

GEOPHYSICS

PAPER—III

Time Allowed : Three Hours

Maximum Marks : 200

QUESTION PAPER SPECIFIC INSTRUCTIONS

**Please read each of the following instructions carefully
before attempting questions**

There are **TEN** questions divided under **TWO** Sections.

Candidate has to attempt **SIX** questions in all.

Question Nos. **1** and **6** are **compulsory**. Out of the remaining **EIGHT** questions, **FOUR** questions are to be attempted choosing **TWO** from each Section.

The number of marks carried by a question/part is indicated against it.

Neat sketches may be drawn to illustrate answers, wherever required. These shall be drawn in the space provided for answering the question itself.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary, and indicate the same clearly.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly.

Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

Answers must be written in **ENGLISH** only.

Constants which may be needed :

Kepler's constant	=	$3.986004418 \times 10^5 \text{ km}^3 \text{ s}^{-2}$
Mean radius of the Earth	=	6378 km
Mass of electron (m_e)	=	$9.11 \times 10^{-31} \text{ kg}$
Charge of electron (e)	=	$1.602 \times 10^{-19} \text{ C}$
Planck's constant (h)	=	$6.62 \times 10^{-34} \text{ J s}$
Boltzmann's constant (k)	=	$1.38 \times 10^{-23} \text{ J/K}$
Permittivity of free space (ϵ_0)	=	$8.854 \times 10^{-12} \text{ F m}^{-1}$

SECTION—A

1. (a) Define average, RMS and stacking velocities for a layered Earth model of n layers. Express them in order of their increasing values. What is move-out time? 5
- (b) What is path radiance? Describe the total radiance recorded by a remote sensing sensor. 5
- (c) State and prove convolution theorem. 1+4=5
- (d) Given a dipole is situated at the centre of the Earth. Calculate the magnetic potential and magnetic field at a distance r from the dipole in spherical polar coordinates (r, θ, ϕ) , where r : radius, θ : colatitude and ϕ : longitude. 5
- (e) A digital filter is given by $y_n = 2x_n - 5x_{n-1} + 1.3y_{n-1}$. The sequence of input values fed to the filter is $x_0 = 5$, $x_1 = 1.8$, $x_2 = -4$, $x_3 = 3$, $x_4 = 2$. Write the output sequence y_0 to y_4 of the above filter. 5
- (f) Find out the expected change in gravity caused by the presence of seawater of depth 1.5 m. Assume density of seawater as 1.05 gm/cm^3 , density of rock between sea surface and sea bottom as 2.5 gm/cm^3 . Explain how terrain correction removes the effect of hill and compensates for valley. 3+2=5
- (g) Briefly explain the wavelength ranges used in remote sensing in which the atmosphere is particularly transmissive. Describe the role of these ranges in designing the remote sensing sensors. 5
- (h) Explain the phenomena describing interaction of photons with radioactive atoms—photoelectric effect, Compton scattering and pair production.
Show in a plot when these processes dominate as a function of energy levels between 10^{-2} MeV and 10^2 MeV of the gamma ray interactions of atoms. 3+2=5
2. (a) Write brief notes on (i) recursive and (ii) non-recursive filters. 5+5=10
- (b) Describe the various elements of visual image interpretation of remote sensing data.
Explain their relative degree of complexity in interpreting the data. 5+5=10

- (c) Consider P waves propagating through thick sediments below seafloor with a velocity gradient. Draw the time-distance ($t-x$) curves and ray path diagrams for reflected and refracted rays in a shallow water column, and diving rays in the deep ocean.

Write the expression for the depth of maximum penetration, and travel time t at a distance x from the source for diving rays.

6+4=10

3. (a) Mention the different direct and indirect indicators of groundwater occurrences on a satellite image. Describe the applications of remote sensing in groundwater mapping.

5+5=10

- (b) Define the rectangular function, $\pi(t)$, in discrete form and evaluate the convolution of $\pi(t)$ with itself. Draw a rough sketch of the convolved output.

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- (c) (i) Why is radon gas considered as an ideal tracer for groundwater studies? Describe the applications of radon gas in hydrogeological investigations.

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- (ii) Why is the sensitivity of air-borne magnetometers greater than those used in ground exploration? Briefly explain the advantages and disadvantages of air-borne magnetics.

2+3=5

4. (a) (i) Describe the relationship between grain size and capillary rise of water in unconsolidated materials. Arrange the following in the order of increasing grain size :

Fine gravels, Very coarse sand, Clay, Silt

2+3=5

- (ii) Differentiate between laminar flow and turbulent flow in porous media. Explain the range of validity of Darcy's law.

2+3=5

- (b) (i) A linear body extends to infinity in $\vec{Y}$ direction. Using Laplace's equation in Cartesian coordinates (X, Y, Z), show that the gravitational potential associated with it at a height h above the sea level decreases exponentially with height than its value at the sea level.

5

- (ii) Define Eötvös correction in gravity. Schematically plot its variation with latitude ($0^\circ-90^\circ$) for a given speed of moving platform and for headings of a moving platform at 5° , 30° and 90° .

5

- (c) Over a homogeneous half-space, show that the apparent resistivity is equal to the true resistivity.

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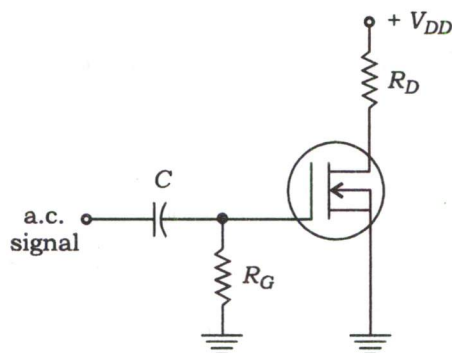
5. (a) (i) Given the absorption coefficient of seawater as 8×10^{-5} dB/wavelength, and intensity of the signal as unity. Calculate the intensity of a pressure signal at a distance of five times its wavelength. Diagrammatically describe the behaviour of an impulse originated at the source as a function of distance. 3+2=5
- (ii) Assume that obliquely incident rays are reflected from two receivers spaced X apart on the surface. Derive the relationship among offset (X), dip of layer (θ) and maximum frequency ($f_{\max}$) of seismic wave that is not aliased. 5
- (b) (i) Define analytic signal and show that its Fourier transform vanishes at negative frequencies. 1+4=5
- (ii) State and prove the time-scaling property of Fourier transform. 5
- (c) (i) The wavelengths of a certain portion of the electromagnetic spectrum range from 20000 nm to 40000 nm. Calculate the lowest frequency (in MHz) associated with that portion of the spectrum.
Calculate the resistivity of the subsurface if an inducing electromagnetic wave of 10 kHz frequency penetrates up to 4 km. 3+2=5
- (ii) Derive the vertical component of the gravity field versus horizontal distance from the centre. 5

SECTION—B

6. (a) (i) Justify that X-ray diffraction is a proof of wave nature of light.
- (ii) The spacing between principal planes of NaCl crystals is $2.82 \text{ \AA}$. It is found that the first-order Bragg reflection occurs at an angle of 10° . Calculate the wavelength of X-rays. 2+3=5
- (b) Draw the 'practical' circuit of a differentiator by using an operational amplifier and briefly explain the working of this circuit. Where do you find the applications of this circuit? 4+1=5
- (c) How do you obtain polarized output from He-Ne laser? Why are external mirrors preferred in making some He-Ne laser resonators? 4+1=5
- (d) (i) What are the types of dispersion in optical fiber communication?
- (ii) A 20 mW laser beam passing through a 50 km optical fiber link experiences loss of 0.4 dB/km. Calculate the output power in dBm. 5

- (e) (i) Realize a NOT gate by using a transistor. 2
- (ii) Show that
- $$\overline{A}BC + B + B\overline{D} + AB\overline{D} + \overline{A}C = B + C \quad 3$$
- (f) (i) What are the features of a Molniya orbit?
- (ii) Compare the applications of the Sun-synchronous and geosynchronous orbits. 5
- (g) The radius of a nucleus is 10^{-14} m. Assume that the maximum value of kinetic energy of an electron emitted by a radioactive nucleus is of the order of 4 MeV. Using Heisenberg uncertainty principle, show that such an electron cannot exist inside the nucleus. 5
- (h) Take $\vec{L} = \vec{r} \times \vec{P}$. Find the three components $\vec{L}_x$, $\vec{L}_y$ and $\vec{L}_z$ of orbital angular momentum operator $\vec{L}$. Evaluate the commutation relation $[L_x, P_y]$. (Here $\vec{P}$ is the momentum operator and $\vec{r}$ is the position operator) 5
7. (a) (i) Explain the concept of forbidden energy gap. With neat sketch, give the order of band gap in metal, semiconductor and insulator. How does the band gap vary with temperature in the above solids? 5
- (ii) If the Einstein's temperature of a material is 157 K, find the value of specific heat at constant volume (C_v) for the material at 100 K in cal/mole/K, using Einstein's formula. Also find the value of Einstein's frequency. 5
- (b) What are the standard techniques for the pulsed operation of a laser? With the help of a suitable illustration, explain the mechanism of producing high energy pulse of width in the 'nanosecond' regime. 3+7=10
- (c) (i) Write down the truth table and draw the logic diagram of a half-adder. Realize a half-adder by using NOR logic. 5
- (ii) A target is moving with a speed of 108 km per hour. A CW radar operating at 4.25 GHz is employed to measure the speed. Obtain the value of Doppler frequency seen by a stationary radar. 5
8. (a) (i) Draw the neat sketch of a particle of mass m confined in a one-dimensional potential box of side L with infinite height. Mention the values of potential at different regions ($x < 0$, $x > L$ and $0 \leq x \leq L$). Write down the relevant Schrödinger equation used to solve for getting quantized energy eigenvalue.
- (ii) An electron is constrained to move in one-dimensional potential box of 0.1 nm side. Obtain the first three eigenvalues in eV. 10

- (b) (i) Discuss briefly the different ways of biasing a depletion type MOSFET.
- (ii) What is the role of R_G in the circuit given below?



7+3=10

- (c) (i) Discuss the importance of studying the laser rate equations.
- (ii) If the duration of a pulse emitted from a 7.5 W ruby laser is 15 ns, find out the number of emitted photons in that pulse. 6+4=10

9. (a) (i) (1) Convert $(98)_{10}$ to its binary equivalent.
- (2) Convert $(10.1011)_2$ to its decimal equivalent. 2+2=4

- (ii) A low Earth orbit satellite is in a circular polar orbit with an altitude, h , of 1000 km. Calculate the velocity of the satellite in the orbit. 6

- (b) (i) What are the limitations of use of time-independent perturbation theory?
- (ii) Derive an expression for first-order correction to energy using the above theory for non-degenerate case. 3+7=10

- (c) (i) Write down the Curie law and Curie-Weiss law. Plot $1/\chi$ versus T using these laws and give physical inferences. (Here χ is the magnetic susceptibility and T is the temperature in Kelvin scale) 5

- (ii) The voltage applied across the a.c. Josephson junction is $8 \mu\text{V}$. Calculate the frequency of the radiation emitted by the junction.
- How can this result be used for practical application? 3+2=5

10. (a) (i) With suitable diagram, explain the lateral confinement technique employed in heterostructure lasers. Where are these lasers mostly used? 5+1=6

- (ii) Discuss the phenomenon of laser spiking that takes place in ruby laser. 4

- (b) (i) Why do you need A/D and D/A converters? Explain the different sequential processes involved for A/D and D/A conversions with suitable diagrams. 5
- (ii) With reference to radar systems, explain (1) range of resolution, (2) Doppler frequency and (3) coherence. 5
- (c) (i) What is Fermi's golden rule for time-dependent perturbation theory? 4
- (ii) Enlist some contrasting points between non-relativistic and relativistic quantum theory. What are the limitations of Klein-Gordon equations? 3+3=6

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