{y-1}{y}\right)=\frac{1-x}{x}$

Therefore, $\frac{dy}{dx}=\frac{y(1-x)}{x(y-1)}$

54.

(d) a^2

Explanation: Given,F(x) is continuous at x = 0.

$\Rightarrow f(x)=\lim_{x\to 0}\frac{\sin^2 ax}{x^2}$

$\Rightarrow f(x)=\lim_{x\to 0}\frac{\sin^2 ax}{x^2}\times\frac{a^2}{a^2}$

$\Rightarrow f(x)=\lim_{x\to 0}\left(\frac{\sin ax}{ax}\right)^2\times a^2$

$\Rightarrow$ f(x) = a^2

$\therefore$ k = a^2

55. **(a)** $\frac{1}{2a}$

Explanation: $\sqrt{x}+\sqrt{y}=\sqrt{a}\ldots\ldots.(1)$

$\Rightarrow \frac{1}{2\sqrt{x}}+\frac{1}{2\sqrt{y}}\frac{dy}{dx}=0$

$\Rightarrow \frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}} \ldots\ldots (2)$

$\Rightarrow \frac{d^2y}{dx^2} = -\frac{\sqrt{x}\frac{1}{2}y^{-\frac{1}{2}}\frac{dy}{dx} - \sqrt{y}\frac{1}{2}x^{-\frac{1}{2}}}{x}$

$= \frac{-\left(\frac{\sqrt{x}}{2\sqrt{y}}\left(-\frac{\sqrt{y}}{\sqrt{x}}\right) - \frac{\sqrt{y}}{2\sqrt{x}}\right)}{x}$

$= \frac{\sqrt{x}+\sqrt{y}}{2x\sqrt{x}} = \frac{\sqrt{a}}{2x\sqrt{x}.} = \frac{\sqrt{a}}{2a\sqrt{a}} = \frac{1}{2a}$

56.

(c) -2

Explanation: At x = 0,

LHL = $\lim_{x \to 0^-} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x} \times \frac{\sqrt{1+kx}+\sqrt{1-kx}}{\sqrt{1+kx}+\sqrt{1-kx}}$

= $\lim_{x \to 0^-} \frac{2kx}{x(\sqrt{1+kx}+\sqrt{1-kx})}$

= $\frac{2k}{\sqrt{1+0}+\sqrt{1-0}}$ = k

RHL = $\lim_{x \to 0^+}$ $(2x^2 + 3x - 2) = -2$

f(0) = -2

∵ It is given that f(x) is continuous at x = 0.

∴ LHL = RHL = f(0) ⇒ k = -2

57.

(c) n^2y

Explanation: $y^{1/n} + y^{-1/n} = 2x$

Differentiating both sides we get

$\frac{y_1}{n}\left(y^{\frac{1}{n}-1} - y^{\frac{-1}{n}-1}\right) = 2$

$\Rightarrow y_1\left(y^{\frac{1}{n}} - y^{\frac{-1}{n}}\right) = 2ny$

Again differentiating both sides we get

$y_2\left(y^{\frac{1}{n}} - y^{\frac{-1}{n}}\right) + \frac{y_1}{n}\left(y^{\frac{1}{n}-1} + y^{\frac{-1}{n}-1}\right) = 2ny_1$

$\Rightarrow ny_2\left(y^{\frac{1}{n}} - y^{\frac{-1}{n}}\right) + \frac{y_1^2}{y}\left(y^{\frac{1}{n}} + y^{\frac{-1}{n}}\right) = 2n^2y_1$

$\Rightarrow nyy_2\left(y^{\frac{1}{n}} - y^{\frac{-1}{n}}\right) + 2xy_1^2 = 2n^2yy_1$

$\Rightarrow nyy_2\frac{2ny}{y_1} + 2xy_1^2 = 2n^2yy_1$

$\Rightarrow \frac{n^2y^2y_2}{y_1^2} + xy_1 = n^2y$

$\Rightarrow y_2\frac{\left(y^{\frac{1}{n}} - y^{\frac{-1}{n}}\right)^2}{4} + xy_1 = n^2y$

$\Rightarrow y_2\frac{\left(y^{\frac{1}{n}} + y^{\frac{-1}{n}}\right)^2 - 4}{4} + xy_1 = n^2y$

$\Rightarrow y_2\frac{4x^2-4}{4} + xy_1 = n^2y$

⇒ $(x^2 - 1)\, y_2 + xy_1 = n^2y$

58.

(c) f(x) and g(x) both are continuous at x = 0

Explanation: Given f(x) = |x| and g(x) = $|x^3|$,

$f(x) = \begin{cases} -x, & x \leq 0 \\ x, & x > 0 \end{cases}$

Checking differentiability and continuity,

LHL at x =0,

$\lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} -(0-h) = 0$

RHL at x =0,

$\lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0}(0+h) = 0$

And f(0)=0

Hence, f(x) is continuous at x =0.

LHD at x =0,

$\lim\limits_{x\to 0^-}\frac{f(x)-f(0)}{x-0}=\lim\limits_{h\to 0}\frac{f(0-h)-f(0)}{0-h-0}$

$=\lim\limits_{h\to 0}\frac{(0-h)-(0)}{-h}=-1$

RHD at x =0,

$\lim\limits_{x\to 0^+}\frac{f(x)-f(0)}{x-0}=\lim\limits_{h\to 0}\frac{f(0+h)-f(0)}{0+h-0}$

$=\lim_{h\to 0}\frac{(0+h)-(0)}{h}=1$

∵ LHD $\neq$ RHD

∴ f(x) is not differentiable at x =0.

$g(x)=\begin{cases}-x^3, \ x\leq 0\\ x^3, \quad x>0\end{cases}$

Checking differentiability and continuity,

LHL at x =0,

$\lim\limits_{x\to 0^-}g(x)=\lim\limits_{h\to 0}g(0-h)=\lim\limits_{h\to 0}-(0-h)^3=0$

RHL at x =0,

$\lim\limits_{x\to 0^+}g(x)=\lim\limits_{h\to 0}g(0+h)=\lim\limits_{h\to 0}(0+h)^3=0$

And g(0)=0

Hence, g(x) is continuous at x =0.

LHD at x =0,

$\lim\limits_{x\to 0^-}\frac{g(x)-g(0)}{x-0}=\lim\limits_{h\to 0}\frac{g(0-h)-g(0)}{0-h-0}$

$=\lim\limits_{h\to 0}\frac{(0-h)^3-(0)}{-h}=0$

RHD at x =0,

$\lim\limits_{x\to 0^+}\frac{g(x)-g(0)}{x-0}=\lim\limits_{h\to 0}\frac{g(0+h)-g(0)}{0+h-0}$

$=\lim\limits_{h\to 0}\frac{(0+h)^3-(0)}{h}=0$

∵ LHD = RHD

∴ g(x) is differentiable at x =0.

59.

(b) – tan x

Explanation: Let y = sin^{3}x and z = cos^{3}x, then , $\frac{dy}{dz}=\frac{\frac{dy}{dx}}{\frac{dz}{dx}}=\frac{3\sin^2 x\cos x}{3\cos^2 x(-\sin x)}=-\tan x$.

Which is the required solution.

60. **(a)** $\frac{1}{2}$

Explanation: Given that y = $\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$

Using 1 - cos x = 2 sin^2 $\frac{x}{2}$ and 1 + cos x = 2 cos^2 $\frac{x}{2}$, we obtain

$y=\tan^{-1}\sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}}=\tan^{-1}\tan\left(\frac{x}{2}\right)=\frac{x}{2}$

Differentiating with respect to x, we

$\frac{dy}{dx}=\frac{1}{2}$

61.

(d) $\frac{-1}{2}$

Explanation: Given that $y=\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$

Using $\cos x=\cos^2\frac{x}{2}-\sin^2\frac{x}{2}$, $\sin x=2\sin\frac{x}{2}\cos\frac{x}{2}$ and $\cos^2\theta+\sin^2\theta=1$

Therefore,

$y=\tan^{-1}\left(\frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}\right)=\tan^{-1}\left(\frac{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}\right)$

$\Rightarrow y=\tan^{-1}\frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}$

Dividing by $\cos\frac{x}{2}$ in numerator and denominator, we get

$y=\tan^{-1}\frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}$

Using $\tan\left(\frac{\pi}{4}-x\right)=\frac{1-\tan x}{1+\tan x}$, we obtain

$y=\tan^{-1}\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)$

$=\frac{\pi}{4}-\frac{x}{2}$

Differentiating with respect to x, we

$\frac{dy}{dx}=-\frac{1}{2}$

62. **(a)** $\frac{-e^{1/x}}{x^2}$

Explanation: Here $y=e^{\frac{1}{x}}$

Taking log both sides, we get

$\log_e y=\frac{1}{x}$ (Since $\log_a b^c = c\log_a b$)

Differentiating with respect to x, we obtain

$\frac{1}{y}\frac{dy}{dx}=-\frac{1}{x^2}$ or $\frac{dy}{dx}=-\frac{1}{x^2}\times y$

Therefore, $\frac{dy}{dx}=-\frac{1}{x^2}\times e^{\frac{1}{x}}$

63. **(a)** $e^x + 2xe^{x^2} + 3x^2e^{x^3} + 4x^3e^{x^4} + 5x^4e^{x^5}$

Explanation: Let $y = e^x + e^{x^2} + ... + e^{x^5}$

On differentiating both sides w.r.t. x, we get

$\frac{d}{dx}(y) = \frac{d}{dx}\left\{e^x+e^{x^2}+e^{x^3}+e^{x^4}+e^{x^5}\right\}$

$= \frac{d}{dx}(e^x) + \frac{d}{dx}\left(e^{x^2}\right) + \frac{d}{dx}\left(e^{x^3}\right) + \frac{d}{dx}\left(e^{x^4}\right) + \frac{d}{dx}\left(e^{x^5}\right)$

$= e^x + e^{x^2}\frac{d}{dx}x^2 + e^{x^3}\frac{d}{dx}x^3 + e^{x^4}\frac{d}{dx}\left(x^4\right) + e^{x^5}\frac{d}{dx}\left(x^5\right)$ [using chain rule]

$= e^x + e^{x^2}(2x) + e^{x^3}\left(3x^2\right) + e^{x^4}\left(4x^3\right) + e^{x^5}\left(5x^4\right)$

$= e^x + 2xe^{x^2} + 3x^2e^{x^3} + 4x^3e^{x^4} + 5x^4e^{x^5}$

64.

(b) $\frac{1}{\sqrt{x}(1-x)}$

Explanation: Given that $y=\log_e\frac{1+\sqrt{x}}{1-\sqrt{x}}$

Differentiating with respect to x, we obtain

$\frac{dy}{dx}=\frac{1-\sqrt{x}}{1+\sqrt{x}}\times\frac{(1-\sqrt{x})\times\frac{1}{2\sqrt{x}}-(1+\sqrt{x})\times-\frac{1}{2\sqrt{x}}}{(1-\sqrt{x})^2}=\frac{1}{(1-x)\sqrt{x}}$

65.

(c) $x^x\{(1 + \log x)^2 + \frac{1}{x}\}$

Explanation: $y = x^x = e^{x\log x}$

$\therefore \frac{dy}{dx} = e^{x\log x}(x\times\frac{1}{x} + \log x)$

$= e^{x\log x}(1 + \log x)$

$\therefore \frac{d^2y}{dx^2} = e^{x\log x}\left(\frac{1}{x}\right) + (1 + \log x)\, e^{x\log x}(1 + \log x)$

$= \frac{x^x}{x} + (1 + \log x)^2\, e^x \log x$

$= x^{x-1} + (1 + \log x)^2 x^x = x^x\{(1 + \log x)^2 + \frac{1}{x}\}$

66. **(a)** $\frac{1}{(2y-1)}$

Explanation: Given:

$\Rightarrow y=\sqrt{x+\sqrt{x+\sqrt{x+}}}\ldots\infty$

We can write it as

$\Rightarrow y=\sqrt{x+y}$

Squaring we get

$\Rightarrow y^2 = x + y$

Differentiating with respect to x,we get

$\Rightarrow 2y\frac{dy}{dx}=1+\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx}=\frac{1}{(2y-1)}$

67. **(a)** $\frac{y(y-x\log y)}{x(x-y\log x)}$

Explanation: given that $x^y = y^y$

Taking log both sides, we obtain

$y \log_e x = c \log_e y$

(Since $\log_a b^c = c \log_a b$)

Differentiating with respect to x, we obtain

$\frac{y}{x}+\log_e x\frac{dy}{dx}=\frac{x}{y}\frac{dy}{dx}+\log_e y$

$\Rightarrow \frac{x-y\log_e x}{y}\frac{dy}{dx}=\frac{y-x\log_e y}{x}$

Hence $\frac{dy}{dx}=\frac{y(y-x\log_e y)}{x(x-y\log_e x)}$

68.

(b) $\frac{1}{t}$

Explanation: Given that, $x = at^2$, $y = 2at$

$\frac{dx}{dt}$ = 2at and $\frac{dy}{dt}$ = 2a

Therefore, $\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{2a}{2at}=\frac{1}{t}$

69.

(d) $\frac{xy}{(1+x^2)}$

Explanation: Given, $y = \sec \tan^{-1} x$

On differentiating both sides w.r.t. x, we get

$\frac{dy}{dx} = \sec \tan^{-1} x \tan \tan^{-1} x \cdot \frac{1}{1+x^2}$

$= y \cdot x \cdot \frac{1}{1+x^2} = \frac{xy}{1+x^2}$ [$\because \tan \tan^{-1} x = x$]

70.

(b) $\frac{-y}{x}$

Explanation: $\frac{-y}{x}$

71.

(c) $\frac{\cos x}{(2y-1)}$

Explanation: Given:

$\Rightarrow y=\sqrt{\sin x+\sqrt{\sin x+\sqrt{\sin x+}}}$

We can write it as

$\Rightarrow y=\sqrt{\sin x+y}$

Squaring we get

$\Rightarrow y^2 = \sin x + y$

Differentiating with respect to x,we get

$\Rightarrow 2y\frac{dy}{dx}=\cos x+\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx}=\frac{\cos x}{(2y-1)}$

72.

(b) -1 and 2

Explanation: Given, f(x) $\begin{bmatrix} ax+3, & x \leq 2 \\ a^2x-1, & x>2 \end{bmatrix}$ continuity at x = 2,

LHL = $\lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2}$ (ax + 3) = 2a + 3

RHL = $\lim\limits_{x \to 2^+} f(x) = \lim\limits_{x \to 2}$ (a^2x - 1) = $2a^2$ - 1

Since, f(x) is continuous for all values of x.

∴ LHL = RHL

⇒ 2a + 3 = $2a^2$ - 1

⇒ $2a^2$ - 2a - 4 = 0

⇒ a^2 - a - 2 = 0

⇒ a^2 - 2a + a - 2 = 0

⇒ a(a - 2) + 1(a - 2) = 0

⇒ (a + 1)(a - 2) = 0

∴ a = -1, 2

73.

(c) $\frac{y^2}{x(1-y\log x)}$

Explanation: Given:

$y = x^{x^{x+\infty}}$

We can write it as

⇒ y = x^y

Taking log of both sides we obtain

log y = y log x

Differentiating with respect to x,we get

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{dy}{dx}\log x + y \cdot \frac{1}{x}$

$\Rightarrow \left(\frac{1}{y} - \log x\right)\frac{dy}{dx} = \frac{y}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x}\left(\frac{y}{1-y\log x}\right)$

$\Rightarrow \frac{dy}{dx} = \frac{y^2}{x(1-y\log x)}$.

Which is the required solution.

74.

(b) a + b

Explanation: $\lim\limits_{x \to 0} \frac{\log(1+ax)-\log(1-bx)}{x} = k$

$\lim\limits_{x \to 0} \frac{\log(1+ax)}{x} - \frac{\log(1-bx)}{x} = k$

$\lim\limits_{x \to 0} \frac{\log(1+ax)}{ax} \times a - \frac{\log(1-bx)}{-bx} \times (-b) = k$

a + b = k

75.

(d) 6

Explanation: $\lim\limits_{x \to 0} \frac{\sin 3x}{x} = \frac{k}{2}$

$\lim\limits_{x \to 0} \frac{\sin 3x}{3x} \times 3 = \frac{k}{2}$

$3 = \frac{k}{2}$

k = 6