Solution

DETERMINANTS WS 1

Class 12 - Mathematics

1.

(c) Idempotent
Explanation: clearly for given matrix $A^2 = A$
Therefore idempotent

2.

(d) if det. A = 0 , (adj A) B =O
Explanation: If det. A = 0, (adj A) B =O$\Rightarrow$ The system AX = B of n equations in n unknowns may be consistent with infinitely many solutions or it may be inconsistent.

3.

(c) $\frac{15}{2}$
Explanation: Δ = $\begin{vmatrix} \log_3 512 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix}$ = $\begin{vmatrix} \log_3 2^9 & \log_4 3^2 \\ \log_3 2^3 & \log_4 3^2 \end{vmatrix}$
= $\begin{vmatrix} 9\log_3 2 & \log_4 3 \\ 3\log_3 2 & 2\log_4 3 \end{vmatrix}$ = 18 $\log_3 2 \times \log_4 3$ - 3 $\log_3 2 \times \log_4 3$
= $\log_3 2 \times \log_4 3$(18 - 3) = $\frac{\ln 2}{\ln 3} \times \frac{\ln 3}{\ln 4} \times 15$ $\left|\because \log_a b = \frac{\ln b}{\ln a}\right|$
= $\frac{\ln 2}{\ln 4} \times 15$ = $\frac{\ln 2}{\ln 2^2} \times 15$ = $\frac{15 \ln 2}{2 \ln 2}$ = $\frac{15}{2}$

4.

(b) adj A
Explanation: adj A

5.

(d) 0
Explanation: 0

6.

(d) ± 6
Explanation: We have $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$
We know that determinant of A is calculated as $|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ = ad - bc
$\Rightarrow$ x(x) – 2(18) = 6(6) – 2(18)
$\Rightarrow$ x^2 - 36 = 36 – 36
$\Rightarrow$ x^2 =36 – 36 + 36
$\Rightarrow$ x^2 = 36
$\Rightarrow$ x = $\pm$6.

7.

(d) $\frac{10}{7}$
Explanation: Let A (2, -3), B (λ, -1) and C (0,4) are given points.
Given points are collinear
$\therefore$ ar($\triangle$ABC) = 0
$\Rightarrow$ $\frac{1}{2}\begin{vmatrix} 2 & -3 & 1 \\ \lambda & -1 & 1 \\ 0 & 4 & 1 \end{vmatrix}$= 0 $\Rightarrow$ |2 (-1 - 4) - λ(-3 - 4) + 0| = 0
$\Rightarrow$|-10 + 7λ| = 0 $\Rightarrow$ 7λ - 10 = 0 $\Rightarrow$ λ = $\frac{10}{7}$

8. **(a)** BA^{-1}
Explanation: $(AB^{-1})^{-1} = (B^{-1})^{-1} A^{-1} = BA^{-1}$

9. **(a)** $\frac{1}{2}$

Explanation: $\Delta = \begin{vmatrix} 0 & \cos\theta & \sin\theta \\ \cos\theta & \sin\theta & 0 \\ \sin\theta & 0 & \cos\theta \end{vmatrix}^2$

= (- cos θ (cos$^2\theta$ - θ) + sin θ (0 - sin$^2\theta$))^2

= (- cos$^3\theta$ - sin$^3\theta$)^2 = (cos^3 θ + sin^3 θ)^2

Now cos 2θ = 0 $\Rightarrow$ 2θ = (2n + 1)$\frac{\pi}{2}$ $\Rightarrow$ θ = (2n + 1)$\frac{\pi}{4}$

$\therefore \Delta = \left(\cos^3 \frac{(2n+1)\pi}{4} + \sin^3 \frac{(2n+1)\pi}{4}\right)^2$

$= \left(\frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}\right)^2 = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$

10. **(a)** 12

Explanation: 12

Explaination:

A$\cdot$(adj A) = $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

we know that A.(AdjA) = I.|A|

$\Longrightarrow \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ = |A| I

$\Longrightarrow 3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ = |A| I

$\Longrightarrow$ 3I = |A| I

$\Longrightarrow$ |A|=3 ---(1)

|Adj A| = |A|$^{3-1}$ [Since order n=3]

|Adj A| = (3)^2=9

|adj(A)| = 9 -------(2)

Now,

|A| + |adj A| = 3 + 9 =12

11. **(a)** $x = -\frac{6}{11},\ y = -\frac{19}{11}$

Explanation: The given system can be written as AX = B where

A = $\begin{bmatrix} 4 & -3 \\ 3 & -5 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$

Here, |A| = $\begin{vmatrix} 4 & -3 \\ 3 & -5 \end{vmatrix}$ = 4(-5) - 3(-3)

= -20 + 9 = -11 $\neq$ 0

Thus, A is non-singular. Therefore, its inverse exists.

Therefore, the given system has a unique solution given by X = A^{-1} B.

$\therefore$ adj (A) = $\begin{bmatrix} -5 & 3 \\ -3 & 4 \end{bmatrix}$

$\therefore$ A^{-1} = $\frac{1}{|A|}(\text{adj}\, A)$ = $\frac{1}{-11}\begin{bmatrix} -5 & 3 \\ -3 & 4 \end{bmatrix}$

Now, X = A^{-1} B = $\frac{1}{-11}\begin{bmatrix} -5 & 3 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 3 \\ 7 \end{bmatrix}$

$= \frac{1}{-11}\begin{bmatrix} -15+21 \\ -9+28 \end{bmatrix}$

$= \frac{1}{-11}\begin{bmatrix} 6 \\ 19 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\frac{6}{11} \\ -\frac{19}{11} \end{bmatrix}$

$\therefore x = -\frac{6}{11},\ and\, y = -\frac{19}{11}$

12.

(d) Null matrix

Explanation: As we know that, A(adj A) = |A| I.

But it is given that A is a singular matrix

Thus, |A| = 0.

Therefore, A(adj A) = 0I = 0, where 0 is the zero matrix.

Hence, if A is a singular matrix, then A(adj A) = $\underline{0}$.

13.

(b) $(A + B)^{-1} = B^{-1} + A^{-1}$

Explanation: Since, A and B are invertible matrices.

So, we can say that

$(AB)^{-1} = B^{-1} A^{-1}$ ...(i)

We know that, $A^{-1} = \frac{1}{|A|}$ (adj A)

$\Rightarrow$ adj A = $|A| \cdot A^{-1}$ ...(ii)

Also, $\det (A)^{-1} = [\det (A)]^{-1}$

$\Rightarrow \det (A)^{-1} = \frac{1}{[\det(A)]}$

$\Rightarrow \det (A) \cdot \det (A)^{-1} = 1$ ...(iii)

Which is true,

So, only option d is incorrect.

14.

(c) AB is non-singular

Explanation: If A and B are non - singular then $|AB| \neq 0$

= AB is non - singular matrix,

As |AB = |A||B|

15.

(b) $\frac{61}{2}$ sq. units

Explanation: The area of triangle is given by

$$\Delta = \frac{1}{2}\begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 5 & 1 & 1 \end{vmatrix}$$

$= \frac{1}{2}$ [3(2 - 1)-8(-4 - 5)+1(-4 - 10)]

$= \frac{1}{2}(3 + 72 - 14) = \frac{61}{2}$ sq. units

16.

(b) a^6

Explanation: $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$

$|A| = a^3$

$|adj\ A| = |A|^{3-1} = |A|$

$|adj\ A| = (a^3)^2 = a^6$

17. **(a)** $C^{-1}BA^{-1}$

Explanation: We know that $(AB)^{-1} = B^{-1} A^{-1}$

Hence, $(AB^{-1}C)^{-1} = C^{-1}(AB^{-1})^{-1} = C^{-1}(B^{-1})^{-1}A^{-1} = C^{-1}BA^{-1}$

18.

(d) μ only

Explanation: The given system of linear equation :-

$x + y + z = \lambda$

$5x - y + \mu z = 10$

2x + 3y – z = 6

The matrix equation corresponding to the above system is :

$$\begin{bmatrix}1 & 1 & 1\\ 5 & -1 & \mu\\ 2 & 3 & -1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}\lambda\\ 10\\ 6\end{bmatrix}$$

Suppose A = $\begin{bmatrix}1 & 1 & 1\\ 5 & -1 & \mu\\ 2 & 3 & -1\end{bmatrix}$

$\therefore$ |A| = $\begin{vmatrix}1 & 1 & 1\\ 5 & -1 & \mu\\ 2 & 3 & -1\end{vmatrix}$ = 1(1-3μ) -1(-5-2μ) +1(15 + 2)

= 1 - 3μ +5 +2μ +17 = 23-μ

For the existence of the unique solution, the value of |A| must not be equal to 0.

Therefore, the existence of the unique solution merely depends on the value of μ.Which is the required solution.

19.

(c) None of these

Explanation: We have,

$A=\begin{bmatrix}2 & \lambda & -3\\ 0 & 2 & 5\\ 1 & 1 & 3\end{bmatrix}$

A^{-1} exists if |A| $\neq$ 0

Now |A| = 2(6 - 5) - λ(-5) - 3(-2) = 8 + 5$\lambda \neq$ 0

$\Rightarrow 5\lambda \neq$ -8

$\Rightarrow \lambda \neq \frac{-8}{5}$

So, A^{-1} exists if and only if $\neq \frac{-8}{5}$

20.

(b) $\begin{bmatrix}1 & 2\\ \frac{1}{2} & \frac{3}{2}\end{bmatrix}$

Explanation: B = I

B = A^{-1} I ....(i)

$A^{-1}=\frac{1}{|A|}\text{adj}\,A$ ...(ii)

|A| = 3 $\times$ 2 - (-4) $\times$ (-1)

= 2

C_{11} = 2, C_{12} = 1

C_{21} = 4, C_{22} = 3

Co-factor matrix $A=\begin{pmatrix}2 & 1\\ 4 & 3\end{pmatrix}$

$\text{Adj}\,A=\begin{pmatrix}2 & 1\\ 4 & 3\end{pmatrix}'$

$=\begin{pmatrix}2 & 4\\ 1 & 3\end{pmatrix}$

Putting in 2

$A^{-1}=\frac{1}{|2|}\begin{pmatrix}2 & 4\\ 1 & 3\end{pmatrix}$

$=\begin{pmatrix}1 & 2\\ \frac{1}{2} & \frac{3}{2}\end{pmatrix}$

Put this value in equation (i)

B = A^{-1} I

= A^{-1}

$=\begin{pmatrix}1 & 2\\ \frac{1}{2} & \frac{3}{2}\end{pmatrix}$

21.

(c) 26

Explanation: f(x) = $\begin{vmatrix} x & -4 & 5 \\ 1 & 1 & -2 \\ 2 & x & 1 \end{vmatrix}$ = x (1 + 2x) - 1 (- 4 - 5x) + 2 (8 - 5)

= x + $2x^2$ + 4 + 5x + 6 = $2x^2$ + 6x + 10

f'(x) = 4x + 6

f'(5) = 20 + 6 = 26

22. **(a)** no solution

Explanation: The given system of equations does not have solution if : $\begin{vmatrix} 1 & 4 & -2 \\ 3 & 1 & 5 \\ 2 & 3 & 1 \end{vmatrix} = 0 \Rightarrow 1(-14) - 4(-7) - 2(7) = 0$

23.

(b) -52

Explanation: For easier calculations, we shall expand the determinant along that row or column which contains maximum number of zeroes.

Note that in the third column, two entries are zero.

24. **(a)** $\Delta \in [2, 4]$

Explanation: Δ = $\begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix}$

= 1(1 + $\sin^2\theta$) - sin θ (- sin θ + sin θ) + 1 (sin2θ + 1)

Δ= 2(1 + $\sin^2\theta$) ...(i)

Now, - 1 $\leq \sin^2\theta \leq$ 1

$\Rightarrow$ 0 + 1 $\leq$ 1+ $\sin^2\theta \leq$ 1 + 1

$\Rightarrow$ 1 $\leq$1 + $\sin^2\theta \leq$ 2

$\Rightarrow$ 2 $\leq$ 2(1 + $\sin^2\theta$) $\leq$ 4

$\Rightarrow$ 2 $\leq \Delta \leq$ 4 [From (i)]

$\Rightarrow \Delta \in$[2, 4]

25.

(b) a = cos 2θ, b = sin 2θ

Explanation: $\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix} \begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

$\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix} \frac{1}{\sec^2\theta} \begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

$\frac{1}{\sec^2\theta} \begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix} \begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

$\begin{bmatrix} \frac{1-\tan^2\theta}{\sec^2\theta} & \frac{-2\tan\theta}{\sec^2\theta} \\ \frac{2\tan\theta}{\sec^2\theta} & \frac{1-\tan^2\theta}{\sec^2\theta} \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

$a = \frac{1-\tan^2\theta}{\sec^2\theta}$ Use tan in terms of sin and cos and simplify, we get,

a = $\cos^2\theta$ - $\sin^2\theta$

a = cos 2θ

and $b = \frac{2\tan\theta}{\sec^2\theta}$ = sin 2θ

26. **(a)** Singular

Explanation: $\begin{vmatrix} ab & b^2 \\ -a^2 & -ab \end{vmatrix} = -a^2b^2 + a^2b^2 = 0$

Therefore, it is Singular.

27.

(c) $\begin{vmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{vmatrix}$

Explanation: $\begin{vmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{vmatrix}$

28.

(d) ± 6

Explanation: We have $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$

$\Rightarrow$ $2x^2$ - 40 = 18 + 14

$\Rightarrow$ $2x^2$ = 72

$\Rightarrow$ x^2 = 36

$\Rightarrow$ x = ± 6

29.

(d) $\frac{1}{13}\begin{vmatrix} 5 & -1 \\ -2 & 3 \end{vmatrix}$

Explanation: A = $\begin{vmatrix} 3 & 1 \\ 2 & 5 \end{vmatrix}$, |A| = 15 - 2 = 13

adj (A) = $\begin{vmatrix} 5 & -1 \\ -2 & 3 \end{vmatrix}$

$\therefore$ A^{-1} = $\frac{1}{13}\begin{vmatrix} 5 & -1 \\ -2 & 3 \end{vmatrix}$

30.

(b) $\begin{vmatrix} 6 & 2 \\ 11/2 & 2 \end{vmatrix}$

Explanation: $\begin{vmatrix} 1 & -4 \\ 3 & -2 \end{vmatrix} X$ = $\begin{vmatrix} -16 & -6 \\ 7 & 2 \end{vmatrix}$ $\Rightarrow$ X = $\begin{vmatrix} 1 & -4 \\ 3 & -2 \end{vmatrix}^{-1} \begin{vmatrix} -16 & -6 \\ 7 & 2 \end{vmatrix}$

$\Rightarrow$ X = $\frac{1}{10}\begin{vmatrix} -2 & 4 \\ -3 & 1 \end{vmatrix}\begin{vmatrix} -16 & -6 \\ 7 & 2 \end{vmatrix}$ = $\frac{1}{10}\begin{vmatrix} 60 & 20 \\ 55 & 20 \end{vmatrix}$ = $\begin{vmatrix} 6 & 2 \\ 11/2 & 2 \end{vmatrix}$

31.

(b) 4

Explanation: A = $\begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix}$, 2A = $\begin{vmatrix} 2 & 4 \\ 8 & 4 \end{vmatrix}$

12A | = 8 - 32 = - 24

IA | = 2 - 8 = 6

$\therefore$ |2A| = 4 $\times$ (-6) = 4|A|

$\Rightarrow$ k = 4

32. **(a)** 1

Explanation: We have, $\begin{vmatrix} x & x+1 \\ x-1 & x \end{vmatrix}$ = x(x) - (x + 1)(x - 1)

= $x^2 - \left(x^2 - 1\right)$

= $x^2 - x^2 + 1$ = 1

33.

(c) $\frac{1}{3}$

Explanation: We have, A = $\begin{bmatrix} 1 & 4 \\ 3 & 15 \end{bmatrix}$

Now, |A| = $\begin{bmatrix} 1 & 4 \\ 3 & 15 \end{bmatrix}$ = 15 -12 = 3

$\therefore$ $|A^{-1}| = |A|^{-1} = (3)^{-1} = \frac{1}{3}$

34. **(a)** do not lie in a straight line

Explanation: Given, the points are

(a + 5, a - 4), (a - 2, a + 3) and (a, a),

$\therefore \Delta=\frac{1}{2}\begin{vmatrix} a+5 & a-4 & 1 \\ a-2 & a+3 & 1 \\ a & a & 1 \end{vmatrix}$

Applying $R_1 \rightarrow R_1 - R_3$ and $R_2 \rightarrow R_2 - R_3$,

we get

$=\frac{1}{2}\begin{vmatrix} 5 & -4 & 0 \\ -2 & 3 & 0 \\ a & a & 1 \end{vmatrix}=\frac{1}{2}[1(15-8)]=\frac{7}{2} \neq 0$

Hence, given points form a triangle i.e. points do not lie in a straight line.

35.

(d) 3

Explanation: The given system of equations does not have solution if $\begin{vmatrix} 1 & 1 & 1 \\ 4 & \lambda & -\lambda \\ 3 & 2 & -4 \end{vmatrix}=0 \Rightarrow \begin{vmatrix} 0 & 0 & 1 \\ 4+\lambda & 2\lambda & -\lambda \\ 7 & 6 & 4 \end{vmatrix}=0$

$\Rightarrow(24+6\lambda-14\lambda)=0 \Rightarrow \lambda=3$

36.

(b) x = $\frac{11}{24}$, y = $\frac{1}{24}$

Explanation: x = $\frac{11}{24}$, y = $\frac{1}{24}$

37.

(b) $\left(A^{\mathrm{T}}\right)^{-1}$

Explanation: $\left(A^{\mathrm{T}}\right)^{-1}$

38.

(b) 3

Explanation: We know that, area of a triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by

$\Delta=\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$\therefore$ Area of triangle with vertices (-3, 0), (3, 0) and (0, k) is given by:

$\Delta=\frac{1}{2}\begin{vmatrix} -3 & 0 & 1 \\ 3 & 0 & 1 \\ 0 & k & 1 \end{vmatrix}$

$\Rightarrow \quad[-3(-k)-0+1(3k)]=\pm 18$

$\Rightarrow \quad 6k=\pm 18$

$\therefore \quad k=\pm \frac{18}{6}=\pm 3$

39.

(c) no solution

Explanation: The given system of equations does not has a solution if: $\begin{vmatrix} 3 & 1 & -1 \\ 5 & 2 & -3 \\ 15 & 6 & -9 \end{vmatrix}=0 \Rightarrow$ 3(-18 + 18) - 1(-45 + 45) -1(30 - 30) = 0

40. **(a)** 1

Explanation: Given: A(adj(A)) = $\begin{vmatrix} k & 0 \\ 0 & k \end{vmatrix}$ = k$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$ = KI

$\Rightarrow$ k = 1

41.

(b) -6, 4

Explanation: -6, 4

42.

(c) -2

Explanation: -2

43.

(b) $\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

Explanation: Let A = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then |A| = $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$.

Now, cofactors of elements of |A| are

$C_{11} = (-1)^{1+1}\ 4 = 4$,

$C_{12} = (-1)^{1+2}\ (3) = -3$,

$C_{21} = (-1)^{2+1}\ (2) = -2$

and $C_{22} = (-1)^{2+2}\ (1) = 1$

Now, adj (A) = $\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$

$= \begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}^T$

$= \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

44.

(d) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm 2A$

Explanation: $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm 2A$

Explanation:

$A = \frac{1}{2}\ [x_1 (y_2 - y_3) - x_2 (y_1 - y_3) + x_3 (y_1 - y_2)]$

$2A = [x_1 (y_2 - y_3) - x_2 (y_1 - y_3) + x_3 (y_1 - y_2)]$

45.

(c)

|A|I

Explanation: Since, we know that

$A^{-1} = \frac{adjA}{|A|}$

pre multiply by A,

$AA^{-1} = \frac{AadjA}{|A|}$

$I = \frac{AadjA}{|A|} \Rightarrow AadjA = |A|\, I$ (since AA^{-1}=I)

46.

(c) $A^2 + B^2$

Explanation: $B = -A^{-1}BA$

$\Rightarrow AB = -AA^{-1}BA$

$\Rightarrow AB = -IBA$

$\Rightarrow AB = -BA$

$\Rightarrow AB + BA = O$ ...(i)

Consider, $(A + B)^2 = A^2 + AB + BA + B^2$ ...($\because AB \neq BA$)

$(A + B)^2 = A^2 + O + B^2$ ...from (i)

$(A + B)^2 = A^2 + B^2$

47.

(b) $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$

Explanation: A = $\begin{vmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{vmatrix}$

A^TA = $\begin{vmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{vmatrix}\begin{vmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{vmatrix}$

= $\begin{vmatrix} \cos^2\alpha+\sin^2\alpha & \cos\alpha\sin\alpha-\sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha-\sin\alpha\cos\alpha & \sin^2\alpha+\cos^2\alpha \end{vmatrix}$

= $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$

48.

(c) unique solution

Explanation: The given system of linear equations is:-

$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$

$\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

$-\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

Let x^2 = p, y^2 = q, x^2 = r

The equation becomes,

$\frac{p}{a^2}+\frac{q}{b^2}-\frac{r}{c^2}=1$

$\frac{p}{a^2}-\frac{q}{b^2}+\frac{r}{c^2}=1$

$-\frac{p}{a^2}+\frac{q}{b^2}+\frac{r}{c^2}=1$

The matrix equation corresponding to the above system of equation is

$$\begin{bmatrix} \frac{1}{a^2} & \frac{1}{b^2} & -\frac{1}{c^2} \\ \frac{1}{a^2} & \frac{-1}{b^2} & \frac{1}{c^2} \\ -\frac{1}{a^2} & \frac{1}{b^2} & \frac{1}{c^2} \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 5 \\ 9 \\ \mu \end{bmatrix}$$

Let A = $\begin{bmatrix} \frac{1}{a^2} & \frac{1}{b^2} & -\frac{1}{c^2} \\ \frac{1}{a^2} & \frac{-1}{b^2} & \frac{1}{c^2} \\ -\frac{1}{a^2} & \frac{1}{b^2} & \frac{1}{c^2} \end{bmatrix}$

$\therefore$ |A| = $\begin{vmatrix} \frac{1}{a^2} & \frac{1}{b^2} & -\frac{1}{c^2} \\ \frac{1}{a^2} & \frac{-1}{b^2} & \frac{1}{c^2} \\ -\frac{1}{a^2} & \frac{1}{b^2} & \frac{1}{c^2} \end{vmatrix}$

Taking common $\frac{1}{a^2}$ from C_1, $\frac{1}{b^2}$ from C_2 and $\frac{1}{c^2}$ from C_3 we get,

|A| = $\frac{1}{a^2}\frac{1}{b^2}\frac{1}{c^2}\begin{vmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{vmatrix}=\frac{-4}{a^2b^2c^2}\neq 0$.

49.

(d) non-singular matrix

Explanation: non-singular matrix

50.

(c) 2^6

Explanation: |A| = d

|adj A| = $|A|^{n-1}$

Here, n = 3, |A| = 8

|adj A| = 8^2

|adj A| = $(2^3)^2 = 2^6$

51.

(d) infinitely many solution

Explanation: Given equations are:

3x - 5y = 7 ...(i)

and 6x - 10y = 14 or 3x - 5y = 7 ...(ii)

Equations (i) and (ii) are same.

Hence it will have infinitely many solutions.

52.

(c) a unique solution

Explanation: a unique solution

The given system of equations can be written in matrix form as follows:

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & -1 & 2 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ -18 \end{bmatrix}$$

AX = B

Here,

A = $\begin{bmatrix} 1 & 1 & 1 \\ 3 & -1 & 2 \\ 3 & 1 & 1 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B = $\begin{bmatrix} 2 \\ 6 \\ -18 \end{bmatrix}$

|A| = 1(- 1 - 2) - 1 (-3 - 6) + 1(3 + 3)

= -3 + 3 + 6

= 6 not equal to 0.

So, the given system of equations has a unique solution.

53.

(b) 27 IAI

Explanation: If A is a matrix of order 3 $\times$ 3, then

|3A| = 3 $\times$ 3 $\times$ 3 |A| = 27 |A|

54.

(b) adj A

Explanation: $A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

|A| = $\cos^2\theta$ - ($-\sin^2\theta$)

= $\cos^2\theta$ + ($\sin^2\theta$)

= 1 ...(i)

We know that $A^{-1} = \frac{1}{|A|}$ adj A

= adj A [From I]

55.

(c) ± 2

Explanation: Since, the area of the $\triangle$ABD is 3 sq units, then we have

$$\frac{1}{2}\begin{vmatrix} 1 & 3 & 1 \\ 0 & 0 & 1 \\ k & 0 & 1 \end{vmatrix} = \pm 3$$

$\Rightarrow \frac{1}{2}[0 + 0 - 1(0 - 3k)] = \pm 3$

$\Rightarrow \frac{3k}{2} = \pm 3$

$\Rightarrow k = \pm 2$

56. **(a)** $\begin{bmatrix} \frac{3}{7} & \frac{1}{7} \\ \frac{-1}{7} & \frac{2}{7} \end{bmatrix}$

Explanation: $A^{-1} = \frac{1}{|\mathbf{A}|} \operatorname{adj} A$ ...(i)

|A| = 3 $\times$ 2 - (1) $\times$ (-1)

= 7

C_{11} = 3, C_{12} = -1

$C_{21} = 1$, $C_{22} = 2$

Co-factor matrix $A = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}$

$\text{Adj } A = \begin{pmatrix} 3 & -1 \\ 1 & 2 \end{pmatrix}'$

$= \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$

Putting in 1

$A^{-1} = \frac{1}{|7|} \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$

$= \begin{pmatrix} \frac{3}{7} & \frac{1}{7} \\ \frac{-1}{7} & \frac{2}{7} \end{pmatrix}$

57.

(b) Non-existent

Explanation: $\begin{bmatrix} 5 & 10 & 3 \\ -2 & -4 & 6 \\ -1 & -2 & b \end{bmatrix}$ is singular matrix. So its determinant value of this matrix is zero.

i.e, $\begin{vmatrix} 5 & 10 & 3 \\ -2 & -4 & 6 \\ -1 & -2 & b \end{vmatrix} = 0$

$\Rightarrow$ 5(-4b + 12) - 10(-2b + 6) + 3(4 - 4) = 0

$\Rightarrow$ -20b + 60 + 20b - 60 = 0

b does not exist

58.

(c) all integral α

Explanation: The given system of equations has unique solution, if:

$\begin{vmatrix} 1 & 1 & 1 \\ 3 & 6 & 1 \\ \alpha & 2 & 3 \end{vmatrix} \neq 0$

$\Rightarrow$ 1(18 - 2) - 1(9 - α) + 1(6 - 6α) $\neq$ 0

$\Rightarrow$ 13 - 5α $\neq$ 0

$\Rightarrow \alpha \neq \frac{13}{5}$. (Since α is not integral value)

Thus, unique solution exists for all integral values of α.

59.

(d) x = 1, y = 1

Explanation: $\begin{pmatrix} x & y \\ 3y & x \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} x \times 1 + y \times 2 \\ 3y \times 1 + x \times 2 \end{pmatrix}$

$= \begin{pmatrix} x + 2y \\ 3y + 2x \end{pmatrix}$

Comparing with R.H.S

x + 2y = 3 ...(i)

2x + 3y = 5 ...(ii)

(i) x 2 - (ii)

2x + 4y - 2x + 3y = 6 - 5

y = 1

Putting y in (i)

x + 2(1) = 3

x = 1

60. **(a)** $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Explanation: $S = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$M_{11} = d \Rightarrow A_{11} = d$

$M_{12} = c \Rightarrow A_{12} = -c$

$M_{21} = b \Rightarrow A_{21} = -b$

$M_{22} = a \Rightarrow A_{22} = a$

$\Rightarrow \mathrm{Ad}(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

61.

(c) $\begin{vmatrix} 4/11 & 1/11 \\ -3/11 & 2/11 \end{vmatrix}$

Explanation: A = $\begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix}$, adj A = $\begin{vmatrix} 4 & 1 \\ -3 & 2 \end{vmatrix}$

|A| = 8 + 3 = 11

$\therefore A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{11}\begin{vmatrix} 4 & 1 \\ -3 & 2 \end{vmatrix} = \begin{vmatrix} 4/11 & 1/11 \\ -3/11 & 2/11 \end{vmatrix}$

62.

(c) 0

Explanation: Determinant value of skew-symmetric matrix is always '0'.

63.

(c) PX = -X

Explanation: Given P' = 2P + I

$\Rightarrow$ (P')' = (2P + I)' = 2P' + I' = 2P' + I

$\Rightarrow$ P = 2(2P + I) + I = 4P + 2I + I

$\Rightarrow$ P = 4P + 3I $\Rightarrow$ -3P = 3I $\Rightarrow$ P = -I

$\therefore$ PX = -IX = -X

64.

(c) One solution

Explanation: For Unique solution ,$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$,

for given system of equations we have:$\frac{2}{1} \neq \frac{1}{-2}, \frac{1}{3} \neq \frac{-2}{5}, \frac{3}{2} \neq \frac{5}{1}$

65.

(d) 0 or 1

Explanation: Since, $A^2 = A \Rightarrow |A|^2 = |A|$

$\therefore$ (Det. A)(Det. A) = Det.A, $\therefore$ det.A [det.A - 1] = 0. $\therefore$ det.A = 0 or 1.

66.

(d) $k \neq 0$

Explanation: In the given question the system of linear equation has unique solution if:

$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & -k \end{vmatrix} \neq 0 \Rightarrow 1(-k+2)\ -1(-2k+3)+1(4-3) \neq 0 \Rightarrow k \neq 0$

67.

(c) $a_1b_2 = a_2b_1$

Explanation: Given that, the three points are collinear.

$\therefore \frac{1}{2}\begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_1+a_2 & b_1+b_2 & 1 \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_1+a_2 & b_1+b_2 & 1 \end{vmatrix} = 0$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$,

we get

$$\begin{vmatrix} a_1 & b_1 & 1 \\ a_2 - a_1 & b_2 - b_1 & 0 \\ a_2 & b_2 & 0 \end{vmatrix} = 0$$

$\Rightarrow \begin{vmatrix} a_2 - a_1 & b_2 - b_1 \\ a_2 & b_2 \end{vmatrix} = 0$ [expanding along C_3]

$\Rightarrow b_2(a_2 - a_1) - d_2(b_2 - b_1) = 0$

$\Rightarrow a_2b_2 - a_1b_1 - a_2b_2 + a_2b_1 = 0$

$\therefore a_1b_2 = a_2b_1$

68.

(d) independent of θ only

Explanation: Let $\Delta = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$

By expanding along first row, we get

$\Delta = x\begin{vmatrix} -x & 1 \\ 1 & x \end{vmatrix} - \sin\theta\begin{vmatrix} -\sin\theta & 1 \\ \cos\theta & x \end{vmatrix} + \cos\theta\begin{vmatrix} -\sin\theta & -x \\ \cos\theta & 1 \end{vmatrix}$

$= x(-x^2 - 1) - \sin\theta(-x \sin\theta - \cos\theta) + \cos\theta(-\sin\theta + x \cos\theta)$

$= -x^3 - x + x \sin^2\theta + \sin\theta \cos\theta - \sin\theta \cos\theta + x \cos^2\theta$

$= -x^3 - x + x(\sin^2\theta + \cos^2\theta) = -x^3 - x + x[\because \sin^2\theta + \cos^2\theta = 1]$

$= -x^3$

Hence, Δ is independent of θ.

69. **(a)** 14641

Explanation: We know that, for a square matrix of order n, if $|A| \neq 0$

$Adj(Adj\ A) = |A|^{n-2} A$ ($\because$ n = 3)

$\therefore Adj(Adj\ A) = |A|^{3-2} A$ ($\because$ n = 3)

$= |A|\ A$

$\therefore |Adj(Adj\ A) = ||A|\ A|| = |A|^3 \det A\ |A|^4$

$= 11^4 = 14641$

70.

(b) singular

Explanation: Given, A = $\begin{bmatrix} 1 & 2 \\ 6 & 12 \end{bmatrix}$

Now, |A| = $\begin{vmatrix} 1 & 2 \\ 6 & 12 \end{vmatrix}$

= 12 - 12 = 0

Hence, A is a singular matrix.

71.

(c) None of these

Explanation: If det (A+B) = 0 implies that A+B a Singular matrix.

72.

(d) (2, 4)

Explanation: (2, 4)

73.

(c) $A^{-1} = |A|^{-1}$

Explanation: $A^{-1} = |A|^{-1}$

74. **(a)** 16

Explanation: Since, A is a square matrix of order 3 i.e. n = 3

we know that

$|adj\ A| = (A)^{n-1}$

$|adj\ (adj\ A)| = |A|^{(n-1)^2}$

$= 2^{(3-1)^2}$ ...($\because$ |A| = 2)

$= 2^4$

= 16

75.

(d) 27

Explanation: We have, $A^2 = 3A \Rightarrow |A^2| = |3A|$

$\Rightarrow |A|.|A| = 3^3|A|$ ($\therefore$ order of matrix A is 3 and |A| is not equal to zero)

$\Rightarrow |A| = 3^3 = 27 \Rightarrow |A| = 27$