

CONTINUITY AND DIFFERENTIABILITY WS 1

Class 12 - Mathematics

1. Derivative of  $\tan^{-1} \left( \frac{x}{\sqrt{1-x^2}} \right)$  with respect to  $\sin^{-1} \left( 2x\sqrt{1-x^2} \right)$  is: [1]
  - a) 2
  - b)  $-\frac{1}{2}$
  - c)  $-\frac{1}{4}$
  - d)  $\frac{1}{2}$
2. If  $y = \log (x + \sqrt{x^2 + a^2})$  then  $\frac{dy}{dx} = ?$  [1]
  - a)  $\frac{1}{2(x+\sqrt{x^2+a^2})}$
  - b)  $\frac{-1}{\sqrt{x^2+a^2}}$
  - c)  $\frac{1}{\sqrt{x^2+a^2}}$
  - d)  $\frac{-1}{2\sqrt{x^2+a^2}}$
3. Let  $f(x) = \begin{cases} e^{1/x}, & x < 0 \\ x, & x \geq 0 \end{cases}$ , then  $\lim_{x \rightarrow 0} f(x)$  [1]
  - a) does not exist
  - b) is equal to 0
  - c) is equal to non – zero real number
  - d) does exist
4. If  $y = \log x^x$ , then the value of  $\frac{dy}{dx}$  is [1]
  - a)  $x^x(1 + \log x)$
  - b)  $\log \frac{e}{x}$
  - c)  $\log \left( \frac{x}{e} \right)$
  - d)  $\log (ex)$
5. If the function  $f(x) = \begin{cases} \frac{1 - \cos 4x}{8x^2}, & x \neq 0 \\ k, & x = 0 \end{cases}$  is continuous  $x = 0$  then  $k = ?$  [1]
  - a)  $\frac{-1}{2}$
  - b)  $\frac{1}{2}$
  - c) 2
  - d) 1
6. Number of points at which  $f(x) = \frac{1}{\log|x|}$  is discontinuous is [1]
  - a) 1
  - b) 3
  - c) 2
  - d) 4
7. The function  $f(x) = \begin{cases} \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$  at  $x = 0$  [1]
  - a) has oscillating discontinuity
  - b) is continuous
  - c) has removable discontinuity
  - d) has jump discontinuity
8. At  $x = 2$ ,  $f(x) = [x]$  is [1]
  - a) Continuous but not differentiable
  - b) None of these
  - c) Continuous as well as differentiable
  - d) Differentiable but not continuous
9. The function  $f(x) = \frac{4-x^2}{4x-x^3}$  is [1]

- a) discontinuous at exactly four points  
b) discontinuous at only one point  
c) discontinuous at exactly two points  
d) discontinuous at exactly three points
10. If  $x^y = e^{x-y}$ , then  $\frac{dy}{dx}$  is [1]  
 a)  $\frac{1-\log x}{1+\log x}$   
 b)  $\frac{1+x}{1+\log x}$   
 c)  $\frac{\log x}{(1+\log x)^2}$   
 d) not defined
11. If  $f(x) = |x| + |x - 2|$ , then [1]  
 a)  $f(x)$  is continuous at  $x = 0$  but not at  $x = 2$   
 b)  $f(x)$  is continuous at  $x = 0$  and at  $x = 2$   
 c)  $f(x)$  is continuous at  $x = -2$  but not at  $x = 0$   
 d)  $f(x)$  is continuous at  $x = 2$  but not at  $x = 0$
12. Let  $f(x) = |\sin x|$  Then [1]  
 a)  $f$  is everywhere differentiable  
 b)  $f$  is everywhere continuous but not differentiable at  $x = (2n + 1)\frac{\pi}{2}, x \in \mathbf{Z}$   
 c)  $f$  is everywhere differentiable  $x = (2n - 1)\frac{\pi}{2}, x \neq \mathbf{Z}$   
 d)  $f$  is everywhere continuous but not differentiable at  $x = n\pi, n \in \mathbf{Z}$
13. The function  $f(x) = \sin^{-1}(\cos x)$  is [1]  
 a) Not differentiable at  $x = 0$   
 b) differentiable at  $x = 0$   
 c) discontinuous at  $x = 0$   
 d) continuous at  $x = 0$
14. The value of  $k$  for which  $f(x) \begin{cases} \frac{3x+4 \tan x}{x}, & \text{when } x \neq 0 \\ k, & \text{when } x = 0 \end{cases}$  is continuous at  $x = 0$ , is [1]  
 a) 3  
 b) 7  
 c) 8  
 d) 4
15. If the function  $f(x) = \begin{cases} \frac{k \cos x}{(\pi - 2x)}, & \text{when } x \neq \frac{\pi}{2} \\ 3, & \text{when } x = \frac{\pi}{2} \end{cases}$  be continuous at  $x = \frac{\pi}{2}$ , then the value of  $k$  is [1]  
 a) 3  
 b) 6  
 c) -3  
 d) -5
16. Let  $f(x) = \begin{cases} x[x - 1], & 0 \leq x < 2 \\ [x](x - 1), & 2 \leq x < 3 \end{cases}$  where  $[x]$  denotes the greatest integer  $\leq x$  then [1]  
 a)  $f(x)$  is differentiable at infinite points  
 b)  $f(x)$  is not differentiable at infinite points  
 c)  $f(x)$  is continuous everywhere  
 d)  $f(x)$  is not continuous at  $x = 1$  and  $x = 2$
17. The function  $f(x) = \begin{cases} x^2 a, & 0 \leq x < 1 \\ a, & 1 \leq x < \sqrt{2} \\ \frac{2b^2 - 4b}{x^2}, & \sqrt{2} \leq x < \infty \end{cases}$  is continuous for  $0 \leq x < \infty$ , then the most suitable values of  $a$  and  $b$  are [1]  
 a)  $a = -1, b = 1$   
 b)  $a = -1, b = 1^+$   
 c)  $a = -1, b = -1$   
 d)  $a = 1, b = 1$
18. If  $y = \log \left( \frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}-x} \right)$  then  $\frac{dy}{dx} = ?$  [1]

a)  $\frac{-2}{\sqrt{1+x^2}}$

b)  $\frac{2\sqrt{1+x^2}}{x^2}$

c)  $-\frac{2\sqrt{1+x^2}}{x^2}$

d)  $\frac{2}{\sqrt{1+x^2}}$

19. If  $y = e^{\tan x}$ , then  $(\cos^2 x) y_2 =$

[1]

a)  $(1 + \sin 2x) y_1$

b)  $(1 + \sin 4x) y_1$

c)  $(1 - \sin 2x) y_1$

d)  $-(1 + \sin 2x) y_1$

20. If  $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ , then  $\frac{dy}{dx}$  is equal to

[1]

a)  $\frac{2}{1+x^2}$

b)  $\frac{-2}{1+x^2}$

c)  $\frac{1}{1+x^2}$

d)  $\frac{2}{1-x^2}$

21.  $f(x) = |\log_e |x||$ , then

[1]

a)  $f(x)$  is continuous and differentiable for all  $x$  in its domain

b)  $f(x)$  is continuous for all  $x$  in its domain but not differentiable at  $x = \pm 1$

c)  $f(x)$  is either continuous or differentiable at  $x = -1$

d)  $f(x)$  is neither continuous nor differentiable at  $x = \pm 1$

22. The values of the constants  $a, b$  and  $c$  for which the function  $f(x) = \begin{cases} (1+ax)^{1/x} & , x < 0 \\ b & , x = 0 \\ \frac{(x+c)^{1/3}-1}{(x+1)^{1/2}-1} & , x > 0 \end{cases}$  may be continuous at  $x = 0$ , are

[1]

a)  $a = \log_e\left(\frac{2}{3}\right), b = \frac{2}{3}, c = -1$

b)  $a = \log_e\left(\frac{4}{3}\right), b = -\frac{4}{3}, c = -1$

c)  $a = \log_e\left(\frac{2}{3}\right), b = -\frac{2}{3}, c = 1$

d)  $a = \log_e\left(\frac{2}{3}\right), b = \frac{2}{3}, c = 1$

23. If  $y = (\sin x) \log^x$  then  $\frac{dy}{dx} = ?$

[1]

a)  $(\sin x)^{\log x} \cdot \left\{ \frac{\cot x + \log \sin x}{x} \right\}$

b)  $(\log x)(\sin x)^{(\log x - 1)} \cos x$

c)  $(2 \sin x)^{\log x} \cdot \left\{ \frac{2 \cot x - \log \sin x}{x} \right\}$

d)  $(\sin x)^{\log x} \cdot \left\{ \frac{x \cot x \log x + \log \sin x}{x} \right\}$

24. The function  $f(x) = \begin{cases} \frac{x^2-x-6}{x+2} & \text{if } x \neq -2 \\ -5, & \text{if } x = -2 \end{cases}$  at  $x = -2$  is

[1]

a) continuous

b) not continuous

c)  $f(-2) = 5$

d)  $f(2) = -5$

25. The value of  $k$  for which function  $f(x) = \begin{cases} x^2, & x \geq 0 \\ kx, & x < 0 \end{cases}$  is differentiable at  $x = 0$  is:

[1]

a) 1

b) 0

c) 2

d) any real number

26. The function  $f$ , where  $f$  is defined by  $f(x) = \begin{cases} -2, & \text{if } x \leq -1 \\ 2x, & \text{if } -1 < x < \infty \end{cases}$  is

[1]

a) continuous at  $R$

b) not continuous at  $x = -1$

c) not continuous at  $R$

d) not continuous at  $x = 1$



- c)  $\{\frac{n\pi}{4} : n = 0, \pm 1, \pm 2, \dots\}$  d)  $\{n\pi : n = 0, \pm 1, \pm 2, \dots\}$
37. If  $f(x) = \frac{1-\sin x}{(\pi-2x)^2}$ , when  $x \neq \frac{\pi}{2}$  and  $f(\frac{\pi}{2}) = \lambda$ , then  $f(x)$  will be continuous function at  $x = \frac{\pi}{2}$ , where  $\lambda =$  [1]
- a) 1/4 b) 1/3  
c) 1/2 d) 1/8
38. The function  $f(x) = e^{-|x|}$  is [1]
- a) continuous everywhere but not differentiable at  $x = 0$  b) continuous and differentiable everywhere  
c) continuous at  $x = 0$  d) not continuous at  $x = 0$
39. If  $y = \cot^{-1}\left(\frac{1-x}{1+x}\right)$  then  $\frac{dy}{dx} = ?$  [1]
- a)  $\frac{1}{(1+x^2)}$  b)  $\frac{-1}{(1+x^2)}$   
c)  $\frac{1}{(1-x^2)^{3/4}}$  d)  $\frac{1}{(1+x^2)^{3/2}}$
40. Let  $f(x) = |\sin x|$ ;  $0 \leq x \leq 2\pi$  then [1]
- a)  $f(x)$  is discontinuous at 3 points b)  $f(x)$  is differentiable function at infinite number of points  
c)  $f(x)$  is non-differentiable at 3 points and continuous everywhere d)  $f(x)$  is discontinuous everywhere
41. Let  $f(x) = \begin{cases} 1-3x, & x < 0 \\ 3, & x = 0 \\ x^2+3, & x > 0 \end{cases}$  then  $f(1)$  is equal to [1]
- a)  $f(x)$  is continuous b)  $f(x)$  has removable discontinuity  
c)  $f(x)$  is right continuous d)  $f(x)$  is continuous from left
42. If  $f(x) = \begin{cases} \frac{1-\sin^2 x}{3\cos^2 x}, & x < \frac{\pi}{2} \\ a, & x = \frac{\pi}{2} \\ \frac{b(1-\sin x)}{(\pi-2x)^2}, & x > \frac{\pi}{2} \end{cases}$  Then,  $f(x)$  is continuous at  $x = \frac{\pi}{2}$ , if [1]
- a)  $a = \frac{1}{3}, b = 2$  b)  $a = \frac{4}{3}, b = \frac{7}{3}$   
c)  $a = \frac{2}{3}, b = \frac{8}{3}$  d)  $a = \frac{1}{3}, b = \frac{8}{3}$
43. If  $x = e^{y+e^y+\dots\text{to}\infty}$ ,  $x > 0$ , then  $\frac{dy}{dx}$  is [1]
- a)  $\frac{1+x}{x}$  b)  $\frac{1-x}{x}$   
c)  $\frac{1}{x}$  d)  $\frac{x}{1+x}$
44. If  $f(x) = \begin{cases} mx+1, & \text{if } x \leq \frac{\pi}{2} \\ \sin x + n, & \text{if } x > \frac{\pi}{2} \end{cases}$  is continuous at  $x = \frac{\pi}{2}$  then [1]
- a)  $m = n = \frac{\pi}{2}$  b)  $n = \frac{m\pi}{2}$   
c)  $m = 1, n = 0$  d)  $m = \frac{n\pi}{2} + 1$
45. If  $y = \frac{\cos x - \sin x}{\cos x + \sin x}$ , then  $\frac{dy}{dx}$  is: [1]
- a)  $-\sec^2\left(\frac{\pi}{4} - x\right)$  b)  $\sec^2\left(\frac{\pi}{4} - x\right)$



56. If  $f(x) = \begin{cases} \frac{1}{4a} & \text{for } -1 \leq x < 0 \\ \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x}, & \text{for } -1 \leq x < 0 \\ 2x^2 + 3x - 2, & \text{for } 0 \leq x \leq 1 \end{cases}$  is continuous at  $x = 0$ , then  $k$  is equal to [1]
- a) -1 b) -3  
c) -2 d) -4
57. If  $y^{1/n} + y^{-1/n} = 2x$ , then  $(x^2 - 1)y_2 + xy_1 =$  [1]
- a) 0 b) 1  
c)  $n^2y$  d)  $-n^2y$
58. Let  $f(x) = |x|$  and  $g(x) = |x^3|$ , then [1]
- a)  $f(x)$  and  $g(x)$  both are differentiable at  $x = 0$  b)  $f(x)$  is differentiable but  $g(x)$  is not differentiable at  $x = 0$   
c)  $f(x)$  and  $g(x)$  both are continuous at  $x = 0$  d)  $f(x)$  and  $g(x)$  both are not differentiable at  $x = 0$
59. Derivative of  $\sin^3x$  w.r.t  $\cos^3x$  is [1]
- a)  $\cot x$  b)  $-\tan x$   
c)  $\tan^3x$  d)  $\tan x$
60. If  $y = \tan^{-1} \sqrt{\frac{1-\cos x}{1+\cos x}}$  then  $\frac{dy}{dx} = ?$  [1]
- a)  $\frac{1}{2}$  b)  $-\frac{1}{2}$   
c)  $\frac{-1}{2}$  d)  $\frac{1}{(1+x^2)}$
61. If  $y = \tan^{-1} \frac{\cos x}{1+\sin x}$  then  $\frac{dy}{dx} = ?$  [1]
- a)  $\frac{1}{2}$  b) 1  
c) 0 d)  $\frac{-1}{2}$
62. If  $y = e^{1/x}$  then  $\frac{dy}{dx} = ?$  [1]
- a)  $\frac{-e^{1/x}}{x^2}$  b)  $e^{1/x} \log x$   
c)  $\frac{1}{x} \cdot e^{(1/x-1)}$  d)  $2e^{1/x} \log x$
63.  $\frac{d}{dx}(e^x + e^{x^2} + \dots + e^{x^5})$  is equal to [1]
- a)  $e^x + 2xe^{x^2} + 3x^2e^{x^3} + 4x^3e^{x^4} + 5x^4e^{x^5}$  b)  $e^x - 2e^x - 3e^x - 4e^x - 5e^x$   
c)  $e^x + 2e^x + 3e^x + 4e^x + 5e^x$  d)  $e^x + 2xe^x + 3xe^{x^2} + 4e^{x^3} + 5xe^{x^4}$
64. If  $y = \log \left( \frac{1+\sqrt{x}}{1-\sqrt{x}} \right)$  then  $\frac{dy}{dx} = ?$  [1]
- a)  $\frac{-1}{x(1-\sqrt{x})^2}$  b)  $\frac{1}{\sqrt{x}(1-x)}$   
c)  $\frac{-\sqrt{x}}{2(1-\sqrt{x})}$  d)  $\frac{\sqrt{x}}{2(1-\sqrt{x})}$
65. If  $y = x^x$  find  $\frac{d^2y}{dx^2}$  [1]
- a) 0 b)  $x^x \left\{ (1 - \log x)^2 + \frac{1}{x} \right\}$

66. If  $y = \sqrt{x + \sqrt{x + \sqrt{x + \dots}}}$  then  $\frac{dy}{dx} = ?$  [1]
- c)  $x^x \left\{ (1 + \log x)^2 + \frac{1}{x} \right\}$  d)  $x^x \left\{ (1 + \log x)^2 - \frac{1}{x} \right\}$
- a)  $\frac{1}{(2y-1)}$  b)  $\frac{1}{(y^2-1)}$
- c)  $\frac{y}{(y^2-1)}$  d)  $\frac{2y}{(y^2-1)}$
67. If  $x^y = y^x$  then  $\frac{dy}{dx} = ?$  [1]
- a)  $\frac{y(y-x \log y)}{x(x-y \log x)}$  b)  $\frac{y(y+x \log y)}{x(x+y \log x)}$
- c)  $\frac{(y-x \log y)}{(x-y \log x)}$  d)  $\frac{y(y-x \log x)}{x(x-y \log y)}$
68. If  $x = at^2$  and  $y = 2at$ , then  $\frac{dy}{dx}$  is equal to [1]
- a) -1 b)  $\frac{1}{t}$
- c) t d)  $\frac{-1}{t^2}$
69. If  $y = \sec \tan^{-1} x$ , then  $\frac{dy}{dx}$  is equal to [1]
- a)  $xy\sqrt{(1+x^2)}$  b)  $\frac{xy}{(1+x)}$
- c)  $\frac{y}{\sqrt{(1+x^2)}}$  d)  $\frac{xy}{(1+x^2)}$
70. If  $x = \sqrt{a^{\sin^{-1} t}}$ ,  $y = \sqrt{a^{\cos^{-1} t}}$ ,  $a > 0$  and  $-1 < t < 1$ , then  $\frac{dy}{dx}$  is: [1]
- a)  $\frac{-x}{y}$  b)  $\frac{-y}{x}$
- c)  $\frac{y}{x}$  d)  $\frac{x}{y}$
71. If  $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \dots}}}$  then  $\frac{dy}{dx} = ?$  [1]
- a)  $\frac{\sin x}{(2y-1)}$  b)  $\frac{\cos x}{(y-1)}$
- c)  $\frac{\cos x}{(2y-1)}$  d)  $\frac{\sin x}{(2y+1)}$
72. If  $f(x) = \begin{cases} ax + 3, & x \leq 2 \\ a^2 x - 1, & x > 2 \end{cases}$ , then the values of a for which f is continuous for all x are [1]
- a) 1 and -2 b) -1 and 2
- c) -1 and -2 d) 1 and 2
73. If  $y = x^{x^{x+\infty}}$  then  $\frac{dy}{dx} = ?$  [1]
- a)  $y x^{y+1}$  b)  $y x^{y-1}$
- c)  $\frac{y^2}{x(1-y \log x)}$  d)  $\frac{y}{x(1+y \log x)}$
74. If  $f(x) = \begin{cases} \frac{\log(1+ax) - \log(1-bx)}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$  and f(x) is continuous at  $x = 0$ , then the value of k is [1]
- a) a - b b) a + b
- c)  $\log a + \log b$  d) ab
75. The function  $f(x) = \begin{cases} \frac{\sin 3x}{x}, & x \neq 0 \\ \frac{k}{2}, & x = 0 \end{cases}$  is continuous at  $x = 0$ , then k = [1]
- a) 3 b) 12

c) 9

d) 6