Solution

GRAVITATION WS 1

Class 11 - Physics

1.

(b) 22.4 km s^{-1}

Explanation: v_e = $\sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G}{R} \cdot \frac{4}{3}\pi R^3 \rho}$

= $\sqrt{\frac{8\pi}{3} G\rho} \times R$

$\therefore \quad v_e \propto R$

$\frac{v_e(\text{ planet })}{v_e(\text{ earth })} = \frac{2R}{R}$ = 2

v_e(planet) = 2 $\times$ 11.2 = 22.4 km s^{-1}

2. **(a)** to balance the effect of earth's gravity

Explanation: The waving of arms helps in keeping the centre of gravity at a suitable position. This helps the man to walk comfortably.

3.

(b) $\frac{T^2}{R^3}$

Explanation: According to 3rd law of Kepler

$T^2 \propto R^3$

$\Rightarrow T^2 = KR^3$

where K is a constant

Thus $\frac{T^2}{R^3}$ does not depends on radius.

4.

(b) $\frac{\text{GMmx}}{\left(a^2+x^2\right)^{\frac{3}{2}}}$

Explanation: Consider a mass segment dM = $\left(\frac{M}{2\pi a}\right) dl = \left(\frac{M}{2\pi a}\right) ad\phi$ on the ring. Whereas $\phi$is the integrated angle. The potential energy between this segment and the mass m is given by

dU = $-G\frac{mdM}{\sqrt{a^2+x^2}} = -G\frac{m\frac{M}{2\pi}d\phi}{\sqrt{a^2+x^2}}$

So that

$U = \int dU = -\int_0^{2\pi} G\frac{m\frac{M}{2\pi}d\phi}{\sqrt{a^2+x^2}}$

$= -G\frac{mM}{\sqrt{a^2+x^2}}$

$F = -\frac{dU}{dx} = -\frac{d}{dx}\left[-G\frac{mM}{\sqrt{a^2+x^2}}\right] = \frac{GMmx}{\left(a^2+x^2\right)^{\frac{3}{2}}}$

5.

(d) maximum

Explanation: From conservation of angular momentum,

Velocity of planet (v) $\propto \frac{1}{\text{Distance of the planet from sun }(r)}$

So, r_P is minimum for perihelion (P).

$\Rightarrow v_P$ is maximum

6.

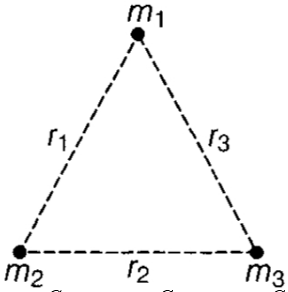
(b) F

Explanation: The gravitational force between two bodies is independent of the nature of the medium between them.

7.

(d) $\left(\frac{-Gm_1m_2}{r_1}\right) + \left(\frac{-Gm_2m_3}{r_2}\right) + \left(\frac{-Gm_1m_3}{r_3}\right)$

Explanation: For a system of particles, all possible pairs are taken and total gravitational potential energy is the algebraic sum of the potential energies due to each pair, applying the principle of superposition. Total gravitational potential energy.



$= \frac{-Gm_1m_2}{r_1} - \frac{Gm_2m_3}{r_2} - \frac{Gm_1m_3}{r_3}$

$= \left(\frac{-Gm_1m_2}{r_1}\right) + \left(\frac{-Gm_2m_3}{r_2}\right) + \left(\frac{-Gm_1m_3}{r_3}\right)$

8. **(a)** 2.67×10^{-9} N

Explanation: In this case, at r = 2.50 m, only a fraction of mass M is located,

so first, we calculate a mass for the position r = 2.50

since density is uniform, So

$\sigma = \sigma'$

$\frac{M}{\frac{4}{3}\pi R^3} = \frac{M'}{\frac{4}{3}\pi r^3}$

Here R = 5 m, r = 2.50 m

$\Rightarrow \frac{1000}{(5)^3} = \frac{M'}{(2.50)^3}$

$\Rightarrow M' = \frac{1000 \times 2.5 \times 2.5 \times 2.5}{125}$

$\Rightarrow$ M′ = 125 kg

We know the gravitational force

F = $\frac{GM'm}{r^2}$

Here G = $6.67 \times 10^{-11} Nm^2kg^{-2}$

M′ = 125 kg m = 2.0 kg

r = 2.5 m

$\Rightarrow F = \frac{6.67 \times 10^{-11} \times 125 \times 2}{(2.5)^2}$

$\Rightarrow F = \frac{1667.5 \times 10^{-11} \times 10^2}{625}$

$\Rightarrow$ F = $2.668 \times 10^{-9} = 2.67 \times 10^{-9}$N

9. **(a)** 8 km/s

Explanation: Near earth's surface,

$v_0 = \sqrt{gR}$ = 7.2 km s^{-1}

= 8 km s^{-1}

10.

(c) deviates considerably from being elliptical due to influence of planets other than earth

Explanation: The gravitational force of attraction due to earth on the moon follows the inverse square law due to which the as seen from the earth, the moon revolves around it in a circular orbit.

When observed from the sun, the moon experiences the gravitational pull due to both, the sun and the moon which results in a net force thus changing the trajectory of the moon, and hence it does not revolve in strictly elliptical due to the influence of both sun and earth.

11. **(a)** decreases from poles to equator

Explanation: Acceleration due to gravity decreases from poles to equator.

12. **(a)** $\frac{R}{2}$

Explanation: $\frac{R}{2}$

13.

(c) 22.4 km/sec

Explanation: $v_e = \sqrt{2gR} = \sqrt{\frac{2GM}{R}}$

if R is $\frac{1}{4}$th then $v_e = 2v_{e-earth} = 2 \times 11.2 = 22.4$ km/sec

14.

(d) increases by 2%

Explanation: g = $\frac{GM}{R^2}$

For constant G and M

$\frac{\Delta g}{g} \times 100 = -2\frac{\Delta R}{R} \times 100$ = -2(-1)% = + 2%

The value of g increases by 2%.

15.

(d) g' = 8g

Explanation: g' = 8g

16.

(b) $g(d) = g(1 - \frac{d}{R_E})$

Explanation: Acceleration due to gravity at the surface of the earth of radius R_E

$g = G\frac{M}{R_E^2} = \frac{4}{3}\pi\rho G R_E$ ...(1)

Acceleration due to gravity at depth d from the surface of the earth

$g(d) = \frac{4}{3}\pi\rho G(R - d)$ ...(2)

From (1) & (2), we get

$g(d) = g(1 - \frac{d}{R_E})$

17. **(a)** the torque is zero

Explanation: As the earth is revolving around the sun in a circular motion (approximately in actual the path of the earth around the sun is elliptical) due to gravitational attraction. When we consider the earth-sun as a single system and we are taking earth as a sphere of uniform density. Then the gravitational force (F) will be of radial nature, i.e. the angle between position vector r and force F is zero. So, torque $|\vec{\tau}| = |\vec{r} \times \vec{F}| = rF\sin 0^\circ$ = 0

18.

(d) 3m

Explanation: $g_c = \frac{4}{3}\pi G R p$

$g_m = \frac{4}{3}\pi G\left(\frac{R}{4}\right)\left(\frac{2}{3}\rho\right) = \frac{1}{6}g_e$

$mg_e h_e = mg_m h_m$

$mg_e \times 0.5 = m \times \frac{1}{6}g_e \times h_w$

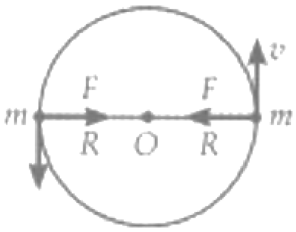
$\therefore h_m = 3.0$ m

19.

(b) $\frac{1}{2}\sqrt{\frac{Gm}{R}}$

Explanation:

The two masses, separated by a distance 2 R, revolve about the common centre of mass O.



Centripetal force = Mutual gravitational attraction

$\frac{mv^2}{R} = \frac{Gm \times m}{(2R)^2}$

or $v^2 = \frac{GM}{4R}$

$\therefore v = \frac{1}{2}\sqrt{\frac{Gm}{R}}$

20.

(b) $\frac{3}{2}\left(\frac{T}{r}\right)\Delta r$

Explanation: $T^2 \propto r^3$

or $T \propto r^{\frac{3}{2}}$
or $T = kr^{\frac{3}{2}}$
$\therefore \quad \frac{dT}{dr} = \frac{3}{2}kr^{\frac{1}{2}}$
= $\frac{3}{2}\left(\frac{T}{r^{\frac{3}{2}}}\right)r^{\frac{1}{2}} = \frac{3}{2}\left(\frac{T}{r}\right)$
$\therefore \quad \Delta T = \frac{3}{2}\left(\frac{T}{r}\right)\Delta r$

21.
(d) $\frac{\Delta A}{\Delta t}$ = constant
Explanation: $\frac{\Delta A}{\Delta t}$= constant

22. **(a)** $\frac{1}{2}mgR$
Explanation: At the surface of the earth,
$U_i = -\frac{GMm}{R}$
At a height equal to the radius of the earth,
$U_f = -\frac{GMm}{R+R} = -\frac{GMm}{2R}$
$\Delta U = U_f - U_i = \frac{GMm}{R}\left(1 - \frac{1}{2}\right)$
= $\frac{gR^2M}{R} \times \frac{1}{2}$ [GM = gR^2]
= $\frac{1}{2}mgR$

23.
(b) mv^2
Explanation: Escape speed = $\sqrt{2} \times$ orbital speed
$v_e = \sqrt{2}v$
At the time of its ejection, kinetic energy of the object will be
K = $\frac{1}{2}mv_e^2 = \frac{1}{2}m(\sqrt{2}v)^2$ = mv^2

24.
(d) 12.86 km
Explanation: Ratio of acceleration due to gravity $\frac{g'}{g} = \frac{978}{980} = 1 - \frac{d}{R}$
or $\frac{d}{R} = 1 - \frac{978}{980} = \frac{2}{980}$ or d = $\frac{2R}{980}$
= $\frac{2 \times 6300}{980}$
= 12.86 km

25.
(b) increases
Explanation: At the equator, $g_e = g - R\omega^2$
When ω = 0, $g_e = g$
The value of g increases if the earth stops rotating.

26. **(a)** v = $\sqrt{gR_e}$, where g = $\frac{GM_e}{R_e^2}$
Explanation: Orbital velocity of satellite, v = $\sqrt{\frac{GM_e}{(R_e+h)}}$
If the satellite is close to the surface of the earth, h = 0
$\Rightarrow$ v = $\sqrt{\frac{GM_e}{R_e}}$ $\Rightarrow$ v = $\sqrt{\left(\frac{GM_e}{R_e^2}\right)R_e}$
= $\sqrt{gR_e}$ [$\because$ g = $\frac{GM_e}{R_e^2}$]

27. **(a)** $\sqrt{\frac{r_2}{r_1}}$
Explanation: v = $\sqrt{\frac{\mathrm{GM}}{r}}$
v is independent of mass of the satellite.
$\frac{\mathrm{v}_1}{\mathrm{v}_2} = \sqrt{\frac{\mathrm{r}_2}{\mathrm{r}_1}}$

28.

(b) doubled

Explanation: Escape speed, v_{min} = $\sqrt{2gR}$

Since we talking about another planet with different radius, gravitational force also changes.

29.

(b) downward at 2.4 m/s^2

Explanation: The acceleration will be downward at 2.4 m/s^2.

30.

(c) $\frac{1}{\sqrt{2}}$

Explanation: Escape velocity from earth's surface,

v = $\sqrt{2gR}$

Let v', be the escape velocity from the platform at height R. As total energy of body at infinity is zero,

$\frac{1}{2}mv_c'^2$ + P.E. of the body on the platform = 0

$\frac{1}{2}mv_c'^2 + \left(-\frac{GMm}{R+R}\right) = 0$

$v_e' = \sqrt{\frac{GM}{R}} = \sqrt{gR}$

But v_e' = fv

$\sqrt{gR} = f \times \sqrt{2gR}$

or f = $\frac{1}{\sqrt{2}}$

31. **(a)** $\sqrt{3}v_{es}$

Explanation: Applying the law of conservation of energy

(K.E. of the body) + (Gravitational potential energy) = K.E of the body in interplanetary space

= $\sqrt{3}v_{es}$

32. **(a)** 5510 m/s (apogee), 8430 m/s (perigee)

Explanation: At Apogee(High Point):

$R_P = R_E + h_P = 6.38 \times 10^6 m + 400 \times 10^3 m$

$= 6.38 \times 10^6 m + 0.40 \times 10^6 m$

At Perigee(Low Point):

$R_a = R_E + h_a = 6.38 \times 10^6 m + 4000 \times 10^3 m$

$= 6.38 \times 10^6 m + 4 \times 10^6 m$

$= 10.38 \times 10^6$ m

$a = \frac{R_p+R_a}{2}$ = $8.58 \times 10^6 m$

Angular momentum, L is conserved because no torque acts on the system

i.e $L = mv_aR_a = mv_pR_p$

Thus $\frac{v_p}{v_a} = \frac{R_a}{R_p}$ = 1.513

Energy is conserved because non - conservative work is zero.

Thus $K_a + U_a = K_p + U_p$

$\Rightarrow K_p - K_a = U_a - U_p$

$\Rightarrow \frac{1}{2}mv_p^2 - \frac{1}{2}mv_a^2 = \frac{-GMm}{R_a} - \left(\frac{-GMm}{R_p}\right)$

$\Rightarrow v_p^2 - v_a^2 = 2GM\left(\frac{1}{R_P} - \frac{1}{R_a}\right)$

But, As we know $\frac{v_p}{v_a} \Rightarrow v_p = 1.531 \times v_a$ - (1)

Thus, $\left[(1.531)^2 - 1\right] v_a^2 = 2GM\left(\frac{1}{R_p} - \frac{1}{R_a}\right)$

Solving for V_a, we get

$v_a = 5.51 \times 10^3$ m/sec = 5510 m/sec

Put value of v_a in eqn(1),

we get $v_p = 8.43 \times 10^3$m/sec = 8430 m/sec

33. **(a)** $\frac{4}{3} \times 10^3$ km

Explanation: By conservation of angular momentum,

$I_1 = I_2$ or $mv_1d_1 = mv_2d_2$

$\therefore v_2 = \frac{v_1 d_1}{d_2}$

34. **(a)** $\sqrt{\frac{2GM_e}{R_e}}$

Explanation: The gravitational potential energy of a body of mass m placed on earth's surface is given by U = $\frac{GM_e m}{R_e}$

Therefore, in order to take a body from the earth's surface to infinity, the work required is $\frac{GM_e m}{R_e}$.

Hence it is evident that if we throw a body of mass m with such a velocity that it's kinetic energy is $\frac{GM_e m}{R_e}$, then it will move outside the gravitational field of earth.

Hence, $\frac{1}{2} m v_e^2 = \frac{GM_e m}{R_e}$ or, $v_e = \sqrt{\frac{2GM_e}{R_e}}$

35.

(c) 9

Explanation: 9

36. **(a)** 3R

Explanation: Acceleration due to gravity at a height h from the surface of earth is

$g' = \frac{g}{\left(1+\frac{h}{R}\right)^2}$ ...(i)

where g is the acceleration due to gravity at the surface of earth and R is the radius of earth. Multiplying by m (mass of the body) on both sides in (i),

we get

m $g' = \frac{mg}{\left(1+\frac{h}{R}\right)^2}$

$\therefore$ Weight of body at height h, $W' = mg'$

Weight of body at surface of earth, W = mg

According to question, $W' = \frac{1}{16} W$

$\therefore \quad \frac{1}{16} = \frac{1}{\left(1+\frac{h}{R}\right)^2}$

$\left(1 + \frac{h}{R}\right)^2 = 16$ or $1 + \frac{h}{R}$ = 4

or $\frac{h}{R}$ = 3 or h = 3 R

37.

(b) 52 N

Explanation: Apparent weight at equator = m(g - a)

g = 10.7m/sec^2

a = centripetal accel. at equator = Rω^2

216 hr = 57600s

$\omega = \frac{2\pi}{T} = \frac{2\pi}{57600s} = 1.1 \times 10^{-4} \text{rad/sec}$

a = Rω^2 = 25 $\times$ 10^{6}m $\times$ (1.1 $\times$ 10^{-4})^2

= 0.297 $\approx$ 0.30m/sec^2

App. weight = m(g - a) = 5(10.7 - 0.3) = 52 N

38.

(d) less than the orbital speed of earth

Explanation: $v_0 = \sqrt{\frac{GM_{sun}}{r}}$

or $v_0 \propto \frac{1}{\sqrt{r}}$

As Jupiter is at a larger distance from the sun than the earth, so the orbital speed of Jupiter is less than that of the earth.

39.

(d) 99.66 N

Explanation: $g_p - g_e = R\omega^2 = 3.37 \times 10^{-2}$ ms^{-2}

$g_e = g_p - 3.37 \times 10^{-2}$ ms^{-2}

= (10 - 0.0337) ms^{-2}

$W_e = mg_e = \frac{100}{10} \times 9.9663$ N = 99.66 N

40. **(a)** Both $\frac{mgh}{\left(1+\frac{h}{R}\right)}$ and $\frac{mghR}{(R+h)}$

Explanation: Both $\frac{mgh}{\left(1+\frac{h}{R}\right)}$ and $\frac{mghR}{(R+h)}$

41. **(a)** will move in orbits like planets and obey Kepler's laws

Explanation: Like any other planet, asteroids also follow Kepler's law and move in elliptical orbits around the planet due to the central gravitational force acting on them.

a small mass of asteroid does not escape the gravitational force due to the sun because of the large mass of the sun.

42.

(b) 1.65 hr

Explanation: Mass of the Earth, $M_e = 6.0 \times 10^{24}$ kg

The radius of the Earth, Re = $6.4 \times 10^6 m$

Universal gravitational constant, G = 6.67×10^{-11} Nm^2 kg^{-2}

Height of the satellite, h = 780 km = 780×10^3 m = 0.78×10^6 m

Time Period of the satellite, T = $2\pi\sqrt{\frac{(R_e+h)^3}{GM_e}}$

$= 2 \times \frac{22}{7} \times \sqrt{\frac{\left(6.4\times10^6+0.78\times10^6\right)^3}{6.67\times10^{-11}\times6.0\times10^{24}}}$

$= 2 \times \frac{22}{7} \times \sqrt{\frac{\left(7.18\times10^6\right)^3}{40\times10^{13}}}$

$= 2 \times \frac{22}{7} \times \sqrt{9 \times 10^5}$

$= 2 \times \frac{22}{7} \times 948$

= $2 \times 22 \times 135.42$

= 5958.85 sec = 1.65 hr

43. **(a)** $\sqrt{2g'R_m}$ where g' = acceleration due to gravity on the moon and R_m = radius of the moon

Explanation: Escape speed from the moon = $\sqrt{2g'R_m}$

where,

g' = acceleration due to gravity on the surface of moon.

R_m = radius of the moon

44.

(d) 2.9×10^{15} kg, 0.0077 m/s^2

Explanation:

i. We can calculate the mass of Toro by mass density relationship:
   Mass = Volume $\times$ Density
   $\Rightarrow M_{toro} = \frac{4}{3}\pi R_{toro}^3 \times \rho_{toro}$
   Here R_{toro} = 5.0 km = 5000 m

   ρtoro = 5.5 g/cm^3 = 55×10^2kg/m3
   $\Rightarrow M_{toro} = \frac{4}{3}\pi(5000)^3 \times 55 \times 10^2$
   $\Rightarrow M_{\text{toro}} = \frac{4}{3} \times \frac{22}{7} \times 125 \times 10^9 \times 55 \times 10^2$
   $\Rightarrow M_{\text{toro}} = 2.9 \times 10^{15}$ kg

ii. we can find the acceleration due to gravity at the surface of Toro by the following relation:
   $g = \frac{GM_{\text{toro}}}{R_{\text{toro}}^2}$

$\Rightarrow g=\frac{6.67\times10^{-11}\times2.9\times10^{15}}{(5000)^2}$

$\Rightarrow g=\frac{19.34\times10^4}{25\times10^6}$

$\Rightarrow$ g = 0.77 $\times$ 10^{-2} = 0.0077 m/sec^2.

45.

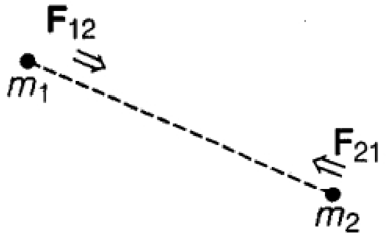
(b) 2 E_0

Explanation: Total energy = $-\frac{GMm}{2r}=E_0$

Potential energy = $-\frac{GMm}{r}=2E_0$

46. **(a)** $\overrightarrow{F_{12}}=-\overrightarrow{F_{21}}$

Explanation: Since, gravitational forces are attractive $\overrightarrow{F_{12}}$ is directed opposite to $\overrightarrow{F_{21}}$ and they are also equal in magnitude.



Hence, $\overrightarrow{F_{21}}=-\overrightarrow{F_{12}}$

or $\overrightarrow{F_{12}}=-\overrightarrow{F_{21}}$

47.

(d) 0.49 m/s^2

Explanation: For earth, g = $\frac{\mathrm{GM}}{\mathrm{R}^2}$ = 9.8 ms^{-2}

For moon, $g'=\frac{G(\frac{M}{80})}{(\frac{R}{2})^2}=\frac{1}{20}\frac{GM}{R^2}$

= $\frac{1}{20}\times$ = 0.49 m/s^2

48. **(a)** Both mg and $\frac{GmM_e}{R_e^2}$

Explanation: Force on particle at surface is

F = mg

where, g = acceleration due to gravity at the earth's surface

Also, g = $\frac{GM_e}{R_e^2}$

$\Rightarrow$ F= mg = $\frac{GmM_e}{R_e^2}$

49.

(d) $\frac{1}{2}$

Explanation: P.E. = $-\frac{\mathrm{CM}m}{r}$

Now $\frac{\mathrm{GMm}}{r^2}=\frac{mv^2}{r}$

$\therefore\quad \mathrm{K.E.}=\frac{1}{2}m\mathrm{v}^2=\frac{\mathrm{GMm}}{2r}$

$\frac{\mathrm{KE}}{\mathrm{P.E}}=\frac{\mathrm{GMm}}{2r}\times\frac{r}{\mathrm{GMm}}=\frac{1}{2}$

50.

(c) d = 2 km

Explanation: g_d = g_k

$8\left(1-\frac{d}{R}\right)-g\left(1-\frac{2h}{R}\right)$

d = 2h = 2 $\times$ 1 km = 2 km

51. **(a)** the speed of P is equal to that of Q

Explanation: Orbital speed does not depend on the mass of the satellite.

52.

(c) 12

Explanation: By conservation of angular momentum,

mvr = constant

m remains constant hence

Vr = constant

i.e., $V_{minimum} \times r_{maximum} = V_{maximum} \times r_{minimum}$

given: $r_{maximum} = 8 \times 10^{12}$ m

$r_{minimum} = 1.6 \times 10^{12}$ m

$V_{maximum}$ = 60 m/s

$V_{minimum}$ = ?

Hence

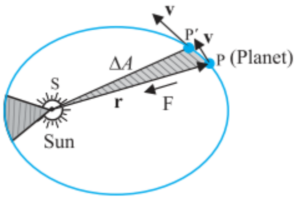
$V_{minimum} = \left[\frac{(60 \times 1.6 \times 1012)}{(8 \times 10^{12})}\right]$ = 12 m/s

53. **(a)** Option iv

Explanation: As the position D lies inside the larger ring, the net gravitational force due to spheres of this ring vill be zero at D.

54.

(d) $\Delta A/\Delta t = L/(2m)$

Explanation: 

The law of areas can be understood as the consequence of conservation of angular momentum which is valid for any central force such that the force on the planet is along the vector joining the sun and the planet. Let the sun be at the origin and let the position and momentum of the planet be denoted by r and p respectively. Then the area swept out by the planet of mass m in the time interval Δt is given by $\Delta A = \frac{1}{2}$ (r × vΔt).

Hence $\frac{\Delta A}{\Delta t} = \frac{1}{2}(\mathbf{r} \times \mathbf{p})/m = \frac{L}{2m}$

where **v** is the velocity, **L** is the angular momentum equal to (**r** × **p**).

55. **(a)** move with a velocity v tangentially to the original orbit

Explanation: In the absence of the gravitational force, the satellite will move with velocity v tangentially to the original orbit.

56.

(b) $\frac{r_1}{r_2}$

Explanation:

$g_1 = \frac{GM_1}{r_1^2} = \frac{G}{r_1^2} \times \frac{4}{3}r_1^3\rho = \frac{4}{3}\pi G r_1 \rho$

$g_2 = \frac{4}{3}\pi G r_2 p$

$\therefore \quad \frac{g_1}{g_2} = \frac{r_1}{r_2}$

57. **(a)** 53.5 N

Explanation: Gravitational Force

F = mg

$\Rightarrow$ F = 5 × 10.7 = 53.5 N

58.

(c) mgR

Explanation: Required K.E. = $\frac{1}{2}mv_e^2$

= $\frac{1}{2}$m × 2gR = mgR

59.

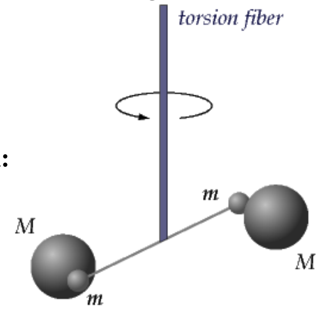
(b) kinetic energy

Explanation: K.E. changes due to the change in the speed of celestial body around the sun.

60.

(c) torque on bar AB having two small lead spheres due to gravitational forces is balanced by the restoring torque of the wire

Explanation:



Cavendish's apparatus for experimentally determining the value of G involved a light, rigid rod about 2-feet long. Two small lead spheres were attached to the ends of the rod and the rod was suspended by a thin wire. When the rod becomes twisted, the torsion of the wire begins to exert a torsional force that is proportional to the angle of rotation of the rod. The more twist of the wire, the more the system pushes backward to restore itself towards the original position.

61.

(b) $\frac{GM_Em}{12RE}$

Explanation: The energy of a satellite in low orbit:

$E_L = \frac{-GMm}{2r} = \frac{-GM_Em}{2(2R_E)} = \frac{-GM_Em}{4R_E}$

The energy of a satellite in high orbit:

$E_H = \frac{-GMm}{2r} = \frac{-GM_Em}{2(3R_E)} = \frac{-GM_Em}{6R_E}$

Energy required to move a satellite from the orbit of radius $2R_E$ to orbit of radius $3R_E$:

$= \Delta E$ = $E_H - E_L$ = $\frac{-GM_Em}{6R_E} - \left(\frac{-GM_Em}{4R_E}\right) \Rightarrow \Delta E = \frac{GM_Em}{12R_E}$

62.

(c) Conservation of angular momentum

Explanation: Kepler's second law states that the radius vector drawn from the sun to a planet sweeps out equal areas in equal intervals of time. It is based on the law of conservation of angular momentum.

63.

(d) $10\sqrt{10}$

Explanation: Distance of two planets from sun (r_1) = 10^{13}m and (r_2) = 10^{12}m

Relation between time period (T) and distance of the planet from the sun is $T^2 \propto r^3$ or $T \propto r^{\frac{3}{2}}$.

Therefore $\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{\frac{3}{2}} = \left(\frac{10^{13}}{10^{12}}\right)^{\frac{3}{2}} = 10^{\frac{3}{2}}$ = $10\sqrt{10}$

64.

(b) $4\sqrt{27}$

Explanation: Using, $\left(\frac{T_2}{T_1}\right)^2 = \left(\frac{R_2}{R_1}\right)^3$, we get

$\left(\frac{T_2}{T_1}\right)^2 = \left(\frac{2R}{R}\right)^3 = \frac{27}{1}$ i.e., $T_2 = \sqrt{27}T = 4\sqrt{27}$ hour

65.

(d) $\left(\frac{n}{n+1}\right)mgR$

Explanation: At distance r from the centre of the earth,

U = $-\frac{GMm}{r}$

At the earth's surface, r = R, so

$U_s = -\frac{GMm}{R}$ = -mgR $\left[g = \frac{GM}{R^2}\right]$

At height h = nR

r = R + nR = R(1 + n)

$U_h = -\frac{GMm}{R(1+n)} = -\frac{mgR}{(1+n)}$

Change in the gravitation P.E.,

ΔU = $U_h - U_s$

$= -\frac{mgR}{1+n} + mgR$
$= \text{mgR}\left(1 - \frac{1}{1+n}\right)$
$= \text{mgR}\left(\frac{n}{n+1}\right)$

66. **(a)** negative
Explanation: For velocity less than escape velocity, the missile is bound due to the gravitational field of the earth. Hence its total energy is negative.

67.
(d) $g(h) = \frac{GM_E}{(R_E+h)^2}$
Explanation: We know by Newton's law of gravitation, the force on the body of mass m, situated at height h to the surface of the earth of mass M_E is given by:
$F = G\frac{M_e m}{(r+h)^2}$ ...(i)
We also know F = mass $\times$ acceleration(here acceleration is the acceleration due to gravity)
$\Rightarrow$ F = mg ...(ii)
Equating (i) & (ii), we get
$g = G\frac{M_e}{(r+h)^2}$

68.
(c) two pens weighing 100 gm at a distance of 0.4 m
Explanation: Gravitational force is given by $F = G\frac{m_1 m_2}{r^2}$. Since $\frac{m_1 m_2}{r^2}$ is smallest in case of two pens and G is constant, so the gravitational force is very small.

69.
(c) 10840 m/sec(perigee),8760 m/sec(apogee)
Explanation: To escape Earth, we need total energy of zero.
(E_{final} = 0 because U $\longrightarrow$ 0 as R $\longrightarrow$ ∞ and K $\longrightarrow$ 0 as v = 0 at R $\longrightarrow$ ∞)
So,
$K_p + U_p = 0$
Looking for the new velocity at perigee;
$\frac{1}{2}mv_{p,escapc}^2 = \frac{GMm}{R_p}$
$v_{p,\text{ escape}} = \sqrt{\frac{2GM}{R_p}} = \sqrt{\frac{2\times6.67\times10^{-11}\times5.97\times10^{24}}{6.78\times10^6}}$
= 1.084 $\times$ 10^4m/sec = 10840 m/sec
The similar calculation at apogee gives
$v_{a,\text{escape}} = \sqrt{\frac{2GM}{R_a}} = \sqrt{\frac{2\times6.67\times10^{-11}\times5.97\times10^{24}}{10.38\times10^6}}$
= 8.76 $\times$ 10^3m/sec = 8760 m/sec

70.
(d) acceleration
Explanation: Acceleration due to gravity is independent of the mass of the body.

71.
(d) 11.2 km s^{-1}
Explanation: The escape velocity of a body does not depend on its angle of projection from the earth's surface.
So, v_e = 11.2 km s^{-1}

72.
(c) $\rho^{\frac{-1}{2}}$
Explanation: The time period T of the artificial satellite of earth depends on average density ρ of earth.

73. **(a)** $\sqrt{\frac{2GM}{R}}$
Explanation: For the particle to escape the gravitational field of the earth,

$\frac{1}{2}mu^2 = \frac{GMm}{R}$

or u = $\sqrt{\frac{2GM}{R}}$

74. **(a)** 12 : 1

Explanation: Total energy of a satellite

E = $-\frac{GMm}{2r}$

$\frac{E_A}{E_B} = \frac{m_A}{m_B} \cdot \frac{r_B}{r_A} = \frac{3}{1} \times \frac{4r}{r}$ = 12 : 1

75.

(c) super dense planetary material

Explanation: Black hole is a super dense planetary material formed due to the continued compression of the core of a star during supernova explosion.