Solution

RELATIONS AND FUNCTIONS WS 1

Class 11 - Mathematics

1.

(c) R $\subseteq$ A $\times$ B

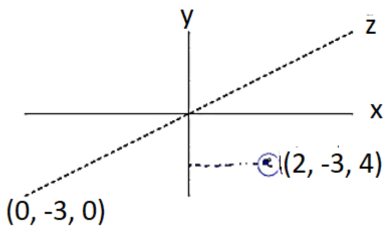
Explanation: Since, R is a relation from set A to set B, therefore it will always be a subset of A $\times$ B.

2.

(d) (0, -3, 0)

Explanation:

for y-axis ..x = 0, y = ?, z = 0



3.

(d) R - {0}

Explanation: Since log x is defined for x≥ 0, therefore domain of log |x| is R - {0}

4.

(c) 8

Explanation: Since A has 3 elements and B has 2 elemets, then number of functions from A to B is $2^3 = 8$

5.

(c) both one-one and onto

Explanation: F(n) = {$\frac{n-1}{2}$, when n is odd; $\frac{-n}{2}$, when n is even

one - one : Let n_1, $n_2 \in$ N

Case I: n_1 is even, n_2 is even

$\therefore$ f(n_1) - f(n_2) $\Rightarrow$ $\frac{-n_1}{2} = \frac{-n_2}{2}$ $\Rightarrow$ n_1 = n_2

Case II: n_1 is odd, n_2 is odd

$\therefore$ f(n_1) - f(n_2) $\Rightarrow$ $\frac{-n-1}{2} = \frac{-n-2}{2}$ $\Rightarrow$ n_1 = n_2

Case III: n_1 is even, n_2 is odd

$\therefore f(n_1) - \frac{-n_1}{2} =$ even as n_1 is even

$\therefore$ f(n_2) is odd, $\frac{n_2-1}{2}$ = even as n_2 is odd

But f(n_1) takes values, -1, -2, -3,..... f(n^2) take values, 0, 1, 2, 3,.....

$\therefore n_1 \neq n_2 \Rightarrow f(n_1) \neq f(n_2)$

Similarly n, is odd, n_2, is even, then $\therefore n_1 \neq n_2 \Rightarrow f(n_1) \neq f(n_2)$

$\Rightarrow$ f is one - one

onto: f(n) = {$\frac{n-1}{2}$, when n is odd; $\frac{-n}{2}$, when n is even}

$\therefore$ f(1) = 0, f(2) = 1, f(5) = 2, f(7) = 3, f(9) - 4, ... f(2)= -1, f(4) = -2, f(6) = -3, f(8) = -4, ...

$\therefore$ Range of f = {......, -2, -1, 0, 1, 2, 3, ...) = z

$\therefore$ F is onto

6. **(a)** ϕ

Explanation: For function to be defined,

$\frac{x-2}{x+2} \geq 0, x \neq -2$

$x \in (-\infty, -2) \cup [2, \infty) \ldots (1)$

and $\frac{1-x}{1+x} \geq 0, x \neq -1$
$\frac{x-1}{x+1} \leq 0$
$x \in (-1, 1] \ldots (2)$
Taking common of both the solutions we get $x \in \phi$

7.

(d) 0.5
Explanation: $e^{f(x)} = \frac{10+x}{10-x}$
$f(x) = \ln\left(\frac{10+x}{10-x}\right)$
$f(x) = kf\left(\frac{200x}{100+x^2}\right)$
$\ln\left(\frac{10+x}{10-x}\right) = k\ln\left(\frac{10+\frac{200x}{100+x^2}}{10-\frac{200x}{100+x^2}}\right)$
$\ln\left(\frac{10+x}{10-x}\right) = k\ln\left(\frac{1000+10x^2+200x}{1000+10x^2-200x}\right)$
$= \mathrm{k}\ln\left(\frac{100+\mathrm{x}^2+20\mathrm{x}}{100+\mathrm{x}^2-20\mathrm{x}}\right)$
$\ln\left(\frac{10+x}{10-x}\right) = k\ln\left(\frac{10+x}{10-x}\right)^2$
$\ln\left(\frac{10+x}{10-x}\right) = \ln\left(\frac{10+x}{10-x}\right)^{2k}$
2k = 1
k = 1/2
= 0.5

8.

(b) $[-3, -2] \cup [2, 3]$
Explanation: $5|x| - x^2 - 6 \geq 0$
$x^2 - 5|x| + 6 \leq 0$
$(|x| - 2)(|x| - 3) \leq 0$
So, $|x| \in [2, 3]$
Therefore, $x \in [-3, -2] \cup [2, 3]$

9. **(a)** {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (1, 2), (2, 3)}
Explanation: R = {(x, y) : $|x^2 - y^2| < 7$}
R = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (1, 2), (2, 3)}

10.

(b) 2 |x|
Explanation: f(2x) = 2(2x) + |2x| = 4x + 2|x|
f(-x) = 2(-x) + |-x|
f(2x) + f(-x) - f(x) = 4x + 2|x| - 2x + |-x| - (2x + |x|)
= 4x + 2|x| - 2x + |x| - 2x - |x| = 2|x|

11.

(c) None of these
Explanation: f(x) = cos (log x)
Now, $f\left(x^2\right) f\left(y^2\right) - \frac{1}{2}\left\{f\left(\frac{x^2}{y^2}\right) + f\left(x^2y^2\right)\right\}$
= cos (log x^2) cos (logy^2) - $\frac{1}{2}\left\{\cos\left(\log\left(\frac{x^2}{y^2}\right)\right) + \cos\left(\log x^2y^2\right)\right\}$
$= \cos(2\log x)\cos(2\log y) - \frac{1}{2}\left\{\cos\left(\log x^2 - \log y^2\right) + \cos\left(\log x^2 + \log y^2\right)\right\}$
$= \cos(2\log x)\cos(2\log y) - \frac{1}{2}$ { cos(2 log x - 2 log y) + cos (2 log x + log y)}
using cosx cos y = 1/2 cos (x + y) + cos (x - y)
= cos (2 log x) cos (2 log y) - cos (2 log x) cos(2 log y)
= 0

12.

(b) {-1, 1}
Explanation: We have $f(x) = \frac{x+2}{|x+2|}$

when x > -2,
f(x) = $\frac{x+2}{x+2} = 1$
When x < -2
We have $= \frac{x+2}{-(x+2)}$ = -1
R(f) = {-1, 1}

13.

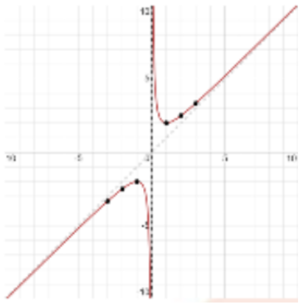
(b) {cos1, cos 2, 1}
Explanation: When -2<x< -1
[x] = -2
f(x) = cos[x] = cos(-2)
= cos2
for -1 < x < 0
[x] = -1
f(x) = cos[x]
= cos(-1)
= cos1
for 0 < x < 1
[x] = 0
f(x) = cos 0 =1
[x] = 1
f(x) = cos 1
Therefore, R(f) = {1, cos 1,cos 2}

14.

(d) none of these
Explanation: f(x) = x + $\frac{1}{x}$
Range of the function can be given by putting values of x and find y.

x	-3	-2	-1	1	2	3
y	-3.33	-2.5	-2	2	2.5	3.33

graph for the above function is



It is clear from the graph that the Range of the function is
$(-\infty, -2] \cup [2, \infty), \{y|y \leq -2, y \geq 2\}$

15. **(a)** {-1, 1}
Explanation: We know that
$|x| = -x$ in$(-\infty, 0)$ and $|x| = x$ in $[0, \infty)$
So, $f(x) = \frac{x}{-x} = -1$ in $(-\infty, 0)$
And $f(x) = \frac{x}{x} = 1$ in $(0, \infty)$
As clearly shown above f(x) has only two values 1 and -1
So, range of f (x) = {-1, 1}

16.

(c) i ϕ 1
Explanation: We have $x \ \phi \ y$ given by, $|x| = y$

i ϕ 1
x = i;
|x| = $\sqrt{1^2}$
= 1
1 = 1
|x| = y.

17.

(b) 0
Explanation: $f(x) = \cos(\log_e x)$
Now, f $\left(\frac{1}{x}\right)$ f $\left(\frac{1}{y}\right) - \frac{1}{2}\left\{f(xy) + f\left(\frac{x}{y}\right)\right\}$
$= \cos\left(\log_e \frac{1}{x}\right) \cos\left(\log_e \frac{1}{y}\right) - \frac{1}{2}\left\{\cos(\log_e xy) + \cos\left(\log_e \frac{x}{y}\right)\right\}$
= cos (- $\log_e$ x) cos (- log y) $-\frac{1}{2}${ cos [(log x) + cos (log y)]+cos [log x-log y]}
= cos (log x) cos (log y)-cos (log x) cos (log y)[using cos x cos y = 1/2 (cos (x + y) + cos (x - y)]
= 0

18.

(b) [3 /4 ,1]
Explanation: f (x) = $\sin^4$ x +1 - $\sin^2$ x
$f(x) = \sin^4 x - \sin^2 x + \frac{1}{4} - \frac{1}{4} + 1$
$f(x) = \left(\sin^2 x - \frac{1}{2}\right)^2 + \frac{3}{4}$
$\left(\sin^2 x - \frac{1}{2}\right)^2 \geq 0$
Minimum value of f(x) = 3/4
0 < $\sin^2$ x < 1
So, maximum value of f(x) = $\left(1 - \frac{1}{2}\right)^2 + \frac{3}{4}$
$= \frac{1}{4} + \frac{3}{4}$
= 1
R(f) =[3/4,1]

19.

(c) A $\times$ B = {(1, a), (1, b), (2, a), (2, b)}
Explanation: The set of all ordered pairs (a, b) such that $a \in A$ and $b \in B$ is called cartesian product of sets A and B.
$\therefore$ A $\times$ B = {(1, a), (1, b), (2, a), (2, b)}

20. **(a)** an equivalence relation
Explanation: Let T be the set of all triangles in the Euclidean plane with R, a relation in T is given by R = {(T_1,T_2): T_1 is congruent to T_2}
(T_1,T_2) $\in$ R if T_1 is congruent to T_2.
Reflexivity: $T_1 \cong T_1 \Rightarrow (T_1,T_1) \in$ R.
Symmetry: $(T_1,T_2) \in$ R $\Rightarrow T_1 \cong T_2 \Rightarrow T_2 \cong T_1 \Rightarrow (T_2,T_1) \in$ R.
Transitivity: $(T_1,T_2) \in$ R and $(T_2,T_3) \in$ R.
$\Rightarrow T_1 \cong T_2$ and $T_2 \cong T_3 \Rightarrow T_1 \cong T_3 \Rightarrow (T_2,T_3) \in$ R.
Therefore, R is an equivalence relation.

21.

(d) 2^{mn} - 1
Explanation: We have, n(A) = m and n(B) = n
n(A $\times$ B) = n(A) $\cdot$ n(B) = mn
Total number of relation from A to B = Number of subsets of A $\times$ B = 2^{mn}
So, total number of non-empty relations = 2^{mn} - 1

22.

(d) 2^{16}

Explanation: No. of elements in the set A = 4 . Therefore , the no. of elements in $A \times A = 4 \times 4 = 16.$ As, the no. of relations in $A \times A$ = no. of subsets of $A \times A = 2^{16}$.

23.

(d) $\frac{1}{\sqrt{2}}$

Explanation: $\frac{1}{\sqrt{2}}$

24. **(a)** ϕ

Explanation: Here, A = {x $\in$ R : x < 0} $\subseteq$ co-domain

f^{-1}(A) Contains those elements in R(domain) whose image is negative.

Since, f(x) = |x|

$\therefore$ no image of any elements of R(domain) is negative

$\therefore$ $f^{-1}(A) = \phi$

25.

(d) $\frac{1}{x}$

Explanation: We have $f(x) = \frac{x-1}{x+1}$ then

$f\left(\frac{1}{f(x)}\right) = \frac{\frac{1}{f(x)}-1}{\frac{1}{f(x)}+1} = \frac{1-f(x)}{1+f(x)}$

$= \frac{1-\frac{x-1}{x+1}}{1+\frac{x-1}{x+1}} = \frac{x+1-x+1}{x+1+x-1} = \frac{2}{2x} = \frac{1}{x}$

26. **(a)** an equivalence relation

Explanation: Given Relation R = {(1, 1), (2, 2), (3, 3)}

Reflexive: If a relation has {(a, b)} as its element, then it should also have {(a, a), (b, b)} as its elements too.

Symmetric: If a relation has (a, b) as its element, then it should also have {(b, a)} as its element too.

Transitive: If a relation has {(a, b), (b, c)} as its elements, then it should also have {(a,c)} as its element too.

Now, the given relation satisfies all these three properties.

Therefore, its an equivalence relation.

27.

(b) (0, ∞)

Explanation: f(x) = a^x, where a > 0

Case 1: When x < 0, then a^x lies between (0, 1)

Case 2: When x $\geq$ 0, then $a^x \geq 1$

Union of above two cases, gives us the Range as (0, ∞)

28. **(a)** symmetric

Explanation: A relation R on a non empty set A is said to be symmetric if xRy $\Leftrightarrow$ yRx, for all x,y $\in$ R. Clearly, $x^2 + y^2 = 1$ is same as $y^2 + x^2 = 1$ for all x,y $\in$R. Therefore, R is symmetric.

29.

(c) $\left[0, \frac{2}{3}\right]$

Explanation: f(x) = $\cos^{-1}$ (3x - 1)

The domain for function $\cos^{-1}$ x is [-1, 1] and range is [0, π]

When a function is multiplied by an integer, the domain of the function is decreased by the same number.

So, domain of $\cos^{-1}$ x is [-1, 1]

Multiply function by 3

$\Rightarrow$ domain of $\cos^{-1}$ 3x is $\left[\frac{-1}{3}, \frac{1}{3}\right]$

$\Rightarrow$ domain of $\cos^{-1}$ (3x - 1) is $\left[\frac{1}{3} - \frac{1}{3}, \frac{1}{3} + \frac{1}{3}\right]$ i.e. $\left[0, \frac{2}{3}\right]$

30.

(c) four points

Explanation: We will solve equations in A and B simultaneously and find values of x and y. The no. of possible ordered pairs from these values will be elements in $A \cap B$.

Now, From B,$x^2 + 9y^2 + y^2 = 144$ and

From $A,\ x^2 + y^2 = 25$

$\therefore 9y^2 + 25 = 144 \Rightarrow 9y^2 = 119$

$\Rightarrow y = \pm\sqrt{\frac{119}{9}}$

$\therefore x^2 + y^2 = 25 \Rightarrow x^2 = 25 = \frac{119}{9} = \frac{106}{9}$

$\Rightarrow x = \pm\sqrt{\frac{106}{9}}$

$\therefore$ x has two value, y has two values

$\therefore$ possible ordered pairs = 4

$\therefore A \cap B$ has 4 elements

31.

(d) {(8,11), (10,13)}

Explanation: Since, y = x – 3;

Therefore, for x = 11, y = 8.

For x = 12, y = 9. [But the value y = 9 does not exist in the given set.]

For x = 13, y =10.

So, we have R = {(11, 8), (13, 10)}

Now, R^{-1} = {(8, 11), (10, 13)}.

32.

(b) 3

Explanation: $f(x) = \frac{x+1}{x-1}$

$f(f(x)) = \frac{f(x)+1}{f(x)-1} = \frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1}$

$= \frac{x+1+x-1}{x+1-x+1} = \frac{2x}{2} = x$

$\mathrm{f(f(f(x))) = f(x) = \frac{x+1}{x-1}}$

$f(f(f(2))) = \frac{2+1}{2-1}$

= 3

33.

(d) Domain = R - {4}, Range = {-1}

Explanation: We have, f(x) $= \frac{4-x}{x-4}$ = -1, for x$\neq$4

34.

(d) 3 f(x)

Explanation: $\mathrm{f(g(x))} = \log\left(\frac{1+g(\mathrm{x})}{1-\mathrm{g(x)}}\right)$

$= \log\left(\frac{1+\frac{3x+x^2}{1+3x^2}}{1-\frac{3x+x^2}{1+3x^2}}\right)$

$= \log\left(\frac{1+3x^2+3x+x^3}{1+3x^2-3x-x^3}\right)$

$= \log\left(\frac{1+x}{1-x}\right)^3 = 3\log\left(\frac{1+x}{1-x}\right)$

f(g)(x) = 3f(x)

35.

(b) {1}

Explanation: As per condition of the relation, x > y.

So, required relation will be : {(2, 1), (3, 1)}

Since we know that Range is the set of elements written after comma in each ordered pair.

Therefore, Range = {1}

36.

(d) 0

Explanation: Since $f(x)=x^3-\frac{1}{x^3}$

$f\left(\frac{1}{x}\right)=\frac{1}{x^3}-\frac{1}{\frac{1}{x^3}}=\frac{1}{x^3}-x^3$

Hence, $f(x)+f\left(\frac{1}{x}\right)=x^3-\frac{1}{x^3}+\frac{1}{x^3}-x^3=0$

37.

(c) 2^{mn}

Explanation: We have n(A) = m, n(B) = n.

∴ Number of relations defined from A to B

= number of possible subsets of A × B = $2^{n(A \times B)} = 2^{mn}$

38.

(d) {1 ,-1 }

Explanation: When -4 < x < 0

f (x) = $-\frac{x}{x}$

= -1

When 0 < x < 4

f(x) = x/x

=1

R(f) = {-1,1}

39.

(b) f (π/2) = 1

Explanation: f $\left(\frac{\pi}{2}\right)=\sin 9\left(\frac{\pi}{2}\right)-\sin 10\left(\frac{\pi}{2}\right)$

= 1-0

=1

40. **(a)** 60 percent

Explanation: Let A denote the set of persons traveling by car, B denotes the set of persons traveling by bus, then

n(A) = 20, n(B) = 50, n(A ∩ B) = 10

∴ n(A ∪ B) = n(A) + n(B) - n(A ∩ B)

= 20 + 50 - 10 = 60

41. **(a)** {b, c}

Explanation: Since the range is represented by the y- coordinate of the ordered pair (x, y). Therefore, the range of the given relation is { b, c }.

42.

(b) transitive but not symmetric

Explanation: Consider the non – empty set consisting of children in a family and a relation R defined as aRb if a is brother of b. Then R is not symmetric, because aRb means a is brother of b, then, it is not necessary that b is also brother of a , it can be the sister of a. Therefore, bRa is not true. Therefore, the relation is not symmetric . Again, if aRb and bRc is true, then aRc is also true. Therefore, R is transitive only.

43.

(c) $(-\infty,-1)\cup(1,4]$

Explanation: We have, f(x)= $\sqrt{4-x}+\frac{1}{\sqrt{x^2-1}}$

f(x) is defined if $4-x\geq 0$ and x^2 - 1 > 0

⇒ $x-4\leq 0$ and (x + 1)(x - 1) > 0

⇒ $x\leq 4$ and (x < -1 or x > 1)

∴ Domain of f $=(-\infty,-1)\cup(1,4]$

44.

(d) 0

Explanation: Because the no. of elements in domain i.e. in A is less than the no. of elements in co-domain i.e. in B. Therefore, no bijection mapping is possible.

45.

(d) Symmetric but neither reflexive nor transitive.

Explanation: The relation R is symmetric only , because if L_1 is perpendicular to L_2 ,then L_2 is also perpendicular to L_1,but no other cases that is reflexive and transitive is not possible.

46. **(a)** $(-\infty, -1] \cup \left[\frac{1}{3}, \infty\right)$

Explanation: We know that, $-1 \leq \cos x \leq 1$

$\Rightarrow -1 \leq -\cos x \leq 1$

$\Rightarrow -2 \leq -2\cos x \leq 2$

$\Rightarrow -1 \leq 1-2\cos x \leq 3$

Now f(x) $= \frac{1}{1-2\cos x}$ is defined if

$-1 \leq 1-2\cos x < 0$ or $0 < 1-2\cos x \leq 3$

$\Rightarrow -1 \geq \frac{1}{1-2\cos x} > -\infty$ or $\infty > \frac{1}{1-2\cos x} \geq \frac{1}{3}$

$\Rightarrow \frac{1}{1-2\cos x} \in (-\infty, -1] \cup \left[\frac{1}{3}, \infty\right)$

47.

(c) $\{(x, y) : x, y \in R, y^2 = x\}$

Explanation: A function is said to exist when we get a unique value for any value of x.

Here, $y^2 = x$ is not a function as for each value of x, we will get 2 values of y which violates the definition of a function.

48. **(a)** (-3, -2) ∪ (-2, -1) ∪ (1, ∞)

Explanation: Given function is f(x) = $\log_{3+x}(x^2 - 1)$

It is obvious that f(x) is defined when $x^2 - 1 > 0$, 3 + x > 0 and 3 + x $\neq$ 1.

Now, $x^2 - 1 > 0 \Rightarrow x^2 > 1$

$\Rightarrow$ x < -1 or x > 1

3 + x > 0 $\Rightarrow$ x > -3

3 + x $\neq$ 1 $\Rightarrow$ x $\neq$ -2

Therefore, domain of the function f(x) = (-3, -2) ∪ (-2, -1) ∪ (1, ∞)

49.

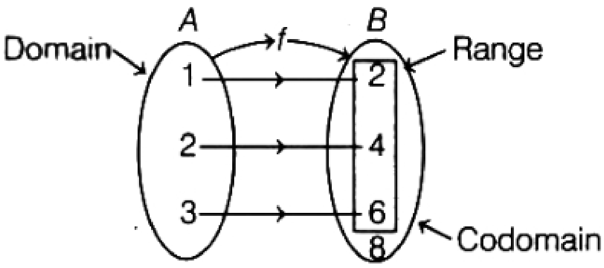
(c) {2, 4, 6}

Explanation: Given, f(x) = 2x, $\forall x \in A$

Value of function at x = 1, f(1) = 2(1) = 2

Value of function at x = 2, f(2) = 2(2) = 4

Value of function at x = 3, f(3) = 2(3) = 6



We Can write it as f = {(1, 2), (2, 4), (3, 6)}

∴ Range of f = {2, 4, 6}

50.

(c) none of these

Explanation: Given set A = {1, 2, 3, 4, 5} and relation R ={(a, b):| a^2-b^2| < 16}

According to the condition |a^2-b^2|<16:

$\Rightarrow$ R = {(1, 1),(1,2), (2, 1), (1,3), (3, 1), (1,4), (4, 1), (2, 3),(2, 2), (3, 2), (4, 2), (2, 4),(3, 3), (4, 3), (5, 4), (3, 4), (4,4), (5,5)}.Which is the required solution.

51.

(c) {(3, 8) (6, 6), (9, 4), (12, 2)}

Explanation: Given that, 2a + 3b = 30

$\Rightarrow$ 3b = 30 - 2a

$\Rightarrow b = \frac{30 - 2a}{3}$

For a = 3, b = 8

a = 6, b = 6

a = 9, b = 4

a = 12, b = 2

$\therefore$ R = {(3, 8) (6, 6), (9, 4), (12, 2)}

52.

(c) $\left\{-\frac{1}{2}\right\}$

Explanation: f : [-2, 2] $\rightarrow$ R is defined by

f(x) $= \begin{cases} -1, & -2 \leqslant x \leqslant 2 \\ x - 1, & 0 \leqslant x \leqslant 2 \end{cases}$

Let x $\leq$ 0 and f(|x|) = x

Now, f(|x|) = x $\Rightarrow$ |x| - 1 = x

$\Rightarrow -x - 1 = x \; [\because \; |x| \geqslant 0]$

$\Rightarrow \; -x - 1 = x \; (\text{ as } x \leqslant 0)$

$\Rightarrow \; 2x = -1 \; \Rightarrow \; x = -\frac{1}{2}$

$\therefore \{x \in [-2, 2] : x \leqslant 0 \; \text{ and } f(|x| = x\} = \left\{-\frac{1}{2}\right\}$

53.

(c) 2^{n^2}

Explanation: A is a set of n elements.

A $\times$ A will have a total of n^2 elements.

Then, the number of relations on A will be 2^{n^2}

54.

(c) R - {3}, {1, -1}

Explanation: The given function is f(x) = $\frac{|x-3|}{x-3}$

This function is well defined for all real numbers other than 3.

$\therefore$ Its domain is R - {3}

Now, f(x) = $\frac{|x-3|}{x-3}$

$= \begin{cases} \frac{x-3}{x-3} : x > 3 \\ \frac{-(x-3)}{x-3} : x < 3 \end{cases} = \begin{cases} 1 : x > 3 \\ -1 : x < 3 \end{cases}$

$\Rightarrow$ Range of function f is {1, -1}

55. **(a)** reflexive, symmetric and transitive

Explanation: By definition of Equivalence Relation, a relation is said to be equivalence if it is reflexive, symmetric and transitive.

56. **(a)** f $(\alpha) = f(\beta)$ = -9

Explanation: $f(x) = 64x^3 + \frac{1}{x^3}$

$= \left(4x + \frac{1}{x}\right)^3 - 3(4x)\left(\frac{1}{x}\right)\left(4x + \frac{1}{x}\right)$

Since, $4x + \frac{1}{x} = 3$ and $\alpha \; and \; \beta$ are its roots,

f(x) = 3^3 - 12(3)

= 27-36

= -9

so,$f(\alpha) = f(\beta) = -9$

57. **(a)** (6, 8) $\in$ R

Explanation: (6, 8) $\in$ R

as b - 2 = 8 - 2 = 6 and b>6 .

58.

(c) $A \times (B \cup C)$

Explanation: A $\times$ (B $\cup$ C) = (A $\times$ B) $\cup$ A $\times$ C

= {a, b} $\times$ {c, d} $\cup$ {a, b} $\times$ {d, c}

= {(a, c), (a, d), (b, c), (b, d)} $\cup$ {(a, d), (a, c), (b, d), (b, c)}

= {(a, c), (a, d), (a, c), (b, c), (b, d), (b, e)}

59.

(c) None of these

Explanation: f(x) = cos (logx)

Now, f(x) f(y) $-\frac{1}{2}\left\{\mathrm{f}\left(\frac{\mathrm{x}}{\mathrm{y}}\right)+\mathrm{f}(\mathrm{xy})\right\}$

= cos(logx) cos(logy) $-\frac{1}{2}\{\cos\left(\log\left(\frac{x}{y}\right)\right)$ + cos (logxy)}

= cos (log x) cos (log y) $-\frac{1}{2}$ {cos(logx - log y) + cos (log x + logy)}

using cosx cosy - 1/2 (cos (x + y) + cos (x - y)

= 0

60.

(d) {8, 27}

Explanation: Given, R = {[a, a^3) : a is a prime number less than 5}

$\Rightarrow$ R = {(2, 8), (3, 27)}

Hence, range of R = {8, 27}.

61. **(a)** $(1, \infty)$

Explanation: f(x) = $e^{\sqrt{x^2-1}}$ · $\log(x-1)$

Domain of the function is defined for

x - 1 > 0 and x^2 - 1 $\geq$ 0

$\Rightarrow$ x > 1 $\Rightarrow$ x2 $\geq$ 1

$\Rightarrow$ -1 $\leq$ x $\geq$ 0

the intersection of above two equations gives $(1, \infty)$

Therefore, domain of f(x) is $(1, \infty)$

62.

(b) $(-\infty, 1] \cup [2, \infty)$

Explanation: $\because$ f : R $\rightarrow$ R defined by

$f(x)=\sqrt{x^2-3x+2}$

Here, x^2 - 3x + 2 $\geq$ 0

(x - 1) (x - 2) $\geq$ 0

x $\leq$ 1 or x $\geq$ 2

$\therefore$ Domain of $f=(-\infty, 1] \cup [2, \infty)$

63.

(c) Domain $=[1, \infty)$, Range $=[0, \infty)$

Explanation: We have, f(x) $=\sqrt{x-1}$

Clearly, f(x) is defined if $x-1 \geq 0$

$\Rightarrow \quad x \geq 1$

$\therefore$ Domain of f $=[1, \infty)$

Now for $x \geq 1, x-1 \geq 0$

$\Rightarrow \quad \sqrt{x-1} \geq 0$

$\Rightarrow$ Range of f $=[0, \infty)$

64. **(a)** $R \subseteq A \times B$

Explanation: Let A and B be two sets. Then a relation R from set A to set B is a subset of A×B.Thus, R is a relation from A to B $\Leftrightarrow R \subseteq A \times B$.

65. **(a)** 0

Explanation: f (x) = cos (log x)

Now, $f(x)f(4)-\frac{1}{2}\left\{f\left(\frac{x}{4}\right)+f(4x)\right\}$

= cos (logx) cos (log 4) - 1/2 { cos $\left(\frac{x}{4}\right)\Big) + \cos(\log 4x)\Big\}$

= cos (log x) cos (log 4) $-\frac{1}{2}\{\cos(\log x - \log 4) + \cos(\log x + \log 4)\}$

Using cox cosy = 1/2 (cos (x + y) + cos (x - y)

= cos (log x) cos (log 4) - cos (log x) cos(4)

= 0

66.

(d) -1

Explanation: gof(e) + fog(π) = g(f(e)) + f(g(π))

= g(1) + f(-1) {∵ e is irrational and π is rational}

= -1 + 0 = -1

67.

(d) reflexive

Explanation: Any relation R is reflexive if xRx for all x ∈ R. Here, (a, a), (b, b), (c, c)∈ R. Therefore, R is reflexive.

68.

(d) $[0, \infty)$

Explanation: A modulus function always gives a positive value

R(f) =[$0, \infty)$

69. **(a)** R - { - 1 / 2 ,1 }

Explanation: Let $y = \frac{x^2 - x}{x^2 + 2x}$

$y(x^2+2x) = x^2-x$

yx(x+2)= x(x-1)

y(x+2)= x-1

$x(y-1) = -(1+2y)$

$x = -\frac{(1+2y)}{y-1}$

Value of x can’t be zero or it cannot be not defined.Therefore,

y = 1, -1/2 are not possible.

So, range = R - { -1/2, 1}

70.

(d) {2, 4, 6}

Explanation: As xRy if x + 2 y = 8 , therefore, domain of the relation R is given by x = 8 – 2y ∈ N. When y = 1, ⇒ x = 6, when y = 2, ⇒ x =4 , when y =3 , ⇒ x = 2. Therefore, domain is { 2, 4, 6 }.

71.

(b) reflexive and symmetric but not transitive.

Explanation: Reflexive: since $1 + a^2 > 0$ ∀ a ∈ S

⇒ 1 + a, a > 0∀ a ∈ S $\Rightarrow (a, a) \in R \forall a$

⇒ R is reflexive.

Symmetric: Let a, b ∈ S Such that (a, b) ∈ R, then,

1 + ab > 0 ⇒ 1 + ba > 0 ⇒ (b, a) ∈ R

⇒ R is symmetric.

Transitivity: Here 1 + 1.2 = 1 + 2 = 3 > 0

⇒ 1 + (-2)(-3) = 1 + 6 = 7 > 0

⇒ (1, 2), (-2, -3) ∈ R

Now, 1 + 1(-3) = 1 - 3 = -2 < 0

∴ (1, -3) ∉ R

∴ R is not a transition.

72.

(d) R and R

Explanation: $f(x) = x^3$

f(x) can assume any value, so domain of f(x) is R

The Range of the function can be positive or negative Real numbers, as the cube of any number depends on the sign of the number, So Range of f(x) is R

73.

(d) $\{(x, y): y=|x|, x, y \in R\}$

Explanation: A function is said to exist when we get a unique value of y for any value of x. If we get 2 values of y for any value of x, then it is not a function.

74. **(a)** two points

Explanation: From A, $x^2 + y^2 = 5$ and from B, 2x = 5y

Now, 2x = 5y $\Rightarrow$ x = $\frac{5}{2y}$

$\therefore$ $x^2 + y^2 = 5 \Rightarrow \left(\frac{5}{2y}\right)^2 + y^2 = 5$

$\Rightarrow 29y^2 = 20 \Rightarrow y = \pm\sqrt{\frac{20}{29}}$

$\Rightarrow 29y^2 = 20 \Rightarrow y = \pm\sqrt{\frac{20}{29}}$

$\therefore$ x = $\frac{5}{2}(\pm\sqrt{\frac{20}{29}})$

$\therefore$ Possible ordered pairs = four

But two ordered pair in which c is positive and y is negative will be rejected as it will not be satisifed by the equation in B. Therefore,

A $\cap$ B contains 2 elements.

75. **(a)** [-1,1] - {0}

Explanation: f(x) = $\frac{\sin^{-1} x}{x}$

Domain of the function is defined for x $\neq$ 0

Domain of $\sin^{-1}$ x is [-1, 1]

Therefore, domain of f(x) is [-1, 1] - 0