CHAPTER 6

APPLICATIONS OF DERIVATIVES

Exercise 1: NCERT Based Topic-wise MCQs

6.1 INTRODUCTION

1. One of the given options is related to chapter "Application of Derivatives" is NCERT Page-194/N-147
(a) Rate of change of quantities
(b) Monotonic functions
(c) Maxima and minima
(d) All of these

6.2 RATE OF CHANGE OF QUANTITIES

2. The altitude of a cone is 20cm and its semi-vertical angle is 30°. If the semi-vertical angle is increasing at the rate of 2° per second, then the radius of the base is increasing at the rate of- NCERT Page-236/N-148
(a) $30\text{cm}/\text{sec}$.
(b) $\frac{160}{3}\text{cm}/\text{sec}$.
(c) $10\text{cm}/\text{sec}$.
(d) $160\text{cm}/\text{sec}$.

3. Water is dripping out from a conical funnel of semi-vertical angle $\frac{\pi}{4}$ at the uniform rate of $2\text{cm}^2/\text{s}$ is the surface area through a tiny hole at the vertex of the bottom. When the slant height of cone is 4cm, then rate of decrease of the slant height of water is NCERT Page-195/N-149
(a) $\frac{\sqrt{2}}{3\pi}\text{cm}/\text{s}$
(b) $\frac{\sqrt{2}}{\pi}\text{cm}/\text{s}$
(c) $\frac{\sqrt{2}}{4\pi}\text{cm}/\text{s}$
(d) None of these

4. A kite is moving horizontally at a height of 151.5 . If the speed of kite is $10\text{m}/\text{s}$, then the rate at which the string is being let out; when the kite is 250m away from the boy who is flying the kite and the height of the boy is 1.5m, is NCERT Page-197/N-148
(a) $4\text{m}/\text{s}$
(b) $6\text{m}/\text{s}$

(c) $7 \mathrm{~m/s}$
(d) $8 \mathrm{~m/s}$

5. The cost of running a bus from A to B, is ₹ $\left(a v+\frac{b}{v}\right)$, where $v \mathrm{~km/h}$ is the average speed of the bus. When the bus travels at 30 $\mathrm{km/h}$, the cost comes out to be ₹75 while at $40 \mathrm{~km/h}$, it is ₹65. Then the most economical speed (in $\mathrm{km/h}$) of the bus is : NCERT Page-196/N-148
(a) 45
(b) 50
(c) 60
(d) 40

6. Water is being filled at the rate of $1 \mathrm{~cm}^3/\mathrm{sec}$ in a right circular conical vessel (vertex downwards) of height 35 cm and diameter $14 \mathrm{~cm}$. When the height of the water level is $10 \mathrm{~cm}$, the rate (in $\mathrm{cm}^2/\mathrm{sec}$) at which the wet conical surface area of the vessel increases is NCERT Page-196/N-148
(a) 5
(b) $\frac{\sqrt{21}}{5}$
(c) $\frac{\sqrt{26}}{5}$
(d) $\frac{\sqrt{26}}{10}$

7. The surface area of a balloon of spherical shape being inflated, increases at a constant rate. If initially, the radius of balloon is 3 units and after 5 seconds, it becomes 7 units, then its radius after 9 seconds is :
(a) 9
(b) 10
(c) 11
(d) 12

8. A point on the parabola $y^2=18 x$ at which the ordinate increases at twice the rate of the abscissa is NCERT Page-196/N-147
(a) $\left(\frac{9}{8}, \frac{9}{2}\right)$
(b) $(2,-4)$
(c) $\left(\frac{-9}{8}, \frac{9}{2}\right)$
(d) $(2,4)$

9. A stone is dropped into a quiet lake and waves moves in circles at the speed of $5 \mathrm{~cm/s}$. If at a instant, the radius of the circular wave is $8 \mathrm{~cm}$, then the rate at which enclosed area is increasing, is
(a) $20 \pi \mathrm{cm}^2/\mathrm{s}$ NCERT Page-196/N-148
(b) $40 \pi \mathrm{cm}^2/\mathrm{s}$
(c) $60 \pi \mathrm{cm}^2/\mathrm{s}$
(d) $80 \pi \mathrm{cm}^2/\mathrm{s}$

10. A particle moves along the curve $6 y=x^3+2$. The point ' P ' on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate, are $(4,11)$ and $\left(-4,-\frac{31}{3}\right)$. NCERT Page-196/N-148

(a) x-coordinates at the point P are ± 4
(b) y-coordinates at the point P are 11 and $\frac{-31}{3}$
(c) Both (a) and (b)
(d) None of the above

11. A ball is dropped from a platform 19.6m high. Its position function is - NCERT Page-197/N-149
(a) $x = -4.9t^2 + 19.6 (0 \le t \le 1)$
(b) $x = -4.9t^2 + 19.6 (0 \le t \le 2)$
(c) $x = -9.8t^2 + 19.6 (0 \le t \le 2)$
(d) $x = -4.9t^2 - 19.6 (0 \le t \le 2)$

12. A lizard, at an initial distance of 21cm behind an insect, moves from rest with an acceleration of 2cm/s^2 and pursues the insect which is crawling uniformly along a straight line at a speed of 20cm/s. Then the lizard will catch the insect after NCERT Page-197/N-149
(a) $20s$
(b) $1s$
(c) $21s$
(d) $24s$

13. The radius of a cylinder is increasing at the rate of 3m/s and its altitude is decreasing at the rate of 4m/s. The rate of change of volume when radius is 4m and altitude is 6m, is
(a) $20\pi\text{m}^3/\text{s}$
(b) $40\pi\text{m}^3/\text{s}$
(c) $60\pi\text{m}^3/\text{s}$
(d) None of these

14. The radius of a sphere initially at zero increases at the rate of 5cm/sec. Then its volume after 1sec is increasing at the rate of : NCERT Page-196/N-149
(a) 50π
(b) 5π
(c) 500π
(d) None of these

15. If a circular plate is heated uniformly, its area expands $3c$ times as fast as its radius, then the value of c when the radius is 6 units, is NCERT Page-197/N-149
(a) 4π
(b) 2π
(c) 6π
(d) 3π

16. A man is moving away from a tower 41.6m high at a rate of 2m/s. If the eye level of the man is 1.6m above the ground, then the rate at which the angle of elevation of the top of the tower changes, when he is at a distance of 30m from the foot of the tower, is
(a) $-\frac{4}{125}\text{rad/s}$

(b) $-\frac{2}{25}$ rad/s
(c) $-\frac{1}{625}$ rad/s
(d) None of these

17. A ladder is resting with the wall at an angle of $30°$. A man is ascending the ladder at the rate of 3ft/sec. His rate of approaching the wall is NCERT Page-236/N-148
(a) 3ft/sec
(b) $\frac{3}{2}$ ft/sec
(c) $\frac{3}{4}$ ft/sec
(d) $\frac{3}{\sqrt{2}}$ ft/sec

18. The rate of increase of bacteria in a certain culture is proportional to the number present. If it doubles in 5 hours then in 25 hours, its number would be NCERT Page-236/N-148
(a) 8 times the original
(c) 32 times the original
(b) 16 times the original
(d) 64 times the original

19. A football is inflated by pumping air in it. When it acquires spherical shape its radius increases at the rate of 0.02cm/s. The rate of increase of its volume when the radius is 10cm is $\pi\text{cm/s}$
(a) 0
(b) 2
(c) 8
(d) 9

6.3 INCREASING AND DECREASING FUNCTIONS

20. The function $f(x) = xe^{x(1-x)}, x \in$ R, is: NCERT Page-202/N-155
(a) increasing in $\left(-\frac{1}{2}, 1\right)$
(b) decreasing in $\left(\frac{1}{2}, 2\right)$
(c) increasing in $\left(-1, -\frac{1}{2}\right)$
(d) decreasing in $\left(-\frac{1}{2}, \frac{1}{2}\right)$

21. Let $f(x) = 2\cos^{-1} x + 4\cot^{-1} x - 3x^2 - 2x + 10, x \in [-1,1]$. If $[a, b]$ is the range of the function then $4a - b$ is equal to: NCERT Page-203/N-155
(a) 11
(b) $11 - \pi$
(c) $11 + \pi$
(d) $15 - \pi$

22. The function $f(x) = \tan\, x - 4x$ is strictly decreasing on NCERT Page-200/N-156

(a) $\left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$

(b) $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

(c) $\left(-\frac{\pi}{3}, \frac{\pi}{2}\right)$

(d) $\left(\frac{\pi}{2}, \pi\right)$

23. If $f(x) = \frac{x}{\sin\, x}$ and $g(x) = \frac{x}{\tan\, x}$, where $0 < x \leq 1$, then in this interval, NCERT Page-201/N-157

(a) both $f(x)$ and $g(x)$ are increasing functions

(b) both $f(x)$ and $g(x)$ are decreasing functions

(c) $f(x)$ is an increasing function

(d) $g(x)$ is an increasing function

24. On which of the following intervals is the function $\mathrm{x}^{100} + \sin\, \mathrm{x} - 1$ decreasing? NCERT Page-200/N-156

(a) $(0, \pi/2)$

(c) $(\pi/2, \pi)$

(b) $(0,1)$

(d) None of these

25. The intervals in which the function f given by $f(x) = \frac{4\sin\, x - 2x - x\cos\, x}{2+\cos\, x}$ is increasing in $x \in (0,2\pi)$ is NCERT Page-203/N-157

(a) $(0, \pi) \cup (2\pi, 4\pi)$

(b) $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right)$

(c) $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \pi\right)$

(d) None of these

26. The interval in which the function $f(x) = \frac{4x^2+1}{x}$ is decreasing is : NCERT Page-203/N-155

(a) $\left(-\frac{1}{2}, \frac{1}{2}\right)$

(b) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(c) $(-1,1)$

(d) $[-1,1]$.

27. The function $f(x) = \log\,(1+x) - \frac{2x}{2+x}$ is increasing on NCERT Page-203/N-155

(a) $(0, \infty)$

(b) $(-\infty, 0)$

(c) $(-\infty, \infty)$

(d) None of these

28. $f(x) = \frac{\log\,(\pi + x)}{\log\,(e+x)}$ is NCERT Page-204/N-155

(a) increasing in $[0, \infty)$

(b) decreasing in $[0, \infty)$
(c) decreasing in $\left[0, \frac{\pi}{e}\right]$& increasing in $\left[\frac{\pi}{e}, \infty\right)$
(d) increasing in $\left[0, \frac{\pi}{e}\right]$& decreasing in $\left[\frac{\pi}{e}, \infty\right)$

29. If $f(x) = 3x^4 + 4x^3 - 12x^2 + 12$, then $f(x)$ is NCERT Page-204/N-156
(a) increasing in $(-\infty, -2)$ and in $(0,1)$
(b) increasing in $(-2,0)$ and in $(1, \infty)$
(c) decreasing in $(-2,0)$ and in $(0,1)$
(d) decreasing in $(-\infty, -2)$ and in $(1, \infty)$

30. Let $f: R \rightarrow R$ and $g: R \rightarrow R$ be two functions defined by $f(x) = \log_e (x^2 + 1) - e^{-x} + 1$ and $g(x) = \frac{1-2e^{2x}}{e^x}$. Then, for which of the following range of α, the inequality
$f\left(g\left(\frac{(\alpha-1)^2}{3}\right)\right) > f\left(g\left(\alpha - \frac{5}{3}\right)\right)$ holds ? NCERT Page-203/N-157
(a) $(2,3)$
(b) $(-2, -1)$
(c) $(1,2)$
(d) $(-1,1)$

31. $f(x) = 4\log_e (x-1) - 2x^2 + 4x + 5, x > 1$, which one of the following is NOT correct?
(a) f is increasing in $(1,2)$ and decreasing in $(2, \infty)$ NCERT Page-201/N-157
(b) $f(x) = -1$ has exactly two solutions
(c) $f'(e) - f''(2) < 0$
(d) $f(x) = 0$ has a root in the interval $(e, e+1)$

32. Let $\lambda *$ be the largest value of λ for which the function $f_\lambda(x) = 4\lambda x^3 - 36\lambda x^2 + 36x + 48$ is increasing for all $x \in R$. Then $f_\lambda * (1) + f_\lambda * (-1)$ is equal to: NCERT Page-203/N-156
(a) 36
(b) 48
(c) 64
(d) 72

33. The function $f(x) = \tan^{-1} (\sin x + \cos x)$ is an increasing function in NCERT Page-204/N-157
(a) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$
(b) $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$
(c) $\left(0, \frac{\pi}{2}\right)$
(d) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

34. If $f(x) = \cos x, g(x) = \cos 2x, h(x) = \cos 3x$ and $I(x) = \tan x$, then which of the following option is correct? NCERT Page-203/N-157
(a) f(x) and g(x) are strictly decreasing in $(0, \pi/2)$

(b) $h(x)$ is neither increasing nor decreasing in $(0, \pi/2)$
(c) I(x) is strictly increasing in $(0, \pi/2)$
(d) All are correct

35. The function $f(x) = 3\cos^4 x + 10\cos^3 x + 6\cos^2 x - 3$, $(0 \leq x \leq \pi)$ is - NCERT Page-202/N-157
(a) Increasing in $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$
(b) Increasing in $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{2\pi}{3}, \pi\right)$
(c) Decreasing in $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$
(d) All of the above

36. Find the intervals in which the function f given by $f(x) = x^2 - 4x + 6$ is strictly increasing:
(a) $(-\infty, 2) \cup (2, \infty)$ NCERT Page-203/N-156
(c) $(-\infty, 2)$
(b) $(2, \infty)$
(d) $(-\infty, 2] \cup (2, \infty)$

37. The value of b for which the function $f(x) = x + \cos x + b$ is strictly decreasing over R is:
(a) $b < 1$ NCERT Page-203/N-157
(b) No value of b exists
(c) $b \leq 1$
(d) $b \geq 1$

38. The number of distinct real roots of the equation $x^7 - 7x - 2 = 0$ is NCERT Page-202/N-156
(a) 5
(b) 7
(c) 1
(d) 3

39. Let $f(x) = 3\sin^4 x + 10\sin^3 x + 6\sin^2 x - 3, x \in \left[-\frac{\pi}{6}, \frac{\pi}{2}\right]$.
Then, f is : NCERT Page-202/N-155
(a) increasing in $\left(-\frac{\pi}{6}, \frac{\pi}{2}\right)$
(b) decreasing in $\left(0, \frac{\pi}{2}\right)$
(c) increasing in $\left(-\frac{\pi}{6}, 0\right)$
(d) decreasing in $\left(-\frac{\pi}{6}, 0\right)$

40. The number of real solutions of $x^7 + 5x^3 + 3x + 1 = 0$ is equal to NCERT Page-200/N-156
(a) 0
(b) 1
(c) 3
(d) 5

6.4 TANGENTS AND NORMALS

41. The slope of the tangent to a curve $C: y = y(x)$ at any point (x, y) on it is $\frac{2e^{2x}-6e^{-x}+9}{2+9e^{-2x}}$. If C passes through the points $\left(0, \frac{1}{2} + \frac{\pi}{2\sqrt{2}}\right)$ and $\left(\alpha, \frac{1}{2}e^{2\alpha}\right)$, then e^{α} is equal to: NCERT Page-210

(a) $\frac{3+\sqrt{2}}{3-\sqrt{2}}$
(b) $\frac{3}{\sqrt{2}}\left(\frac{3+\sqrt{2}}{3-\sqrt{2}}\right)$
(c) $\frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)$
(d) $\frac{\sqrt{2}+1}{\sqrt{2}-1}$

42. Let S be the set of all the natural numbers, for which the line $\frac{x}{a} + \frac{y}{b} = 2$ is a tangent to the curve $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$ at the point $(a, b), ab \neq 0$. Then: NCERT Page-208

(a) $S = \phi$
(b) $n(S) = 1$
(c) $S = \{2k : k \in N\}$
(d) $S = N$

43. The point(s) on the curve $y = x^3 - 11x + 5$ at which the tangent is $y = x - 11$ is/are: NCERT Page-209 I

(a) $(-2,19)$
(b) $(2,-9)$
(c) $(\pm 2,19)$
(d) $(-2,19)$ and $(2,-9)$

44. The points on the curve $\frac{x^2}{9} + \frac{y^2}{16} = 1$ at which the tangents are parallel to y-axis are: NCERT Page-209

(a) $(0, \pm 4)$
(b) $(\pm 4,0)$
(c) $(\pm 3,0)$
(d) $(0, \pm 3)$

45. The point at which the normal to the curve $y = x + \frac{1}{x}, x > 0$ is perpendicular to the line $3x - 4y - 7 = 0$ is: NCERT Page-210

(a) $(2,5/2)$
(b) $(\pm 2,5/2)$
(c) $(-1/2,5/2)$
(d) $(1/2,5/2)$

46. The tangent at the point (x_1, y_1) on the curve $y = x^3 + 3x^2 + 5$ passes through the origin, then (x_1, y_1) does NOT lie on the curve: NCERT Page-208

(a) $x^2+\frac{y^2}{81}=2$
(b) $\frac{y^2}{9}-x^2=8$
(c) $y=4x^2+5$
(d) $\frac{x}{3}-y^2=2$

47. If the angle made by the tangent at the point (x_0, y_0) on the curve $x=12(t+\sin t\cos t), y=12(1+\sin t)^2, 0<t<\frac{\pi}{2}$, with the positive x-axis is $\frac{\pi}{3}$, then y_0 is equal to NCERT Page-208
(a) $6(3+2\sqrt{2})$
(b) $3(7+4\sqrt{3})$
(c) 27
(d) 48

48. If the tangent at $P(1,1)$ on $y^2=x(2-x)^2$ meets the curve again at Q, then Q is NCERT Page-208
(a) $(2,2)$
(b) $(-1,-2)$
(c) $\left(\frac{9}{4}, \frac{3}{8}\right)$
(d) None of these

49. If the curves $x^2=9A(9-y)$ and $x^2=A(y+1)$ intersect orthogonally, then the value of A is
(a) 3 NCERT Page-209
(b) 4
(c) 5
(d) 7

50. The curve $y-e^{xy}+x=0$ has a vertical tangent at the point: NCERT Page-209
(a) $(1,1)$
(b) atnopoint
(c) $(0,1)$
(d) $(1,0)$

51. The equation of all lines having slope 2 which are tangent to the curve $y=\frac{1}{x-3}, x\neq 3$, is NCERT Page-207
(a) $y=2$
(b) $y=2x$
(c) $y=2x+3$
(d) None of these

52. The slope of the normal to the curve - NCERT Page-210
(a) $x=a\cos^3\theta, y=a\sin^3\theta$ at $\theta=\frac{\pi}{4}$ is 0
(b) $x=1-a\sin\theta, y=b\cos^2\theta$ at $\theta=\frac{\pi}{2}$ is $\frac{a}{2b}$
(c) Both (a) and (b) are true
(d) Both (a) and (b) are not true

53. The point of intersection of the tangents drawn to the curve $x^2y = 1 - y$ at the points where it is met by the curve $xy = 1 - y$, is given by NCERT Page-208

(a) $(0, -1)$
(b) $(1,1)$
(c) $(0,1)$
(d) None of these

54. The curve given by $x + y = e^{xy}$ has a tangent parallel to the Y-axis at the point NCERT Page-209

(a) $(0,1)$
(b) $(1,0)$
(c) $(1,1)$
(d) None of these

55. If $y = (4x - 5)$ is a tangent to the curve $y^2 = px^3 + q$ at $(2,3)$, then NCERTPage-208

(a) $p = -2, q = -7$
(b) $p = -2, q = 7$
(c) $p = 2, q = -7$
(d) $\mathrm{p} = 2, \mathrm{q} = 7$

56. The two curves $x^3 - 3xy^2 + 2 = 0$ and $3x^2y - y^3 - 2 = 0$ intersect at an angle of NCERT Page-209

(a) $\frac{\pi}{4}$
(b) $\frac{\pi}{3}$
(c) $\frac{\pi}{2}$
(d) $\frac{\pi}{6}$

57. The angle of intersection to the curve $y = x^2, 6y = 7 - x^3$ at $(1,1)$ is : NCERT Page-209

(a) $\frac{\pi}{2}$
(b) $\frac{\pi}{4}$
(c) $\frac{\pi}{3}$
(d) π

58. If tangent to the curve $x = at^2, y = 2at$ is perpendicular to x-axis, then its point of contact is .

(a) (a, a) NCERT Page-208
(b) $(0, a)$
(c) $(0,0)$
(d) $(a, 0)$

59. The slope of the tangent to the curve $x = 3t^2 + 1, y = t^3 - 1$ at $x = 1$ is: NCERT Page-207

(a) $\frac{1}{2}$
(b) 0
(c) -2
(d) ∞

60. If the line $y = 4 + kx, k > 0$, is the tangent to the parabola $y = x - x^2$ at the point P and V is the vertex of the parabola, then the slope of the line through P and V is : NCERT Page-208

(a) $\frac{3}{2}$
(b) $\frac{26}{9}$
(c) $\frac{5}{2}$
(d) $\frac{23}{6}$

61. The condition that the curves $ax^2 + by^2 = 1$ and $a_1x^2 + b_1y^2 = 1$ may cut each other orthogonally is NCERT Page-207

(a) $\frac{a-a_1}{aa_1} = \frac{b-b_1}{bb_1}$
(b) $\frac{a+a_1}{aa_1} = \frac{b+b_1}{bb_1}$
(c) $\frac{a-a_1}{a+a_1} = \frac{b-b_1}{b+b_1}$
(d) None of these

62. What is the x-coordinate of the point on the curve $f(x) = \sqrt{x}(7x - 6)$, where the tangent is parallel to x-axis? NCERT Page-208

(a) $-\frac{1}{3}$
(b) $\frac{2}{7}$
(c) $\frac{6}{7}$
(d) $\frac{1}{2}$

63. The distance between the point $(1,1)$ and the tangent to the curve $y = e^{2x} + x^2$ drawn at the point $x = 0$ is NCERT Page-209

(a) $\frac{1}{\sqrt{5}}$
(b) $\frac{-1}{\sqrt{5}}$
(c) $\frac{2}{\sqrt{5}}$
(d) $\frac{-2}{\sqrt{5}}$

64. The equation of one of the tangents to the curve $y = \cos(x + y), -2\pi \leq x \leq 2\pi$ that is parallel to the line $x + 2y = 0$, is NCERT Page-207

(a) $x + 2y = 1$
(b) $x + 2y = \pi/2$
(c) $x + 2y = \pi/4$
(d) None of these

65. The locus of all the points on the curve $y^2 = 4a\left(x + a\sin\frac{x}{a}\right)$ at which the tangent is parallel to x-axis is : NCERT Page-208

(a) $y = 4a$

(b) $y = 4ax$
(c) $y^2 = 4ax$
(d) $y^2 = 4a^2 \sin \frac{x}{a}$

66. Angle formed by the positive Y-axis and the tangent to $y = x^2 + 4x - 17$ at $\left(\frac{5}{2}, \frac{-3}{4}\right)$ is NCERT Page-209
(a) $\tan^{-1} 9$
(b) $\frac{\pi}{2} - \tan^{-1} 9$
(c) $\frac{\pi}{2} + \tan^{-1} 9$
(d) $\frac{\pi}{2}$

67. The points at which the tangent passes through the origin for the curve $y = 4x^3 - 2x^5$ are
NCERT Page-208
(a) $(0,0), (2,1)$ and $(-1,-2)$
(b) $(0,0), (2,1)$ and $(-2,-1)$
(c) $(2,0), (2,1)$ and $(-3,1)$
(d) $(0,0), (1,2)$ and $(-1,-2)$

68. The angle of intersection of the curve $y^2 = x$ and $x^2 = y$ is NCERT Page-207
(a) $\tan^{-1}\left(\frac{3}{2}\right)$
(b) $\tan^{-1}\left(\frac{3}{4}\right)$
(c) $\tan^{-1}\left(\frac{1}{2}\right)$
(d) $\tan^{-1}\left(\frac{1}{5}\right)$

69. The angle at which the curve $y = ke^{kx}$ intersects the y-axis is : NCERT Page-209
(a) $\tan^{-1}(k^2)$
(b) $\cot^{-1}(k^2)$
(c) $\sin^{-1}\left(\frac{1}{\sqrt{1+k^4}}\right)$
(d) $\sec^{-1}\sqrt{1+k^4}$

70. The number of tangents to the curve $x^{3/2} + y^{3/2} = 2a^{3/2}, a > 0$, which are equally inclined to the axes, is NCERT Page-208
(a) 2
(b) 1
(c) 0
(d) 4

6.5 APPROXIMATIONS

71. If the radius of a sphere is measured as 9cm with an error of 0.03cm, then find the approximating error in calculating its volume. NCERT Page-215
(a) $2.46\pi \mathrm{cm}^3$
(b) $8.62\pi \mathrm{cm}^3$
(c) $9.72\pi \mathrm{cm}^3$
(d) $7.6\pi \mathrm{cm}^3$

72. If the error k% is made in measuring the radius of a sphere, then percentage error in its volume is
(a) k% NCERT Page-215
(b) 3k%
(c) 2k%
(d) $\frac{k}{3}$%

73. The approximate value of $(0.007)^{113}$ NCERT Page-214
(a) $\frac{23}{120}$
(b) $\frac{27}{120}$
(c) $\frac{19}{120}$
(d) $\frac{17}{120}$

74. If $f(x) = x^3 - 7x^2 + 15$, then the approximate value of $f(5.001)$ is
(a) 34.995
(b) -30.995
(c) 24.875
(d) None of these

75. If the error committed in measuring the radius of sphere, then ... will be the percentage error in the surface area.
(a) 1%
(b) 2%
(c) 3%
(d) 4%

76. The approximate change in the volume V of a cube of side x meters caused by increasing the side by 2%, is
(a) $1.06x^3 m^3$ NCERT Page-214
(b) $1.26x^3 m^3$
(c) $2.50x^3 m^3$
(d) $0.06x^3 m^3$

77. The approximate value of $\{(3.92)^2 + 3(2.1)^4\}^{1/6}$ is NCERT Page-215
(a) 2.466
(b) 3.567
(c) 1.562
(d) 2.577

78. There is an error of 0.04cm in the measurement of the diameter of a sphere. When the radius is 10cm, the percentage error in the volume of the sphere is NCERT Page-215
(a) ± 1.2
(b) ± 1.0
(c) ± 0.6
(d) ± 0.8

79. If the radius of a spherical balloon increases by 0.2%. Find the percentage increase in its volume
NCERT Page-215
(a) 0.8%
(b) 0.12%
(c) 0.6%
(d) 0.3%

6.6 MAXIMA AND MINIMA

80. A wire of length 22m is to be cut into two pieces. One of the pieces is to be made into a square and the other into an equilateral triangle. Then, the length of the side of the equilateral triangle, so that the combined area of the square and the equilateral triangle is minimum, is: NCERT Page-231/N-160
(a) $\frac{22}{9+4\sqrt{3}}$
(b) $\frac{66}{9+4\sqrt{3}}$
(c) $\frac{22}{4+9\sqrt{3}}$
(d) $\frac{66}{4+9\sqrt{3}}$

81. Let $f: \mathbb{R} \to \mathbb{R}$ be a function defined by
$f(x) = (x-3)^{n_1}(x-5)^{n_2}, n_1, n_2 \in N$. Then, which of the following is not true? NCERT Page-224/N-166
(a) For $n_1 = 3, n_2 = 4$, there exists $\alpha \in (3,5)$ where f attains local maxima.
(b) For $n_1 = 4, n_2 = 3$, there exists $\alpha \in (3,5)$ where f attains local minima.
(c) For $n_1 = 3, n_2 = 5$, there exists $\alpha \in (3,5)$ where f attains local maxima.
(d) For $n_1 = 4, n_2 = 6$, there exists $\alpha \in (3,5)$ where f attains local maxima.

82. Let $y = y(x)$ be the solution of the differential equation $(x+1)y' - y = e^{3x}(x+1)^2$, with $y(0) = \frac{1}{3}$.
Then, the point $x = -\frac{4}{3}$ for the curve $y = y(x)$ is : NCERT/ Page-230/N-167
(a) not a critical point
(b) a point of local minima

(c) a point of local maxima
(d) a point of inflection

83. Consider a cuboid of sides $2,4x$ and $5x$ and a closed hemisphere of radius r. If the sum of their surface areas is constant k, then the ratio $x:r$, for which the sum of their volumes is maximum, is :
(a) $2:5$ NCERT Page-224/N-167
(b) $19:45$
(c) $3:8$
(d) $19:15$

84. The sum of the absolute minimum and the absolute maximum values of the function $f(x) = |3x - x^2 + 2| - x$ in the interval $[-1,2]$ is : NCERT Page-230/N-166
(a) $\frac{\sqrt{17}+3}{2}$
(b) $\frac{\sqrt{17}+5}{2}$
(c) 5
(d) $\frac{9-\sqrt{17}}{2}$

85. The lengths of the sides of a triangle are $10 + x^2, 10 + x^2$ and $20 - 2x^2$. If for $x = k$, the area of the triangle is maximum, then $3\mathrm{k}^2$ is equal to: NCERT Page-226/N-167
(a) 5
(b) 8
(c) 10
(d) 12

86. The sum of absolute maximum and absolute minimum values of the function
$f(x) = |2x^2 + 3x - 2| + \sin x \cos x$ in the interval $[0,1]$ is : NCERT Page-224/N-168
(a) $3 + \frac{\sin(1)\cos^2\left(\frac{1}{2}\right)}{2}$
(b) $3 + \frac{1}{2}(1 + 2\cos(1))\sin(1)$
(c) $5 + \frac{1}{2}(\sin(1) + \sin(2))$
(d) $2 + \sin\left(\frac{1}{2}\right)\cos\left(\frac{1}{2}\right)$

87. Let the maximum area of the triangle that can be inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{4} = 1, a > 2$, having one of its vertices at one end of the major axis of the ellipse and one of its sides parallel to the y-axis, be $6\sqrt{3}$. Then the eccentricity of the ellispe is: NCERT Page-236/N-170
(a) $\frac{\sqrt{3}}{2}$
(b) $\frac{1}{2}$
(c) $\frac{1}{\sqrt{2}}$
(d) $\frac{\sqrt{3}}{4}$

88. If the function f be given by $f(x) = |x|, x \in R$, then NCERT Page-224/N-167
(a) point of minimum value of f is $x = 1$
(b) f has no point of maximum value in R
(c) Both (a) and (b) are true
(d) Both (a) and (b) are not true

89. The function $f(x) = 1 + x(\sin x)[\cos x], 0 < x \le \frac{\pi}{2}$ (where [. . is GI.F.) NCERT Page-223/N-168
(a) is continuous on $\left(0, \frac{\pi}{2}\right)$
(b) is strictly increasing in $\left(0, \frac{\pi}{2}\right)$
(c) is strictly decreasing in $\left(0, \frac{\pi}{2}\right)$
(d) has global maximum value 2

90. The difference between the greatest and least values of the function $f(x) = \sin 2x - x$, on $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ is NCERT Page-222/ N-173
(a) $\frac{\pi}{2}$
(b) π
(c) $\frac{3\pi}{2}$
(d) $\frac{\pi}{4}$

91. The maximum value of $\frac{\ln x}{x}$ in $(2, \infty)$ is
(a) 1
(b) e
(c) 2/e
(d) 1/e

92. If at $x = 1$, the function $x^4 - 62x^2 + ax + 9$ attains its maximum value on the interval $[0,2]$, then the value of a is NCERT Page-225/N-172
(a) 110
(b) 10
(c) 55
(d) None of these

93. If for a function $f(x), f'(a) = 0, f''(a) = 0, f'''(a) > 0$, then at $\mathrm{x} = \mathrm{a}, \mathrm{f(x)}$ is NCERT Page-223/N-166
(a) Minimum
(c) Not an extreme point
(b) Maximum
(d) Extreme point

94. On the interval $[0,1]$ the function $x^{25}(1-x)^{75}$ takes its maximum value at the point NCERT Page-223/N-167
(a) 0
(b) $\frac{1}{4}$

(c) $\frac{1}{2}$
(d) $\frac{1}{3}$

95. The maximum area of rectangle inscribed in a circle of diameter R is NCERT Page-227/N-169
(a) R^2
(b) $\frac{R^2}{2}$
(c) $\frac{R^2}{4}$
(d) $\frac{R^2}{8}$

96. If sum of two numbers is 3 , the maximum value of the product of first and the square of second is
(a) 4 NCERT Page-224/N-170
(b) 3
(c) 2
(d) 1

97. If xy^4 attains maximum value at the point (x, y) on the line passing through the points $(50+\alpha, 0)$ and $(0,50+\alpha), \alpha>0$, then (x, y) also lies on the line : NCERT Page-223/N-171
(a) $y=4x$
(b) $x=4y$
(c) $y=4x+\alpha$
(d) $x=4y-\alpha$

98. Let S be the set of all integral values of α for which the sum of squares of two real roots of the quadratic equation $3x^2+(\alpha-6)x+(\alpha+3)=0$ is minimum. Then S :
(a) is an empty set NCERT Page-222/N-172
(b) is a singleton
(c) contains exactly two elements
(d) contains more than two elements

99. The curve $y(x)=ax^3+bx^2+cx+5$ touches the x-axis at the point $P(-2,0)$ and cuts the y-axis at the point Q, where y' is equal to 3 . Then the local maximum value of $y(x)$ is: NCERT Page-224/N-167
(a) $\frac{27}{4}$
(b) $\frac{29}{4}$
(c) $\frac{37}{4}$
(d) $\frac{9}{2}$

100. If $A>0, B>0$ and $A+B=\pi/3$, then the maximum value of $\tan A \tan B$ is NCERT Page-225/N-169
(a) $\frac{1}{\sqrt{3}}$
(b) $\frac{1}{3}$

(c) 3
(d) $\sqrt{3}$

101. Let $f(x)=4x^3-11x^2+8x-5, x\in R$. Then f : NCERT Page-224/N-167
(a) has a local minima at $x=\frac{1}{2}$
(b) has a local minima at $x=\frac{3}{4}$
(c) is increasing in $\left(\frac{1}{2},\frac{3}{4}\right)$
(d) is decreasing in $\left(\frac{1}{2},\frac{4}{3}\right)$

102. Let the eccentricity of the ellipse $x^2+a^2y^2=25a^2$ be b times the eccentricity of the hyperbola $x^2-a^2y^2=5$, where a is the minimum distance between the curves $y=e^x$ and $y=\log_e x$. Then $a^2+\frac{1}{b^2}$ is equal to: NCERT Page-225/N-169
(a) $\frac{3}{2}$
(b) $\frac{5}{2}$
(c) 3
(d) 5

103. Let $f(x)$ be a function defined as follows:
$f(x)=\sin(x^2-3x), x\leq 0$; and $6x+5x^2, x>0$
Then at $x=0, f(x)$ NCERT Page-225/N-167
(a) has a local maximum
(c) is discontinuous
(b) has a local minimum
(d) None of these

104. A right circular cylinder which is open at the top and has a given surface area, will have the greatest volume if its height h and radius r are related by NCERT Page-224/ $\mathrm{N}-170$
(a) $2\mathrm{h}=\mathrm{r}$
(b) $\mathrm{h}=4\mathrm{r}$
(c) $\mathrm{h}=2\mathrm{r}$
(d) $\mathrm{h}=\mathrm{r}$

105. A wire $34\mathrm{cm}$ long is to be bent in the form of a quadrilateral of which each angle is $90°$. What is the maximum area which can be enclosed inside the quadrilateral? NCERT Page-224/N-170
(a) $68\mathrm{cm}^2$
(b) $70\mathrm{cm}^2$
(c) $71.25\mathrm{cm}^2$
(d) $72.25\mathrm{cm}^2$

106. Find the greatest value of the function NCERT Page-225/N-170

$f(x)=\frac{\sin 2x}{\sin\left(x+\frac{\pi}{4}\right)}$ on the interval $\left[0,\frac{\pi}{2}\right]$

(a) 1
(b) 2
(c) 3
(d) None of these

107. The local minimum value of the function f given by $f(x)=3+|x|, x\in R$ is NCERT Page-224/N-166

(a) 1
(b) 2
(c) 3
(d) 0

108. The minimum value of the function $y=x^4-2x^2+1$ in the interval $\left[\frac{1}{2},2\right]$ is

NCERT Page-225/N-167

(a) 0
(b) 2
(c) 8
(d) 9

109. If the absolute maximum value of the function $f(x)=(x^2-2x+7)e^{(4x^3-12x^2-180x+31)}$ in the interval $[-3,0]$ is $f(\alpha)$, then: NCERT Page-230/N-167

(a) $\alpha=0$.
(c) $\alpha\in(-1,0)$
(b) $\alpha=-3$
(d) $\alpha\in(-3,-1)$

110. The maximum value of the function $y=-x^2$ in the interval $[-1,1]$ is NCERT Page-224/N-168

(a) 0
(b) 2
(c) 8
(d) 9

Exercise 2 : NCERT Exemplar & JEE Main

NCERT EXEMPLAR QUESTIONS

1. Each side of an equilateral triangle expands at the rate of 2 cm/s. What is the rate of increase of area of the triangle when each side is 10cm ? NCERT Page-196/N-169

(a) $10\sqrt{2}\mathrm{cm}^2/\mathrm{s}$
(b) $10\sqrt{3}\mathrm{cm}^2/\mathrm{s}$
(c) $10\mathrm{cm}^2/\mathrm{s}$
(d) $5\sqrt{3}\mathrm{cm}^2/\mathrm{s}$

2. Aladder, 5m long, standing on a horizontal floor, leans against a vertical wall. If the top of the ladder slides downwards at the rate of 10cm/s, then the rate at which the angle between the floor and the ladder is decreasing when lower end of ladder is 2 m from the wall is NCERT Page-236/N-169

(a) $\frac{1}{10}\text{rad/s}$
(b) $\frac{1}{20}\text{rad/s}$
(c) 20rad/s
(d) 10rad/s

3. The curve $y = x^{\frac{1}{5}}$ at $(0,0)$ has NCERT Page-210

(a) a vertical tangent (parallel to y-axis)
(b) a horizontal tangent (parallel to x-axis)
(c) no oblique tangent
(d) no tangent

4. The equation of normal to the curve $3x^2 - y^2 = 8$ which is parallel to the line $x + 3y = 8$ is NCERT Page-210

(a) $3x - y = 8$
(b) $3x + y + 8 = 0$
(c) $x + 3y \pm 8 = 0$
(d) $x + 3y = 0$

5. If the curve $ay + x^2 = 7$ and $x^3 = y$, cut orthogonally at $(1,1)$, then the value of a is NCERT Page-210

(a) 1
(b) 0
(c) -6
(d) 6

6. If $y = x^4 - 10$ and x changes from 2 to 1.99 , then what is the change in y ? NCERT Page-196/N-149

(a) 0.32
(b) 0.032
(c) 5.68
(d) 5.968

7. The equation of tangent to the curve $y(1 + x^2) = 2 - x$, where it crosses X-axis is NCERT Page-210

(a) $x + 5y = 2$
(b) $x - 5y = 2$
(c) $5x - y = 2$
(b) $x - 5y = 2$

8. The points at which the tangent to the curve $y = x^3 - 12x + 18$ are parallel to X-axis are NCERT Page-209

(a) $(2, -2), (-2, -34)$
(b) $(2,34), (-2,0)$
(c) $(0,34), (-2,0)$
(d) $(2,2), (-2,34)$

9. The tangent to the curve $y = e^{2x}$ at the point $(0,1)$ meets X-axis at NCERT Page-209
(a) $(0,1)$
(b) $\left(-\frac{1}{2},0\right)$
(c) $(2,0)$
(d) $(0,2)$

10. The slope of tangent to the curve $x = t^2 + 3t - 8, y = 2t^2 - 2t - 5$ at the point $(2,-1)$ is
NCERT Page-210
(a) $\frac{22}{7}$
(b) $\frac{6}{7}$
(c) $\frac{-6}{7}$
(d) -6

11. The two curves $x^3 - 3xy^2 + 2 = 0$ and $3x^2y - y^3 - 2 = 0$ intersect at an angle of NCERT Page-210
(a) $\frac{\pi}{4}$
(b) $\frac{\pi}{3}$
(c) $\frac{\pi}{2}$
(d) $\frac{\pi}{6}$

12. The interval on which the function $f(x) = 2x^3 + 9x^2 + 12x - 1$ is decreasing, is NCERT Page-203/N-167
(a) $[-1,\infty)$
(b) $[-2,-1]$
(c) $(-\infty,-2]$
(d) $[-1,1]$

13. If $f: R \to R$ be definend by $f(x) = 2x + \cos\, x$, then f NCERT Page-203/N-167
(a) has a minimum at $x = \pi$
(b) has a maximum at $x = 0$
(c) is a decreasing function
(d) is an increasing function

14. If $y = x(x-3)^2$ decreases for the values of x given by NCERT Page-202/N-168
(a) $1 < x < 3$
(b) $x < 0$
(c) $x > 0$
(d) $0 < x < \frac{3}{2}$.

15. The function $f(x) = 4\sin^3\, x - 6\sin^2\, x + 12\sin\, x + 100$ is strictly NCERT Page-203/N-167
(a) increasing in $\left(\pi,\frac{3\pi}{2}\right)$
(b) decreasing in $\left(\frac{\pi}{2},\pi\right)$
(c) decreasing in $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$

(d) decreasing in $\left[0, \frac{\pi}{2}\right]$

16. Which of the following function is decreasing on $\left(0, \frac{\pi}{2}\right)$? NCERT Page-203/N-175
   (a) $\sin 2x$
   (b) $\tan x$
   (c) $\cos x$
   (d) $\cos 3x$

17. The function $f(x) = \tan x - x$ NCERT Page-203/N-177
   (a) always increases
   (b) always decreases
   (c) never increases
   (d) sometimes increases and sometimes decreases

18. If x is real, then the minimum value of $x^2 - 8x + 17$ is NCERT Page-218/N-173
   (a) -1
   (b) 0
   (c) $\frac{N}{1}$
   (d) 2

19. The smallest value of the polynomial $x^3 - 18x^2 + 96x$ in $[0,9]$ is NCERT Page-218/N-173
   (a) 126
   (b) 0
   (c) 135
   (d) 160

20. The function $f(x) = 2x^3 - 3x^2 - 12x + 4$, has NCERT Page-214/N-172
   (a) two points of local maximum
   (b) two points of local minimum
   (c) one maxima and one minima
   (d) no maxima or minima

21. The maximum value of $\sin x \cdot \cos x$ is NCERT Page-219/N-175
   (a) $\frac{1}{4}$
   (b) $\frac{1}{2}$
   (c) $\sqrt{2}$
   (d) $2\sqrt{2}$

22. At $x = \frac{5\pi}{6}, f(x) = 2\sin 3x + 3\cos 3x$ is
   (a) maximum NCERT Page-222/N-175
   (b) minimum
   (c) zero
   (d) neither maximum nor minimum

23. The maximmum slope of curve $y = -x^3 + 3x^2 + 9x - 27$ is
(a) 0
(b) 12
(c) 16
(d) 32

24. The function $f(x) = x^x$ has a stationary point at NCERT Page-224/N-175
(a) $x = e$
(b) $x = \frac{1}{e}$
(c) $x = 1$
(d) $x = \sqrt{e}$

25. The maximum value of $\left(\frac{1}{x}\right)^x$ is NCERT Page-224/N-173
(a) e
(b) e^e
(c) $e^{\frac{1}{e}}$
(d) $\left(\frac{1}{e}\right)^{\frac{1}{e}}$

JEE MAIN

26. Consider $f(x) = \tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right), x \in \left(0, \frac{\pi}{2}\right)$. A normal to $y = f(x)$ at $x = \frac{\pi}{6}$ a so passes through the point : NCERT Page-210
(a) $\left(\frac{\pi}{6}, 0\right)$
(b) $\left(\frac{\pi}{4}, 0\right)$
(c) $(0,0)$
(d) $\left(0, \frac{2\pi}{3}\right)$

27. A wire of length 2 units is cut into two parts which are bent respectively to form a square of side = x units and a circle of radius = r units. If the sum of the areas of the square and the circle so formed is minimum, then: NCERT Page-226/N-169
(a) $x = 2r$
(c) $2x = (\pi + 4)r$
(b) $2x = r$
(d) $(4 - \pi)x = \pi r$

28. The normal to the curve $y(x-2)(x-3) = x + 6$ at the point where the curve intersects the y-axis passes through the point: NCERT Page-210 |
(a) $\left(\frac{1}{2}, \frac{1}{3}\right)$

(b) $\left(-\frac{1}{2}, -\frac{1}{2}\right)$
(c) $\left(\frac{1}{2}, \frac{1}{2}\right)$
(d) $\left(\frac{1}{2}, -\frac{1}{3}\right)$

29. Twenty metres of wire is available for fencing off a flowerbed in the form of a circular sector. Then the maximum area (in sq. m) of the flower-bed, is : NCERT Page-236/N-169
(a) 30
(b) 12.5
(c)10
(d) 25

30. The eccentricity of an ellipse whose centre is at the origin is $\frac{1}{2}$. If one of its directices is $x = -4$, then the equation of the normal to it at $\left(1, \frac{3}{2}\right)$ is : NCERT Page-236
(a) $x + 2y = 4$
(c) $4x - 2y = 1$
(b) $2y - x = 2$
(d) $4x + 2y = 7$

31. Let $f(x) = x^2 + \frac{1}{x^2}$ and $g(x) = x - \frac{1}{x}, x \in R - \{-1,0,1\}$. If $h(x) = \frac{f(x)}{g(x)}$, then the local minimum value of $h(x)$ is : NCERT Page-224/N-168
(a) -3
(b) $-2\sqrt{2}$
(c) $2\sqrt{2}$
(d) 3

32. If the curves $y^2 = 6x, 9x^2 + b^2 = 16$ intersect each other at right angles, then the value of b is:
(a) $\frac{7}{2}$ NCERT Page-236
(b) 4
(c) $\frac{9}{2}$
(d) 6

33. The maximum volume (in cu.m) of the right circular cone having slant height 3m is:
(a) 6π NCERT Page-240/N-169
(b) $3\sqrt{3}\pi$
(c) $\frac{4}{3}\pi$
(d) $2\sqrt{3}\pi$

34. If θ denotes the acute angle between the curves, $y = 10 - x^2$ and $y = 2 + x^2$ at a point of their intersection, then $|\tan \theta|$ is equal to: ⬚ NCERT Page-236

(a) $\frac{4}{9}$
(b) $\frac{8}{15}$
(c) $\frac{7}{17}$
(d) $\frac{8}{17}$

35. If the tangent to the curve, $y = x^3 + ax - b$ at the point $(1, -5)$ is perpendicular to the line, $-x + y + 4 = 0$, then which one of the following points lies on the curve? NCERT Page-209 |
(a) $(-2,1)$
(b) $(-2,2)$
(c) $(2,-1)$
(d) $(2,-2)$

36. Let S be the set of all values of x for which the tangent to the curve $y = f(x) = x^3 - x^2 - 2x$ at (x, y) is parallel to the line segment joining the points $(1, f(1))$ and $(-1, f(-1))$, then S is equal to:
NCERT Page-208 |
(a) $\left\{\frac{1}{3}, 1\right\}$
(b) $\left\{-\frac{1}{3}, -1\right\}$
(c) $\left\{\frac{1}{3}, -1\right\}$
(d) $\left\{-\frac{1}{3}, 1\right\}$

37. Let $f(x)$ be a polynomial of degree 5 such that $x = \pm 1$ are its critical points. If $\lim_{x\to 0}\left(2 + \frac{f(x)}{x^3}\right) = 4$, then which one of the following is not true ? NCERT Page-230/N-170
(a) f is an odd function.
(b) $f(1) - 4f(-1) = 4$.
(c) $x = 1$ is a point of maxima and $x = -1$ is a point of minima of f.
(d) $x = 1$ is a point of minima and $x = -1$ is a point of maxima of f.

38. Let $f(x)$ be a polynomial of degree 3 such that $f(-1) = 10, f(1) = -6, f(x)$ has a critical point at $x = -1$ and $f'(x)$ has a critical point at $x = 1$.
Then $f(x)$ has a local minima at $x =$ NCERT Page-224/N-169

39. Let $f(x) = 3\sin^4 x + 10\sin^3 x + 6\sin^2 x - 3, x \in \left[-\frac{\pi}{6}, \frac{\pi}{2}\right]$.
Then, f is : NCERT Page-239/N-156
(a) increasing in $\left(-\frac{\pi}{6}, \frac{\pi}{2}\right)$ (b) decreasing in $\left(0, \frac{\pi}{2}\right)$
(c) increasing in $\left(-\frac{\pi}{6}, 0\right)$ (d) decreasing in $\left(-\frac{\pi}{6}, 0\right)$

40. The number of distinct real roots of the equation $x^5(x^3 - x^2 - x + 1) + x(3x^3 - 4x^2 - 2x + 4) - 1 = 0$ is NCERT Page-200 |

41. Let the function $f(x) = 2x^2 - \log_e x, x > 0$, be decreasing in $(0, a)$ and increasing in $(a, 4)$. A tangent to the parabola $y^2 = 4ax$ at a point P on it passes through the point $(8a, 8a - 1)$ but does not pass through the point $\left(-\frac{1}{a}, 0\right)$.
If the equation of the normal at P is $\frac{x}{\alpha} + \frac{y}{\beta} = 1$, then $\alpha + \beta$ is equal to NCERT Page-210

42. Let $f(x) = \begin{cases} x^3 - x^2 + 10x - 7, & x \leq 1 \\ -2x + \log_2(b^2 - 4), & x > 1. \end{cases}$ NCERT/Page-224/N-169
Then the set of all values of b, for which $f(x)$ has maximum value at $x = 1$, is:
(a) $(-6, -2)$
(b) $(2,6)$
(c) $[-6, -2) \cup (2,6]$
(d) $[-\sqrt{6}, -2] \cup [2, \sqrt{6}]$

43. Let quadratic curve passing through the point $(-1,0)$ and touching the line $y = x$ at $(1,1)$ be $y = f(x)$. Then the x-intercept of the normal to the curve at the point $(\alpha, \alpha + 1)$ in the first quadrant is
NCERT Page 210 |

44. $\max_{0 \leq x \leq \pi} \left\{x - 2\sin x \cos x + \frac{1}{3}\sin 3x\right\} =$ NCERT Page-230/N-167
(a) $\frac{5\pi+2+3\sqrt{3}}{6}$
(b) $\frac{\pi+2-3\sqrt{3}}{6}$
(c) π
(d) 0

Exercise 3 : Skill Enhancer MCQs

1. Let the function $g: (-\infty, \infty) \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ be given by $g(u) = 2\tan^{-1}(e^u) - \frac{\pi}{2}$. Then, g is
(a) even and is strictly increasing in $(0, \infty)$
(b) odd and is strictly decreasing in $(-\infty, \infty)$
(c) odd and is strictly increasing in $(-\infty, \infty)$
(d) neither even nor odd, but is strictly increasing in $(-\infty, \infty)$

2. If A denotes the arithmetic mean of the real numbers $a_1, a_2, \ldots\ldots, a_n$, then $\sum_{i=1}^{n}(x - a_i)^2$ has a minimum at.
(a) $x = 0$
(b) $x = 1$
(c) $x = A$
(d) $x = A + 1$

3. Let $f(x)$ and $g(x)$ be two continuous functions defined from $R \to R$, such that $f(x_1) > f(x_2)$ and $g(x_1) > g(x_2) \forall x_1 > x_2$. Then the solution set of $f\left(g(\alpha^2 - 2\alpha)\right) > f(g(3\alpha - 4))$, is

(a) $\alpha \in [1,4)$
(b) $\alpha \in (1,4)$
(c) $\alpha \in (1,4]$
(d) None of these

4. $f(x)$ is cubic polynomial with $f(2) = 18$ and $f(1) = -1$. Also $f(x)$ has local maxima at $x = -1$ and $f'(x)$ has local minima at $x = 0$, then
(a) the distance between $(-1,2)$ and (a $f(a)$), where $x = a$ is the point of local minima is $2\sqrt{5}$
(b) $f(x)$ is increasing for $x \in [1,2\sqrt{5}]$
(c) $f(x)$ has local minima at $x = 2$
(d) the value of $f(0) = 15$

5. Tangent at $P_1(2,3)$ on the curve $3y = x^3 + 1$ meets the curve again at P_2. The tangent at P_2 meets the curve at P_3 and so on. If the sum of the ordinates for $P_1, P_2, P_3, \ldots . P_{60}$ be S then $S + \left(\frac{2^{183}-8}{27}\right)$ is equal to 5k, when k is equal to
(a) 3
(b) 4
(c) 5
(d) 6

6. For the curve $y = 3\sin\ \theta\cos\ \theta, x = e^{\theta}\sin\ \theta, 0 \leq \theta \leq \pi$, the tangent is parallel to x-axis when θ is:
(a) $\frac{3\pi}{4}$
(b) $\frac{\pi}{2}$
(c) $\frac{\pi}{4}$
(d) $\frac{\pi}{6}$

7. If $f(x) = (\sin^2\ x - 1)^n(2 + \cos^2\ x)$, then $x = \frac{\pi}{2}$ is a point of (where, $n \in N$)
(a) local maximum, if n is odd
(b) Both (a) and (b)
(c) Neither (a) nor (b)
(d) local maximum, if n is even

8. The largest term in the sequence, $a_n = \frac{n^2}{n^3+200}$ is
(a) a_6
(b) a_7
(c) a_8
(d) None of these

9. If $f(x)$ is differentiable and strictly increasing function,

then the value of $\lim_{x\to 0} \frac{f(x^2)-f(x)}{f(x)-f(0)}$ is
(a) 1 (b) 0 (c) -1
(d) 2

10. Domain of the function $f(x)$ if $3^x + 3^{f(x)} =$ minimum of $\phi(t)$ where $\phi(t) =$ minimum of $\{12t^3 - 15t^2 + 36t - 25, 2|\sin\ t|; 2 \le t \le 4\}$ is
(a) $(-\infty, 1)$
(b) $(-\infty, \log_3\ e)$
(c) $(0, \log_3\ 2)$
(d) $(-\infty, \log_3\ 2)$

11. The greatest of the numbers $1, 2^{1/2}, 3^{1/3}, 4^{1/4}, 5^{1/5}, 6^{1/6}$ and $7^{1/7}$ is
(a) $2^{1/2}$
(b) $3^{1/3}$
(c) $7^{1/4}$
(d) All but 1 are equal

12. $f(x) = \begin{cases} 2 - |x^2 + 5x + 6|; & x \neq -2 \\ a^2 + 1; & x = -2 \end{cases}$ then the range of a so that $f(x)$ has maxima at $x = -2$ is
(a) $|a| \ge 1$
(b) $|a| < 1$
(c) $a > 1$
(d) $a < 1$

13. Let f and g be functions from the interval $[0, \infty)$ to the interval $[0, \infty), f$ being an increasing and g being a decreasing function. If $\mathrm{f\{g(0)\}} = 0$ then
(a) $\mathrm{f\{g(x)\}} \ge \mathrm{f\{g(0)\}}$
(b) $\mathrm{f\{g(2)\}} = 0, \mathrm{g\{f(x)\}} \le \mathrm{g\{f(0)\}}$
(c) $\mathrm{f\{g(2)\}} = -1$
(d) None of these

14. If composite function $f_1(f_2(f_3(\ldots..(f_n(x))\ldots..)$ is an increasing function and if r of $f_i's$ are decreasing functions while rest are increasing, then maximum value of $r(n-r)$ is :
(a) $\frac{n^2-1}{4}$ when n is an even number
(b) $\frac{n^2}{4}$ when n is an odd number
(c) Both (a) and (b)
(d) Neither (a) nor (b)

15. The set of all values of a for which the function $f(x) = (a^2 - 3a + 2)(\cos^2\ x/4 - \sin^2\ x/4) + (a - 1)x + \sin\ 1$ does not possess critical points is
(a) $[1, \infty)$
(c) $(-2,4)$
(b) $(0,1) \cup (1,4)$
(d) $(1,3) \cup (3,5)$

Exercise 4: Numeric Value Answer Questions

1. A water tank has the shape of a light circular cone with axis vertical and vertex downwards. Its semivertical angle is $\tan^{-1}\frac{3}{4}$. Water is poured in it at a constant rate of 6 cubic meter per hour. The rate (in square meter per hour), at which the wet curved surface area of the tank is increasing, when the depth of water in the tank is 4 meters, is

2. If the tangent to the curve $y = x^3 - x^2 + x$ at the point (a, b) is also tangent to the curve $y = 5x^2 + 2x - 25$ at the point $(2, -1)$, then $|2a + 9b|$ is equal to

3. A water tank has the shape of an inverted right circular cone, whose semi-vertical angle is $\tan^{-1}\left(\frac{1}{2}\right)$. Water is poured into it at a constant rate of 5 cubic meter per minute. Then the rate (in m/min.), at which the level of water is rising at the instant when the depth of water in the tank is 10m; is equal to $\frac{k}{\pi}$, then k is

4. If a curve passes through the point $(1, -2)$ and has slope of the tangentat anypoint (x, y) on itas $\frac{x^2-2y}{x}$, alsothe curvepasses through the point $(\sqrt{3}a, 0)$, then value of a is

5. The height of a right circular cylinder of maximum volume inscribed in a sphere of radius 3 is equal to $m\sqrt{3}$, then the value of m is

6. The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local minimum at

7. If the tangent to the curve $y = \frac{x}{x^2-3}, x \in R, (x \neq \pm\sqrt{3})$, at a point $(\alpha, \beta) \neq (0,0)$ on it is parallel to the line $2x + 6y - 11 = 0$, then calculate $|6\alpha + 2\beta|$.

8. Let M and N be the number of points on the curve $y^5 - 9xy + 2x = 0$, where the tangents to the curve are parallel to x-axis and y-axis, respectively. Then the value of $M + N$ equals

9. . Let l be a line which is normal to the curve $y = 2x^2 + x + 2$ at a point P on the curve. If the point $Q(6,4)$ lies on the line l and O is origin, then the area of the triangle OPQ is equal to

10. If the curves $\frac{x^2}{a^2} + \frac{y^2}{12} = 1$ and $y^3 = 8x$ intersect at right angles, then the value of a^2 is equal to

Answer Keys

EXCERSISE- 1

1	(d)	12	(c)	23	(c)	34	(d)	45	(a)	56	(c)	67	(d)	78	(c)	89	(a)	100	(b)
2	(b)	13	(d)	24	(d)	35	(a)	46	(d)	57	(a)	68	(b)	79	(c)	90	(b)	101	(d)
3	(c)	14	(c)	25	(b)	36	(b)	47	(c)	58	(c)	69	(b)	80	(b)	91	(d)	102	(d)
4	(d)	15	(a)	26	(a)	37	(b)	48	(c)	59	(b)	70	(b)	81	(c)	92	(d)	103	(b)
5	(c)	16	(a)	27	(a)	38	(d)	49	(b)	60	(c)	71	(c)	82	(b)	93	(c)	104	(d)
6	(c)	17	(b)	28	(b)	39	(d)	50	(d)	61	(a)	72	(b)	83	(b)	94	(b)	105	(d)
7	(a)	18	(c)	29	(b)	40	(b)	51	(d)	62	(b)	73	(a)	84	(a)	95	(b)	106	(a)
8	(a)	19	(c)	30	(a)	41	(b)	52	(d)	63	(c)	74	(b)	85	(c)	96	(a)	107	(c)
9	(d)	20	(a)	31	(c)	42	(d)	53	(c)	64	(b)	75	(d)	86	(b)	97	(a)	108	(a)
10	(c)	21	(b)	32	(d)	43	(b)	54	(b)	65	(c)	76	(d)	87	(a)	98	(a)	109	(b)
11	(b)	22.	(a)	33	(b)	44	(c)	55	(c)	66	(b)	77	(a)	88	(b)	99	(a)	110	(a)

- EXCERSISE -2 (NCERT Exemplar & JEE MAIN)

1	(b)	6	(a)	11	(c)	16	(c)	21	(b)	26	(d)	31	(c)	36	(d)	41	(45)		
2	(b)	7	(a)	12	(b)	17	(a)	22	(d)	27	(a)	32	(c)	37	(c)	42	(c)		
3	(b)	8	(d)	13	(d)	18	(c)	23	(b)	28	(c)	33	(d)	38	(3)	43	(11)		
4	(c)	9	(b)	14	(a)	19	(b)	24	(b)	29	(d)	34	(b)	39	(d)	44	(a)		
5	(d)	10	(b)	15	(b)	20	(c)	25	(c)	30	(c)	35	(d)	40	(3)				

Exercise-3: (Skill Enhancer MCOS)

1	(c)	3	(b)	5	(c)	7	(c)	9	(c)	11	(b)	13	(b)	15	(b)				
2	(c)	4	(b)	6	(c)	8	(b)	10	(d)	12	(a)	14	(d)						

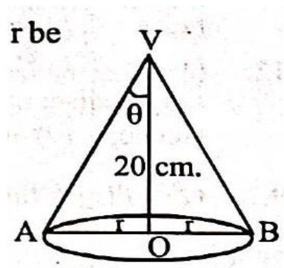
EXCERSISE-4- NUMERICAL VALUE TYPE QUESTIONS

1	5	2	195	3	(0.2)	4	(1)	5	2	6	(2)	7	(19)	8	(2)	9	13	10	4

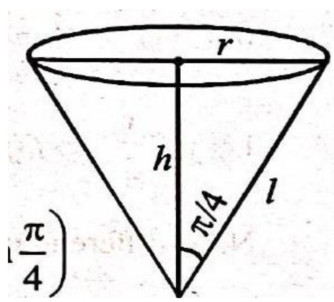
HINTS AND SOLUTIONS

EXERCISE -1

1. (d)
2. (b) Let θ be the semi-vertical angle and r be the radius of the cone at time t.
Then, $r = 20\tan\ \theta$
$$\frac{dr}{dt} = 20\sec^2\ \theta \frac{d\theta}{dt}$$
$$\Rightarrow \frac{dr}{dt} = 20 \times \sec^2\ 30° \times 2 = \frac{160}{3}\text{cm/sec}$$

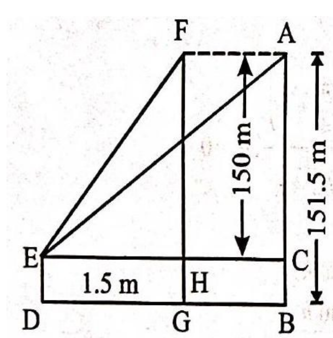


3. (c) It represents the surface area, then
$$\frac{ds}{dt} = 2\text{cm}^2/\text{s}$$
$$s = \pi r l = \pi l \cdot \sin\frac{\pi}{4} l = \frac{\pi}{\sqrt{2}} l^2$$



Therefore, $\frac{ds}{dt} = \frac{2\pi}{\sqrt{2}} l \cdot \frac{dl}{dt} = \sqrt{2}\pi l \cdot \frac{dl}{dt}$
When $\frac{ds}{dt} = 2\text{cm}^2/\text{s}, l = 4\text{cm}$
$$\frac{dl}{dt} = \frac{1}{\sqrt{2}\pi \cdot 4} \cdot 2 = \frac{1}{2\sqrt{2}\pi} = \frac{\sqrt{2}}{4\pi}\text{cm/s}$$

4. (d)



Let AB be the height of the kite and DE be the height of the boy.

Let $\mathrm{DB} = \mathrm{x} = \mathrm{EC} \therefore \frac{\mathrm{dx}}{\mathrm{dt}} = 10\mathrm{m/s}$

Let $AE = y \because AB = 15.15\mathrm{m}$

$\therefore \mathrm{AC} = \mathrm{AB} - \mathrm{BC} = 151.5\mathrm{m} - 1.5\mathrm{m} = 150\mathrm{m}$

Also, $\mathrm{AC}^2 + \mathrm{EC}^2 = \mathrm{AE}^2$ (by Pythagoras theorem)

$\Rightarrow 150^2 + \mathrm{x}^2 = \mathrm{y}^2$

Differentiating both sides w.r.t. t, we have

$0 + 2x\frac{dx}{dt} = 2y\frac{dy}{dt} \Rightarrow x\frac{dx}{dt} = y\frac{dy}{dt}$

Now, when $y = 250\mathrm{m}; \mathrm{x} = \sqrt{\mathrm{y}^2 - (150)^2}$

$= \sqrt{62500 - 22500} = 200\mathrm{m}$

$\therefore 200 \times 10 = 250 \times \frac{\mathrm{dy}}{\mathrm{dt}} \Rightarrow \frac{\mathrm{dy}}{\mathrm{dt}} = \frac{2000}{250} = 8\mathrm{m/s}$

5. (c) Let cost $\mathrm{C} = av + \frac{b}{v}$

According to given question, $30a + \frac{b}{30} = 75$

$40a + \frac{b}{40} = 65$

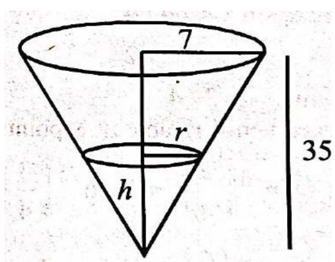
On solving (i) and (ii), we get $a = \frac{1}{2}$ and $b = 1800$

Now, $C = av + \frac{b}{v} \Rightarrow \frac{dC}{dv} = a - \frac{b}{v^2}$

$\frac{dC}{dv} = 0 \Rightarrow a - \frac{b}{v^2} = 0 \Rightarrow v = \sqrt{\frac{b}{a}} = \sqrt{3600}$

$\Rightarrow v = 60\mathrm{kmph}$

6. (c)



From figure $\frac{r}{h} = \frac{7}{35} \Rightarrow h = 5r$

Given $\frac{dV}{dt} = 1 \Rightarrow \frac{d}{dt}\left(\frac{\pi r^2 h}{3}\right) = 1$

$\Rightarrow \frac{d}{dt}\left(\frac{5\pi}{3}r^3\right) = 1 \Rightarrow r^2\frac{dr}{dt} = \frac{1}{5\pi}$

Let wet conical surface area $= S$

$= \pi rl = \pi r\sqrt{h^2 + r^2}$

Differentiate w.r.t ' t '

$= \sqrt{26}\pi r^2 \Rightarrow \frac{dS}{dt} = 2\sqrt{26}\pi r\frac{dr}{dt}$

Put $h = 10$ in eq. (i), Then, $r = 2$

Put the value of $\frac{dr}{dt}$ in eq. (ii),

Therefore, $\frac{dS}{dt}=\frac{2\sqrt{26}}{10}$

7. (a) Let ' r ' be the radius of spherical balloon and S is Surface area. $S=4\pi r^2$

Differentiate both sides w.r.t. ' t '.

$\frac{dS}{dt}=8\pi r\times\frac{dr}{dt}=k$ (constant)

Take integral both sides,

$4\pi r^2=kt+C$ (C is constant of integration)

Put the values of ' t ' (& ' r ') in equation (i).

At $t=0, r=3\Rightarrow 36\pi=C$; At $t=5, r=7\Rightarrow k=32\pi$

Put the values of C and k in eq. (i)

$4\pi r^2=32\pi t+36\pi\Rightarrow r^2=8t+9$; Put $t=9; r^2=81\Rightarrow r=9$

8. (a) $y^2=18x\Rightarrow 2y\frac{dy}{dx}=18\Rightarrow\frac{dy}{dx}=\frac{9}{y}$

Given $\frac{dy}{dx}=2\Rightarrow\frac{9}{y}=2\Rightarrow y=\frac{9}{2}$

Putting in $y^2=18x\Rightarrow x=\frac{9}{8}\therefore$ Req. point is $\left(\frac{9}{8},\frac{9}{2}\right)$

9. (d) Let r be the radius of the circular wave and A be the area, then $A=\pi r^2$

Therefore, the rate of change of area (A) with respect to time (t) is given by

$\frac{dA}{dt}=\frac{d}{dt}(\pi r^2)\Rightarrow\frac{dA}{dt}=2\pi r\frac{dr}{dt}$

(by Chain rule)

It is given that waves move in circles at the speed of 5cm/

s. So, $dr/dt=5\text{cm}/\text{s}$

$\therefore$ ⬚ $\frac{dA}{dt}=2\pi r\times 5=10\pi r\text{m}/\text{s}$

Thus, when $r=8\text{cm}, \frac{dA}{dt}=10\pi(8)=80\pi\text{cm}^2/\text{s}$

Hence, when the radius of the circular wave is 8cm, the enclosed area is increasing at the rate of $80\text{cm}^2/\text{s}$.

10. (c) Given, $6y=x^3+2$

On differentiating w.r.t. t, we get

$6\frac{dy}{dt}=3x^2\frac{dx}{dt}\Rightarrow 6\times 8\frac{dx}{dt}=3x^2\frac{dx}{dt}$

$\Rightarrow 3x^2=48\Rightarrow x^2=16$

$\Rightarrow x=\pm 4$

When $x=4$, then $6y=(4)^3+2$

$\Rightarrow 6y=64+2\Rightarrow y=\frac{66}{6}=11$

When $x=-4$, then $6y=(-4)^3+2$

$\Rightarrow 6y=-64+2\Rightarrow y=\frac{-62}{6}=\frac{-31}{3}$

Hence, the required points on the curve are $(4,11)$ and $\left(-4,\frac{-31}{3}\right)$

11. (b) We have, $a=\frac{d^2x}{dt^2}=-9.8$

The initial conditions are $x(0) = 19.6$ and $v(0) = 0$
So, $v = \frac{dx}{dt} = -9.8t + v(0) = -9.8t$
$\therefore \mathrm{x} = -4.9\mathrm{t}^2 + \mathrm{x}(0) = -4.9\mathrm{t}^2 + 19.6$
Now, the domain of the function is restricted since the ball hits the ground after a certain time. To find this time we set $x = 0$ and solve for $t; 0 = -4.9t^2 + 19.6 \Rightarrow t = 2$

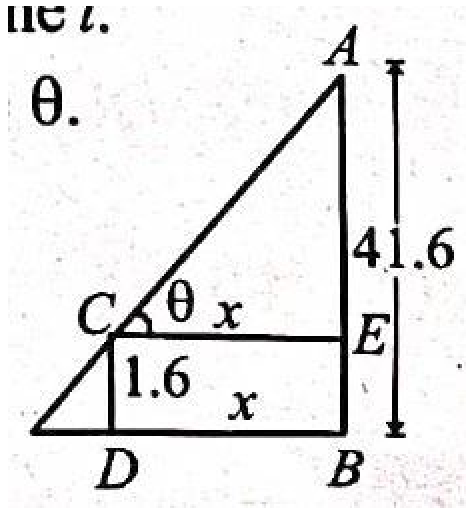
12. (c) Let the lizard catches the insect after time t then distance covered by lizard $= 21\mathrm{cm} +$ distance covered by insect
$\Rightarrow \frac{1}{2} f t^2 = 4 \times t + 21 \Rightarrow \frac{1}{2} \times 2 \times t^2 = 20 \times t + 21$
$\Rightarrow t^2 - 20t - 21 = 0 \Rightarrow t = 21\mathrm{sec}$

13. (d) Let h and r be the height and radius of cylinder.
Given that, $\frac{\mathrm{dr}}{\mathrm{dt}} = 3\mathrm{m/s}, \frac{\mathrm{dh}}{\mathrm{dt}} = -4\mathrm{m/s}$
Let volume of cylinder, $\mathrm{V} = \pi \mathrm{r}^2 \mathrm{h}$
$\Rightarrow \frac{\mathrm{dV}}{\mathrm{dt}} = \pi \left[\mathrm{r}^2 \frac{\mathrm{dh}}{\mathrm{dt}} + \mathrm{h} \cdot 2\mathrm{r} \frac{\mathrm{dr}}{\mathrm{dt}}\right]$
At $\mathrm{r} = 4\mathrm{m}$ and $\mathrm{h} = 6\mathrm{m}$
$\therefore \frac{\mathrm{dV}}{\mathrm{dt}} = \pi[-64 + 144] = 80\pi \mathrm{m}^3/\mathrm{s}$

14. (c) Let ' r ' be the radius and V be the volume of the sphere.
Given : Radius increases at the rate of $5\mathrm{cm/sec}$.
$\therefore \frac{\mathrm{dr}}{\mathrm{dt}} = 5\mathrm{cm/sec}$
Now, $V = \frac{4}{3}\pi r^3 \therefore \frac{dV}{dt} = \frac{4}{3}\pi(3r^2)\frac{dr}{dt} = 4\pi r^2(5) = 20\pi r^2$
Now, after one second, $\mathrm{r} = 5$
$\therefore \frac{\mathrm{dV}}{\mathrm{dt}}$ after $1\mathrm{sec} = 20\pi(5)^2 = 500\pi$.

15. (a) Let A sq. units in the area measure when the radius is r units. their $A = \pi r^2$
Differentiate both side w.r.t ' t ' $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$
We have, $\frac{dA}{dt} = 3c \frac{dr}{dt}$
From eqn (i), we get $3c \cdot \frac{dr}{dt} = 2\pi r. \frac{dr}{dt} \Rightarrow 3c = 2\pi r$ Now, $c = \frac{2}{3}\pi(6) = 4\pi$ when $r = 6$

16. (a) Let CD be the position of man at any time t.
Let BD be x. Then $EC = x$. Let $\angle ACE$ be θ.
Given $AB = 41.6\mathrm{m}, CD = 1.6\mathrm{m}$,
and $\frac{dx}{dt} = 2\mathrm{m/s}$.



$AE = AB - EB = AB - CD = 41.6 - 1.6 = 40\text{m}$

We have to find $\frac{d\theta}{dt}$ when $x = 30\text{m}$.

From $\triangle AEC, \tan\theta = \frac{AE}{EC} = \frac{40}{x}$

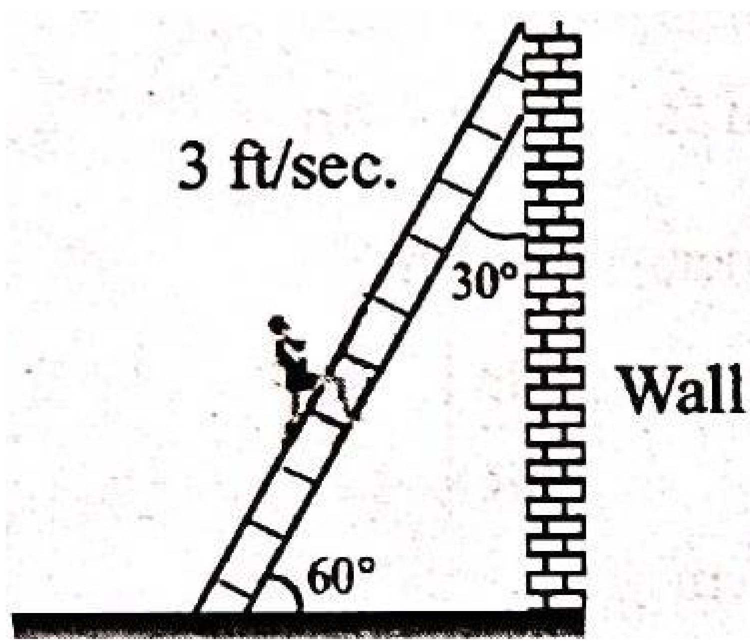
Differentiating w.r.t. to $t, \sec^2\theta \frac{d\theta}{dt} = \frac{-40}{x^2}\frac{dx}{dt}$

or $\sec^2\theta \frac{d\theta}{dt} = \frac{-40}{x^2} \times 2$

or $\frac{d\theta}{dt} = \frac{-80}{x^2}\cos^2\theta = -\frac{80}{x^2}\frac{x^2}{x^2+40^2} = -\frac{80}{x^2+40^2}$.

When $x = 30\text{m}, \frac{d\theta}{dt} = -\frac{80}{30^2+40^2} = -\frac{4}{125}\text{rad/s}$.

17. (b)



His rate of approaching the wall $= 3 \times \cos 60° = \frac{3}{2}\text{ft/sec}$.

18. (c) Let N be the no. of bacteria at time t.

Let N_0 be the initial original no. of bacteria.

Then, $\frac{d}{dt}N \propto N \frac{d}{dt}N = kN \Rightarrow \frac{dN}{N} = kdt$

$\Rightarrow \int \frac{dN}{N} = k\int dt \Rightarrow \log\ N = kt + c$

At $t = 0, N = N_0 \Rightarrow \log\ N_0 = 0 + c \Rightarrow c = \log\ N_0$

$\therefore \log\ N = kt + \log\ N_0 \Rightarrow \log \frac{N}{N_0} = kt$

When $t = 5$ hrs. $N = 2N_0 \therefore \log\left(\frac{2N_0}{N_0}\right) = 5k$

$\Rightarrow k = \frac{\log\ 2}{5} \therefore \log \frac{N}{N_0} = \frac{\log\ 2}{5}t$

When $t = 25\text{hrs}, \log \frac{N}{N_0} = \frac{\log\ 2}{5^\circ} \times 25$

$\Rightarrow \log \frac{N}{N_0} = 5\log\ 2\log \frac{N}{N_0} = \log\ 2^5 \Rightarrow \frac{N}{N_0} = 32$

$\therefore N = 32N_0$ Therefore, 32 times the original.

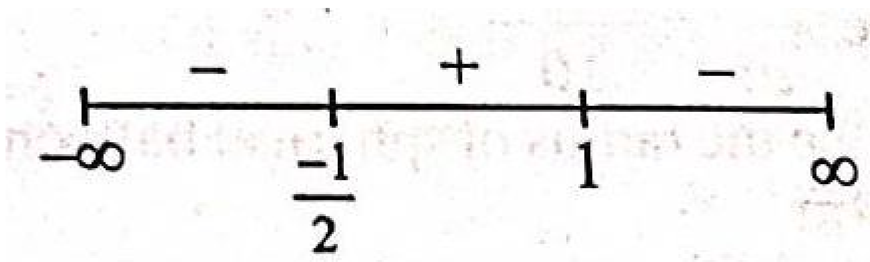
19.

(c) $v = \frac{4}{3}\pi r^3, \frac{dv}{dt} = \frac{4}{3}\pi \frac{d}{dt}r^3 = \frac{4}{3}\pi 3r^2 \cdot \frac{dr}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$ when $r = 10\text{cm}; \frac{dv}{dt} = 4\pi(10)^2.(0.02) = 8\pi\text{cm}^3/\text{s}.$

20. (a) $f(x) = xe^{x(1-x)}$

$f'(x) = -e^{x(1-x)}(2x+1)(x-1) = 0$

$x = 1, -\frac{1}{2}$



$\therefore$ f(x) is increasing in $\left(-\frac{1}{2}, 1\right)$

21. (b) Given $f(x) = 2\cos^{-1}(x) + 4\cot^{-1}(x) - 3x^2 - 2x + 10$

Hence $f'(x) = \frac{-2}{\sqrt{1-x^2}} - \frac{4}{1+x^2} - 6x - 2$

$= -2\left[\frac{1}{\sqrt{1-x^2}} + \frac{2}{1+x^2} + 3x + 1\right]$

$f'(x) < 0 \Rightarrow f(x)$ is a decreasing function

$f(1) = 5 + \pi$

$f(-1) = 5 + 5\lambda$

Range : $[a, b] \equiv [\pi + 5, 5\pi + 5]$

$a = \pi + 5, b = 5\pi + 5 \Rightarrow 4a - b = 11 - \pi$

22. (a) $f(x) = \tan\ x - 4x \Rightarrow f'(x) = \sec^2\ x - 4$

When $\frac{-\pi}{3} < x < \frac{\pi}{3}, 1 < sec\ x < 2$

Therefore, $1 < \sec^2\ x < 4$

$\Rightarrow -3 < (\sec^2\ x - 4) < 0$

Thus, for $\frac{-\pi}{3} < x < \frac{\pi}{3}, f'(x) < 0$

Hence, f is strictly decreasing on $\left(\frac{-\pi}{3}, \frac{\pi}{3}\right)$

23. (c) We have $f(x) = \frac{x}{\sin x}, 0 < x \leq 1$

$\therefore f'(x) = \frac{\sin x - x\cos x}{\sin^2 x} = \frac{\cos x(\tan x - x)}{\sin^2 x}$

We know that $\tan x > x$ for $0 < x < \pi/2$

or $f'(x) > 0$ for $0 < x <\leq 1$

Hence, $f(x)$ is an increasing function. $g(x) = \frac{x}{\tan x}$

$\therefore g'(x) = \frac{\tan x - x\sec^2 x}{\tan^2 x} = \frac{\sin x\cos x - x}{\sin^2 x}$

$= \frac{\sin 2x - 2x}{2\sin^2 x} = \frac{\sin \theta - \theta}{2\sin^2 (\theta/2)}$, where $\theta \in (0,2)$.

We know that $\sin \theta < \theta \forall \theta > 0$.

Thus, $g'(x) < 0$, i.e., $g(x)$ is a decreasing function.

24. (d) $f(x) = x^{100} + \sin x - 1 \Rightarrow f'(x) = 100x^{99} + \cos x$.

If $0 < x < \pi/2$, then $\mathrm{f}'(\mathrm{x}) > 0$,

therefore $f(x)$ is increasing on $(0, \pi/2)$.

If $0 < x < 1$, then $100x^{99} > 0$ and $\cos x > 0$

[$\because x$ lies between 0 and 1 radian]

$\Rightarrow f'(x) = 100x^{99} + \cos x > 0 \Rightarrow f(x)$ is increasing on $(0,1)$.

If $\pi/2 < x < \pi$, then $100x^{99} > 100[\because x > 1, \therefore x^{99} > 1]$

$\Rightarrow 100\mathrm{x}^{99} + \cos \mathrm{x} > 0$

$[\because \cos x \geq -1, \therefore 100x^{99} + \cos x > 99]$

$\Rightarrow \mathrm{f}'(\mathrm{x}) > 0 \Rightarrow f(x)$ is increasing on $(\pi/2, \pi)$.

25. (b) $f(x) = \frac{4\sin x - 2x - x\cos x}{2 + \cos x} = \frac{4\sin x}{2 + \cos x} - x$

$\therefore f'(x) = \frac{4\cos x(2 + \cos x) - (-\sin x)(4\sin x)}{(2 + \cos x)^2} - 1$

$= \frac{4 + 8\cos x}{(2 + \cos x)^2 - 1} = \frac{\cos x(4 - \cos x)}{(2 + \cos x)^2}$

Now, $f'(x) = 0$ or $\cos x = 0$ (As $\cos x \neq 4$)

or $x = \frac{\pi}{2}, \frac{3\pi}{2}$; $\cos x > 0$ for $x \in (0, \pi/2) \cup (3\pi/2, 2\pi)$

and $\cos x < 0$ for $x \in (\pi/2, \pi) \cup (\pi, 3\pi/2)$

Thus, f(x) is increasing for $x \in (0, \pi/2) \cup (3\pi/2, 2\pi)$

26. (a) Given $\mathrm{f}(\dot{\mathrm{x}}) = \frac{4\mathrm{x}^2 + 1}{\mathrm{x}}$ Thus $\mathrm{f}'(\mathrm{x}) = 4 - \frac{1}{\mathrm{x}^2}$

f(x) will be decreasing if $\mathrm{f}'(\mathrm{x}) < 0$

Thus $4 - \frac{1}{x^2} < 0 \Rightarrow \frac{1}{x^2} > 4 \Rightarrow \frac{-1}{2} < x < \frac{1}{2}$

Thus interval in which f(x) is decreasing, is $\left(-\frac{1}{2}, \frac{1}{2}\right)$.

27. (a) Given $f(x) = \log(1 + x) - \frac{2x}{2 + x}$

$f'(x) = \frac{1}{1 + x} - \frac{(2 + x)(2) - 2x}{(2 + x)^2} = \frac{1}{1 + x} - \frac{4}{(2 + x)^2}$

$= \frac{(2 + x)^2 - 4 - 4x}{(1 + x)(2 + x)^2} = \frac{x^2}{(1 + x)(2 + x)^2} > 0$ for all $x \in (0, \infty)$

Thus, given function f(x) is increasing on $(0, \infty)$.

28. (b) We have $e < \pi$ and

$$f'(x) = \frac{\frac{1}{\pi+x}\log(e+x) - \frac{1}{e+x}\log(\pi+x)}{\{\log(e+x)\}^2}$$

$$= \frac{(e+x)\log(e+x) - (\pi+x)\log(\pi+x)}{(\pi+x)(e+x)\{\log(e+x)\}^2}$$

In $[0, \infty)$, denominator > 0 and numerator < 0, since, $e + x < \pi + x$. Hence, f(x) is decreasing in $[0, \infty)$.

29. (b) Given : $f(x) = 3x^4 + 4x^3 - 12x^2 + 12$

Differentiating with respect to x, we get

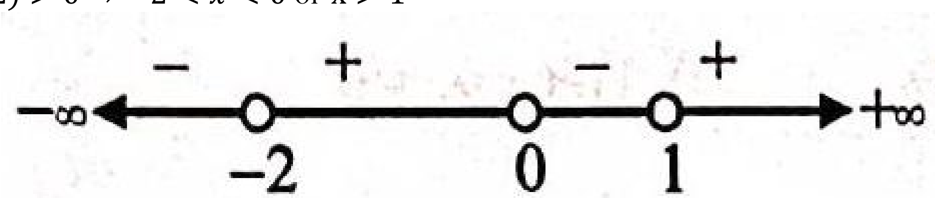
$f'(x) = 12x^3 + 12x^2 - 24x$

For $f(x)$ to be increasing

$f'(x) > 0 \Rightarrow 12x^3 + 12x^2 - 24x > 0$

$\Rightarrow 12x(x^2 + x - 2) > 0 \Rightarrow 12x(x-1)(x+2) > 0$

$\Rightarrow x(x-1)(x+2) > 0 \Rightarrow -2 < x < 0$ or $x > 1$



It means $x \in (-2,0) \cup (1, \infty)$.

Hence $f(x)$ is increasing in $(-2,0)$ and $(1, \infty)$

30. (a) Given function is $f(x) = \log_e(x^2+1) - e^{-x} + 1$

Differentiate w.r.t. x both sides.

$\Rightarrow f'(x) = \dfrac{2x}{x^2+1} + e^{-x} > 0, \forall x \in R$

For every real value $f(x)$ is strictly increasing.

Given function $g(x) = \frac{1-2e^{2x}}{e^x} = e^{-x} - 2e^x$

Differentiate w.r.t. x both sides,

$\Rightarrow g'(x) = -(2e^x + e^{-x}) < 0, \forall x \in R$

$\Rightarrow g$ is decreasing.

Take $f\left(g\left(\frac{(\alpha-1)^2}{3}\right)\right) > f\left(g\left(\alpha - \frac{5}{3}\right)\right)$

$\Rightarrow g\left(\dfrac{(\alpha-1)^2}{3}\right) > g\left(\alpha - \dfrac{5}{3}\right)$

$\Rightarrow \dfrac{(\alpha-1)^2}{3} < \alpha - \dfrac{5}{3}$

$\Rightarrow \alpha^2 - 5\alpha + 6 < 0 \Rightarrow (\alpha-2)(\alpha-3) < 0 \Rightarrow \alpha \in (2,3)$

31. (c) Given function is, $f(x) = 4\log_e(x-1) - 2x^2 + 4x + 5, x > 1$

Differentiate w.r.t ' x '.

$f'(x) = \dfrac{4}{x-1} - 4(x-1)$

Now, check optionwise

Take $1 < x < 2 \Rightarrow f'(x) > 0$

Take $x > 2 \Rightarrow f'(x) < 0$
So, option (a) is correct.
Take $f(x) = -1$.
We have
$\log_e (x-1)^2 = (x-3)(x+1)$
Therefore, it has two solutions
Take $f(e) > 0, f(e+1) < 0$
$f(e).f(e+1) < 0$
So, option (d) is correct.
Now, put $x = e$ in eq. (i) and again diff. eq. (i).
$$f'(e) - f''(2) = \frac{4}{e-1} - 4(e-1) + 8 > 0$$
Therefore, option (c) is incorrect.

32. (d) Given tunction is
$f_\lambda(x) = 4\lambda x^3 - 36\lambda x^2 + 36x + 48$
Differentiate w.r.t. x
$f'_\lambda(x) = 12\lambda x^2 - 72\lambda x + 36$
$f'_\lambda(x) = 12(\lambda x^2 - 6\lambda x + 3) \geq 0$
For $\lambda > 0, D$ should be less or equal to $0. D \leq 0$
$36\lambda^2 - 4 \times \lambda \times 3 \leq 0 \Rightarrow 9\lambda^2 - 3\lambda \leq 0$
$\Rightarrow 3\lambda(3\lambda - 1) \leq 0$; Here, $\lambda = 0, \lambda = \frac{1}{3}$.
Then, λ lies in the interval $\left[0, \frac{1}{3}\right]$. So, $\lambda_{\max} = \frac{1}{3}$
Put the value of λ in the given function.
$$f(x) = \frac{4}{3}x^3 - 12x^2 + 36x + 48$$
Put $x = 1, -1$ in above equation.
$\therefore f(1) + f(-1) = 72$

33. (b) Since, $f(x) = \tan^{-1}(\sin x + \cos x)$
$$\therefore f'(x) = \frac{1}{1 + (\sin x + \cos x)^2}(\cos x - \sin x)$$
$$= \frac{\sqrt{2}\cos\left(x + \frac{\pi}{4}\right)}{1 + (\sin x + \cos x)^2}$$
$f(x)$ is increasing if $f'(x) > 0 \Rightarrow cos\left(x + \frac{\pi}{4}\right) > 0$
$$\Rightarrow \square - \frac{\pi}{2} < x + \frac{\pi}{4} < \frac{\pi}{2} \Rightarrow -\frac{3\pi}{2} < x + \frac{\pi}{4}$$
Hence, $f(x)$ is increasing when $x \in \left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$.

34. (d) (a) Let $f(x) = \cos x$, then $f'(x) = -\sin x$.
In interval $\left(0, \frac{\pi}{2}\right)$, $f'(x) < 0$
Therefore, $f(x)$ is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$
(b) Let $f(x) = \cos 2x \Rightarrow f'(x) = -2\sin x2x$
In interval $\left(0, \frac{\pi}{2}\right)$, $f'(x) < 0$

Because $\sin 2x$ will either lie in the first or second quadrant which will give a positive value.

Therefore, $f(x)$ is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$

(c) Let $f'(x) = \cos 3x$

$\Rightarrow f'(x) = -3\sin 3x \cdot$ In Interval $\left(0, \frac{\pi}{3}\right), f'(x) < 0$

Because $\sin 3x$ will either lie in the first or second quadrant which will give a positive value.

Therefore, $f(x)$ is strictly decreasing on $\left(0, \frac{\pi}{3}\right)$.

When $x \in \left(\frac{\pi}{3}, \frac{\pi}{2}\right)$, then $f'(x) > 0$

Because $\sin 3x$ will lie in the third quadrant.

Therefore, $f(x)$ is not strictly decreasing on $\left(0, \frac{\pi}{2}\right)$

(d) Let $f(x) = \tan x \Rightarrow f'(x) = \sec^2 x$.

In Interval $x \in \left(0, \frac{\pi}{2}\right), f'(x) > 0$

Therefore, $f(x)$ is not strictly decreasing on $\left(0, \frac{\pi}{2}\right)$

35. (a) $f'(x) = -12\cos^3 x\sin x - 30\cos^2 x\sin x - 12\cos x\sin x = -6\sin x\cos x(\cos x + 2)(2\cos x + 1)$

$f'(x) = 0$, for $x = 0, \frac{\pi}{2}, \frac{2\pi}{3}, \pi$

Clearly, $f'(x) > 0$ for $\frac{\pi}{2} < x < \frac{2\pi}{3}$

And $f'(x) < 0$; for $0 < x < \frac{\pi}{2}$ or $\frac{2\pi}{3} < x < \pi$

36.

(b) $f(x) = x^2 - 4x + 6$

$f'(x) = 2x - 4$

Let $f'(x) = 0 \Rightarrow x = 2$

$\overline{+}$

$\rightarrow$

$-\infty$

$\Rightarrow f(x)$ is strictly increasing in $(2, \infty)$

37. (b) $f'(x) = 1 - \sin x = \left(\sin \frac{x}{2} - \cos \frac{x}{2}\right)^2$

$\Rightarrow f'(x) > 0 \forall x \in R$

$\Rightarrow$ no value of b exists

38. (d) Given equation is $x^7 - 7x - 2 = 0$

$x^7 - 7x = 2$

$f(x) = x^7 - 7x$ and $y = 2$

Differentiate w.r.t. x.

$f'(x) = 7(x^6 - 1) = 7(x^2 - 1)(x^4 + x^2 + 1)$

$f'(x) = 0 \Rightarrow x = \pm 1$

A fumction $f(\bar{x})$ is strictly decreasing at $[-1,1]$ and increasing at some points of $x < -1$ and $x > 1$. $y = 2$ intersects at 3 points.

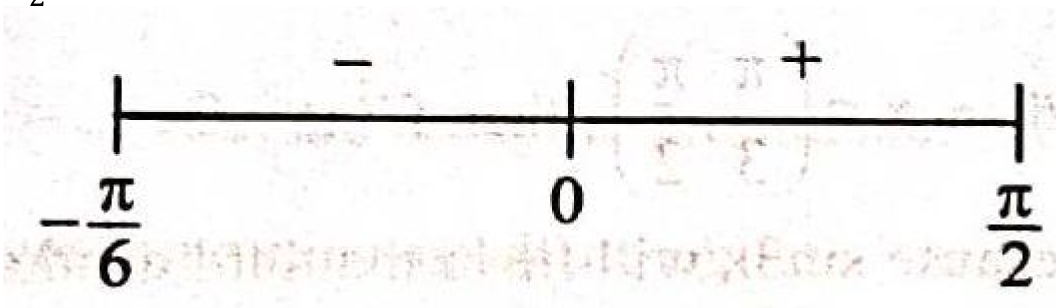
$f(x) = 2$ has 3 real distinct solution.

39. (d) Given that

$f(x) = 3\sin^4 x + 10\sin^3 x + 6\sin^2 x - 3, x \in \left[-\frac{\pi}{6}, \frac{\pi}{2}\right]$

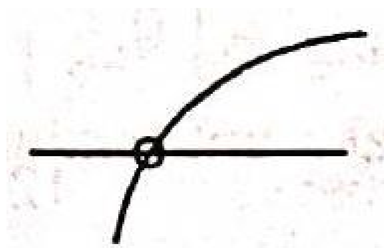
$f'(x) = 12\sin^3 x\cos x + 30\sin^2 x\cos x + 12\sin x\cos x$

$= 6\sin x\cos x(2\sin^2 x + 5\sin x + 2)$
$= 6\sin x\cos x(2\sin x + 1)(\sin x + 2) = 0$
$\Rightarrow x = -\frac{\pi}{6}, 0, \frac{\pi}{2}$



Decreasing in $\left(-\frac{\pi}{6}, 0\right)$

40. (b) Given function is $f(x) = x^7 + 5x^3 + 3x + 1$
Differentiate w.r.t.x.
$f(x) = 7x^6 + 15x^2 + 3 > 0$
$\therefore f(x)$ is strictly increasing function



It intersects only at one point, then no. of real solution is 1

41. (b) $\frac{dy}{dx} = \frac{2e^{2x} - 6e^{-x} + 9}{2 + 9e^{-2x}}$
$\frac{dy}{dx} = \frac{-6e^x}{2e^{2x} + 9} + e^{2x} \Rightarrow y = \frac{e^{2x}}{2} - \tan^{-1}\left(\frac{\sqrt{2e^x}}{3}\right) + c$
If C passes through the point $\left(0, \frac{1}{2} + \frac{\pi}{2\sqrt{2}}\right)$
$c = -\frac{\pi}{4} - \tan^{-1}\frac{\sqrt{2}}{3}$
Again C passes through the point $\left(\alpha, \frac{1}{2}e^{2\alpha}\right)$
then $e^\alpha = \frac{3}{\sqrt{2}}\left(\frac{3+\sqrt{2}}{3-\sqrt{2}}\right)$

42. (d) Given curve is
$\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$
Now, slope of tangent at (a, b)
n. $\left(\frac{x}{a}\right)^{n-1} \cdot \frac{1}{a} + n\left(\frac{x}{b}\right)^{n-1} \cdot \frac{1}{b}\frac{dy}{dx} = 0$
$\left.\frac{dy}{dx}\right|_{(a,b)} = -\frac{b}{a}$
$\therefore$ Equation of tangent

$$y-b=-\frac{b}{a}(x-a)$$
$\Rightarrow ay-ab=-bx+ab$
$\frac{x}{a}+\frac{y}{b}=2 \forall n \in N$
43. (b) $y=x^3-11x+5 \Rightarrow \frac{dy}{dx}=3x^2-11$
Slope of line $y=x-11$ is 1
$\Rightarrow 3x^2-11=1 \Rightarrow x=\pm 2$
$\therefore$ point is $(2,-9)$ as $(-2,19)$ does not satisfy given line
44.
(c) $\frac{x^2}{9}+\frac{y^2}{16}=1 \Rightarrow \frac{2x}{9}+\frac{2y}{16}\frac{dy}{dx}=0$
$\Rightarrow$ Slope of tangent $=\frac{dy}{dx}=-\frac{16x}{9y}$
Since tangent of curve is parallel to y-axis
$\Rightarrow \frac{9y}{16x}=0 \Rightarrow y=0$ and $x=\pm 3$
$\therefore$ points $=(\pm 3,0)$
45. (a) $f(x)=x+\frac{1}{x}, x>0$
$$\Rightarrow f'(x)=1-\frac{1}{x^2}=\frac{x^2-1}{x^2}, x>0$$
Since normal to $f(x)$ is $\perp$ to given line $3x-4y-7=0$
$$\Rightarrow\left(\frac{x^2}{1-x^2}\right)\times\frac{3}{4}=-1 \quad (\because m_1 \cdot m_2=-1)$$
$\Rightarrow x^2=4 \Rightarrow x=\pm 2$
But $x>0, \therefore x=2$
Therefore point $=\left(2,\frac{5}{2}\right)$
46. (d) Given equation of curve is $y=x^3+3x^2+5$.
Slope of tangent, $\frac{dy}{dx}=3x^2+6x$
Satisfy the point (x_1,y_1) in the slope. Then the equation of tangent represented as,
$y-y_1=(3x_1^2+6x_1)(x-x_1)$
Put $(x,y)=(0,0)$
$-y_1=(3x_1^2+6x_1)(-x_1)$
$y_1=(3x_1^3+6x_1^2)$
Here, (x_1,y_1) lies on the curve $y=x^3+3x^2+5$
$y_1=x_1^3+3x_1^2+5$
From equations (i) and (ii), $2y_1=3x_1^2+15$
Hence the equation of curve $y=\frac{3}{2}x^2+\frac{15}{2}$
is symmetrical about y-axis and it will not because intersect the curve $\frac{x}{3}-y^2=2$ it is symmetrical about x-axis.
47.
(c) Given equation of curve
$x=12(t+\sin t\cos t), y=12(1+\sin t)^2$ Differentiate w.r.t ' t ',

$$\frac{dx}{dt}=12(1+\cos^2 t-\sin^2 t)$$

$$\frac{dx}{dt}=12(1+\cos 2t) \text{ and } \frac{dy}{dt}=24(1+\sin t)\cos t$$

$$\frac{dy}{dx}=\frac{2(1+\sin t)\times\cos t}{1+\cos 2t}\Rightarrow\frac{2(1+\sin t)\cos t}{2\cos^2 t}=\sqrt{3}$$

48. (c) $y^2=x(2-x)^2\Rightarrow y^2=x^3-4x^2+4x$

$$\Rightarrow 2y\frac{dy}{dx}=3x^2-8x+4\Rightarrow\frac{dy}{dx}=\frac{3x^2-8x+4}{2y}$$

$$\Rightarrow\left[\frac{dy}{dx}\right]_P=\frac{3-8+4}{2}=-\frac{1}{2}$$

∴ Equation of tangent at P is: $y-1=-\frac{1}{2}(x-1)$

$\Rightarrow x+2y-3=0$

Using $y=\frac{3-x}{2}$ in (i), we get: $\left(\frac{3-x}{2}\right)^2=x^3-4x^2+4x$

$\Rightarrow 4x^3-17x^2+22x-9=0$

which has two roots 1,1

(Because of(ii) being tangent at $(1,1)$).

Sum of 3 roots $=\frac{17}{4}$ ∴ rrd root $=\frac{17}{4}-2=\frac{9}{4}$

Then, $y=\frac{3-\frac{9}{4}}{2}=\frac{3}{8}$ ∴ Q is $\left(\frac{9}{4},\frac{3}{8}\right)$

49. (b) If two curves intersect each other orthogonally, then the slopes of corresponding tangents at the point of intersection are perpendicular.

Let the point of intersection be (x_1,y_1).

Given curves:

$x^2=9A(9-y)$

and $x^2=A(y+1)$

Differentiating w.r.t. tox both sides equations (i) and (ii) respectively, we get

$$2x=-9A\frac{dy}{dx}\Rightarrow\left(\frac{dy}{dx}\right)_{(x_1,y_1)}=-\frac{2x_1}{9A}\Rightarrow m_1=-\frac{2x_1}{9A}$$

and $2x=A\frac{dy}{dx}\Rightarrow\left(\frac{dy}{dx}\right)_{(x_1,y_1)}=\frac{2x_1}{A}\Rightarrow m_2=\frac{2x_1}{A}$

$$m_1m_2=-1\Rightarrow\frac{4x_1^2}{9A^2}=1\Rightarrow 4x_1^2=9A^2$$

Solving equations (i) and (ii), we find $y_1=8$

Substituting $y_1=8$ in equation (ii), we get $x_1^2=9A$

From equations (iii) and (iv), we get $A=4$

50. (d) Given: curve $y-e^{xy}+x=0\Rightarrow y=e^{xy}-x$

Differentiate w.r.t. x, we have

$$\therefore\frac{dy}{dx}=e^{xy}\left[x\cdot\frac{dy}{dx}+y\cdot 1\right]-1$$

$$\Rightarrow\frac{dy}{dx}=\frac{dy}{dx}\cdot xe^{xy}+y\cdot e^{xy}-1$$

$$\Rightarrow\frac{dy}{dx}\cdot(1-xe^{xy})=ye^{xy}-1\Rightarrow\frac{dy}{dx}=\frac{ye^{xy}-1}{1-xe^{xy}}$$

But for vertical tangent $\frac{dx}{dy}=0 \Rightarrow \frac{ye^{xy}-1}{1-x^{xy}}=\frac{1}{0}$

$\Rightarrow 1-x\cdot e^{xy}=0 \Rightarrow e^{xy}=\frac{1}{x}$

This equation is satisfied at point $(1,0)$.

51. (d) The equation of the given curve is $y=\frac{1}{x-3}, x\neq 3$. The slope of the tangent to the given curve at any point (x,y) is given by $\frac{dy}{dx}=\frac{-1}{(x-3)^2}$

For tangent having slope 2 , we must have $2=\frac{-1}{(x-3)^2}$

$\Rightarrow 2(x-3)^2=-1 \Rightarrow (x-3)^2=-\frac{1}{2}$

which is not possible as square of a real number cannot be negative.

Hence, there is no tangent to the given curve having slope 2.

52. (d) (a) Given, $x=a\cos^3\theta$ and $y=a\sin^3\theta$.

On differentiating x and y both w.r.t θ, we get

$\frac{\mathrm{dx}}{\mathrm{d}\theta}=3a\cos^2\theta(-\sin\theta)=-3a\cos^2\theta\sin\theta$

and $\frac{dy}{d\theta}=3a\sin^2\theta\cos\theta$

$\therefore \frac{dy}{dx}=\frac{\frac{dy}{dx}}{\frac{dx}{d\theta}}=\frac{3a\sin^2\theta\cos\theta}{-3a\cos^2\theta\sin\theta}=-\frac{\sin\theta}{\cos\theta}=-\tan\theta$

$\therefore$ Slopeofnormal at the point $\theta=\frac{\pi}{4}$ is $-\left(\frac{\mathrm{dx}}{\mathrm{dy}}\right)_{\theta=\frac{\pi}{4}}$

$=-\left(\frac{1}{\mathrm{dy}/\mathrm{dx}}\right)_{\theta=\frac{\pi}{4}}=\frac{-1}{\left(\frac{\mathrm{dy}}{\mathrm{dx}}\right)_{(\theta=\pi/4)}}$

$=\frac{-1}{-\tan(\pi/4)}=\frac{-1}{-1}=1$

(b) It is given that $x=1-a\sin\theta$ and $y=b\cos^2\theta$

On differentiating x and y w.r.t. θ, we get

$\frac{dx}{d\theta}=\frac{d}{d\theta}[1-a\sin\theta]=-a\cos\theta$ and $\frac{dy}{d\theta}=\frac{d}{d\theta}[b\cos^2\theta]$

$=2b\cos\theta(-\sin\theta)=-2b\cos\theta\sin\theta$

$\therefore \frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{-2b\cos\theta\sin\theta}{-a\cos\theta}=\frac{2b}{a}\sin\theta$

$\therefore$ Slope of normal at the point $\theta=\frac{\pi}{2}$, is

$=\frac{-1}{\left(\frac{\mathrm{dy}}{\mathrm{dx}}\right)_{\theta=\frac{\pi}{2}}}=\frac{-1}{\frac{2\mathrm{b}}{\mathrm{a}}\sin\left(\frac{\pi}{2}\right)}=\frac{-\mathrm{a}}{2\mathrm{b}}$

So, both (a) and (b) are not true.

53. (c) Solving the two equations, we get $x^2y=xy \Rightarrow xy(x-1)=0 \Rightarrow x=0, y=0, x=1$.

Since $y = 0$ does not satisfy the two equations. So, we neglect it. Putting $x = 0$ in the either equation, we get $y = 1$. Now,
putting $x = 1$ in one of the two equations we obtain $y = \frac{1}{2}$.
Thus, the two curve interset at $(0,1)$ and $\left(1, \frac{1}{2}\right)$.
Now, $x^2 y = 1 - y \Rightarrow x^2\left(\frac{dy}{dx}\right) + 2xy = -\left(\frac{dy}{dx}\right)$
$\Rightarrow \frac{dy}{dx} = -\frac{2xy}{x^2+1} \Rightarrow \left(\frac{dy}{dx}\right)_{(0,1)} = 0$ and $\left(\frac{dy}{dx}\right)_{(1,1/2)} = -\frac{1}{2}$.
The equations of the required tangents are
$y - 1 = 0(x - 0)$ and $y - 1/2 = 1/2(x - 1)$
$\Rightarrow y = 1$ and $x + 2y - 2 = 0$
These two tangents intersect at $(0,1)$.

54. (b) $\because x + y = e^{xy}$
Differentiating w.r.t. x, we get
$1 + \frac{dy}{dx} = e^{xy}\left[y + x\frac{dy}{dx}\right] \Rightarrow \frac{dy}{dx}(1 - xe^{xy}) = ye^{xy} - 1$
$\Rightarrow \frac{dy}{dx} = \frac{ye^{xy}-1}{1-xe^{xy}} \because \frac{dy}{dx} = \infty$, as tangent is parallel to Y-axis
$\Rightarrow 1 - xe^{xy} = 0$
$\therefore xe^{xy} = 1$
This holds, when $x = 1$ and $y = 0$

55. (c) Given: tangent $y = 4x - 5$
$\therefore$ Slope $m = 4$
Curve $y^2 = px^3 + q$
$\Rightarrow 2y \cdot \frac{dy}{dx} = 3px^2 \Rightarrow \frac{dy}{dx} = \frac{3pxx^2}{2y}$
$\Rightarrow \left(\frac{dy}{dx}\right)_{(2,3)} = \frac{3p(2)^2}{2(3)}$
$\Rightarrow 4 = \frac{12p}{6}$ [using (i)]
$\Rightarrow p = 2$
On putting the value of $p = 2, x = 2$ and $y = 3$ in equation (ii), we get, $(3)^2 = 2 \times (2)^3 + q$
$\Rightarrow 16 + q = 9 \Rightarrow q = -7$, So, $p = 2$ and $q = -7$

56. (c) $x^3 - 3xy^2 + 2 = 0$
differentiating w.r.t. x: $3x^2 - 3x(2y)\frac{dy}{dx} - 3y^2 = 0$
$\Rightarrow \frac{dy}{dx} = \frac{3x^2-3y^2}{6xy}$ and $3x^2y - y^3 - 2 = 0$
differentiating w.r.t. $x \Rightarrow 3x^2\frac{dy}{dx} + 6xy - 3y^2\frac{dy}{dx} = 0$
$\Rightarrow \frac{dy}{dx} = -\left(\frac{6xy}{3x^2 - 3y^2}\right)$
Now, product of slope $= \frac{3x^2-3y^2}{6xy} \times -\left(\frac{6xy}{3x^2-3y^2}\right) = -1$
$\therefore$ they are perpendicular. Hence, angle $= \pi/2$

57. (a) Let m_1 and m_2 be slope of curve $y = x^2$ and $6y = 7 - x^3$ respectively.

Now, $y = x^2 \Rightarrow \frac{dy}{dx} = 2x \Rightarrow \left(\frac{dy}{dx}\right)_{(1,1)} = 2$ i.e. $m_1 = 2$

and $6y = 7 - x^3 \Rightarrow 6\frac{dy}{dx} = -3x^2 \Rightarrow \frac{dy}{dx} = -\frac{3}{6}x^2 = -\frac{1}{2}x^2$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(1,1)} = -\frac{1}{2}(1)^2 = -\frac{1}{2}$ $\therefore m_2 = -\frac{1}{2}$

$\therefore m_1 m_2 = 2, -\frac{1}{2} = -1$

$\therefore$ Angle of intersection is $90°$ i.e. $\frac{\pi}{2}$

58. (c) Given $x = at^2, y = 2at$

Note: When tangent to the curve is perpendicular to x-axis, then $\frac{dy}{dx} = \infty$

Now, $\frac{dx}{dt} = 2$ at and $\frac{dy}{dt} = 2a$, So $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{2a}{2at} = \infty$

$\Rightarrow \frac{1}{t} = \infty$ so, $t = \frac{1}{\infty} = 0$

So, the point of contact will be

$x = a.(0)^2 = 0$ and $y = 2a.(0) = 0$

59. (b) Given curve is $x = 3t^2 + 1$

$\therefore \frac{dx}{dt} = 6t$

Second curve is $y = t^3 - 1$

$\therefore \frac{dy}{dt} = 3t^2$

$\therefore \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = 3t^2 \times \frac{1}{6t} = \frac{t}{2}$

But from (i) when $x = 1$

we have $1 = 3t^2 + 1 \Rightarrow 3t^2 = 0 \Rightarrow t = 0$

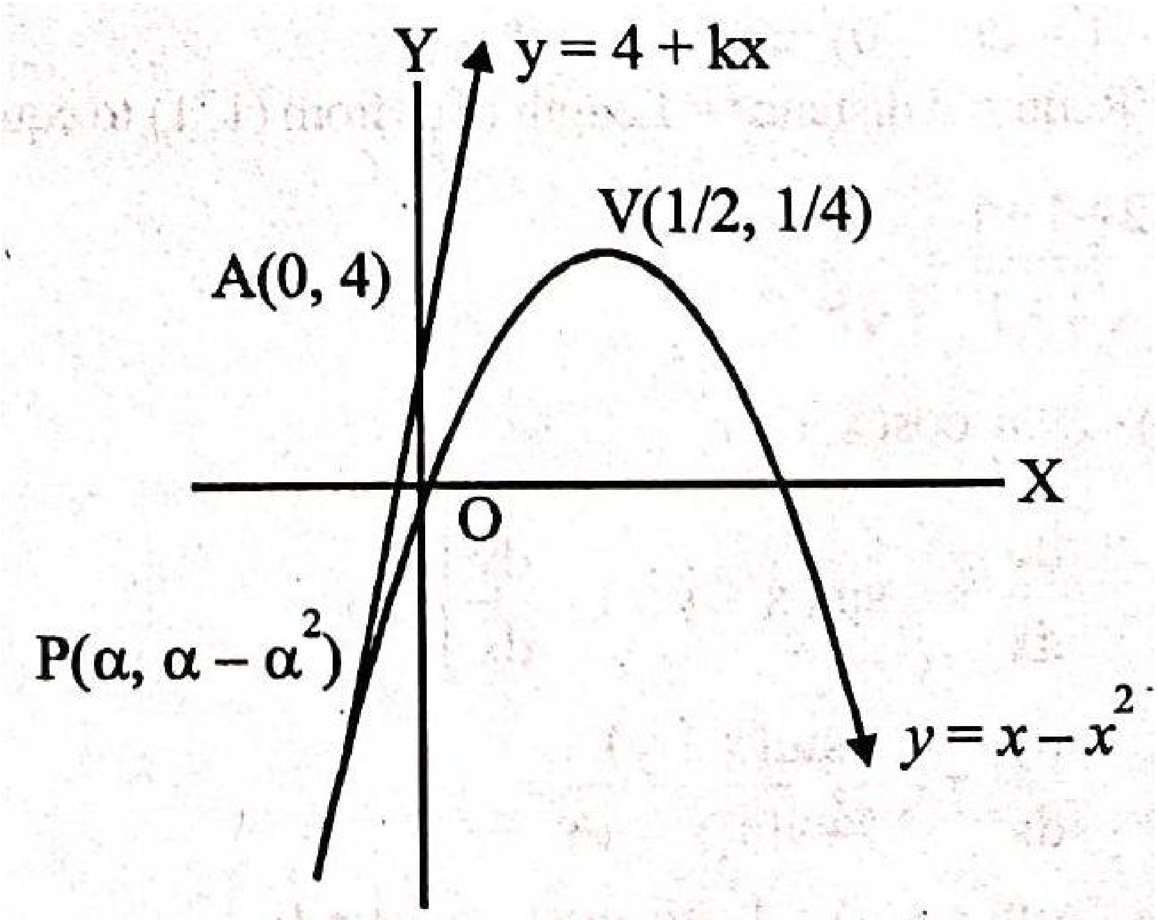
$\therefore$ When $x = 1$ then $t = 0$ $\therefore \frac{dy}{dx} = 0$

Hence, slope of the tangent to the curve $= 0$

60.

(c) Point P satisfies the equation of curve.

Let abscis a of point P is α then $y = \alpha - \alpha^2$.



Slope of tangent at $P =$ Slope of line AP

$(1-2x)|_{(\alpha,\alpha-\alpha^2)} = (1-2\alpha) = \dfrac{\alpha-\alpha^2-4}{\alpha}$

$\Rightarrow \alpha = \pm 2$

Take $\alpha = -2 \Rightarrow P(-2,-6)$

Slope of $PV = \frac{\frac{1}{4}+6}{\frac{1}{2}+2} = \frac{5}{2}$

61. (a) The given curves $areax^2 + by^2 = 1$
and $a_1x^2 + b_1y^2 = 1$
From (i) $2ax + 2by\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{ax}{by} = m_1$ (say)
From (ii) $2a_1x + 2b_1y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{a_1x}{b_1y} = m_2$ (say)
Since the curves are orthogonal, $m_1m_2 = -1$
$\therefore \left(\frac{-ax}{by}\right)\left(-\frac{a_1x}{b_1y}\right) = -1 \Rightarrow a_1x^2 = -bb_1y^2$
Solving (i) and (ii) we get $(a-a_1)x^2 = (b_1-b)y^2$
Dividing by(iii), $\frac{a-a_1}{aa_1} = \frac{b-b_1}{bb_1}$

62. (b) $f(x) = \sqrt{x}(7x-6) = 7x^{3/2} - 6x^{1/2}$
$f'(x) = 7 \times \dfrac{3}{2}x^{1/2} - 6 \times \dfrac{1}{2}x^{-1/2}$

When tangent is parallel to x axis then $f'(x)=0$

or, $\frac{21}{2}x^{1/2}-3x^{-1/2}=0$ or $\frac{21}{2}\sqrt{x}=\frac{3}{\sqrt{x}}$

or, $7x=2 \Rightarrow x=\frac{2}{7}$

63. (c) Putting $x=0$ in $y=e^{2x}+x^2$ we get $y=1$

∴ ⬚ The given point is $P(0,1)$

$y=e^{2x}+x^2\frac{dy}{dx}=2e^{2x}+2x \Rightarrow \left[\frac{dy}{dx}\right]_P=2$

∴ Equation of tangent at P to equation (i) is

$y-1=(x-0) \Rightarrow 2x-y+1=0$

∴ Required distance = Length of ⊥ from $(1,1)$ to equation (ii).

$=\frac{2-1+1}{\sqrt{4+1}}=\frac{2}{\sqrt{5}}$.

64. (b) $y=\cos(x+y)$

∴ ⬚ $\frac{dy}{dx}=-\sin(x+y)\left\{1+\frac{dy}{dx}\right\}$

∴ ⬚ $\frac{dy}{dx}=-\frac{\sin(x+y)}{1+\sin(x+y)}=-\frac{1}{2}$

$\Rightarrow \sin(x+y)=1$, so $\cos(x+y)=0$

∴ ⬚ from (i), $y=0$ and $(x+y)=2n\pi+\frac{\pi}{2}$

Tangent at $\left(\frac{\pi}{2},0\right)$ is $x+2y=\frac{\pi}{2}$

65. (c) We have $y^2=4a\left(x+a\sin\frac{x}{a}\right)$

Differentiating w.r. to $x, 2y\frac{dy}{dx}=4a\left[1+\cos\frac{x}{a}\right]$

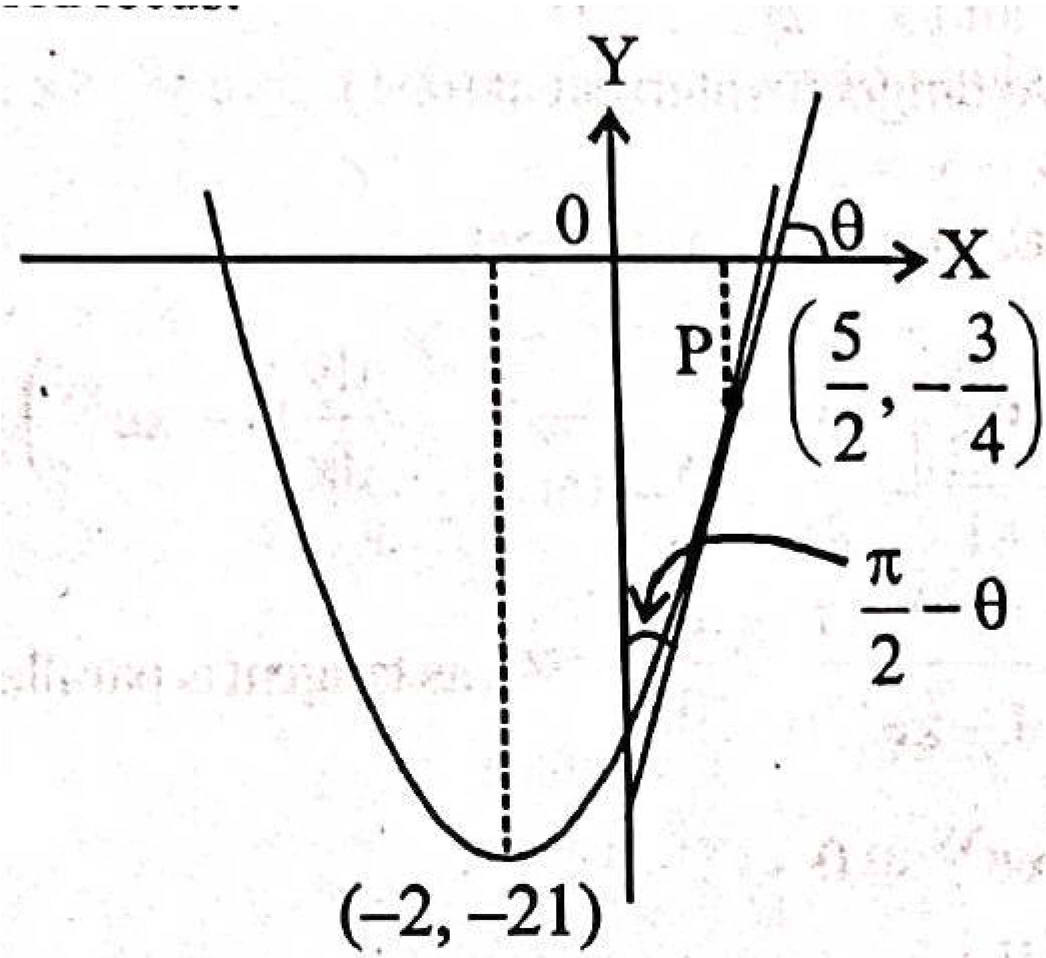
Since, the tangent is parallel to x - axis, $\frac{dy}{dx}=0$

This gives

$4a\left(1+\cos\frac{x}{a}\right)=0 \Rightarrow \cos\frac{x}{a}=-1 \Rightarrow \sin\frac{x}{a}=0$

∴ From (i) $y^2=4a(x+0) \Rightarrow y^2=4ax$, which is the required locus.

66. (b)



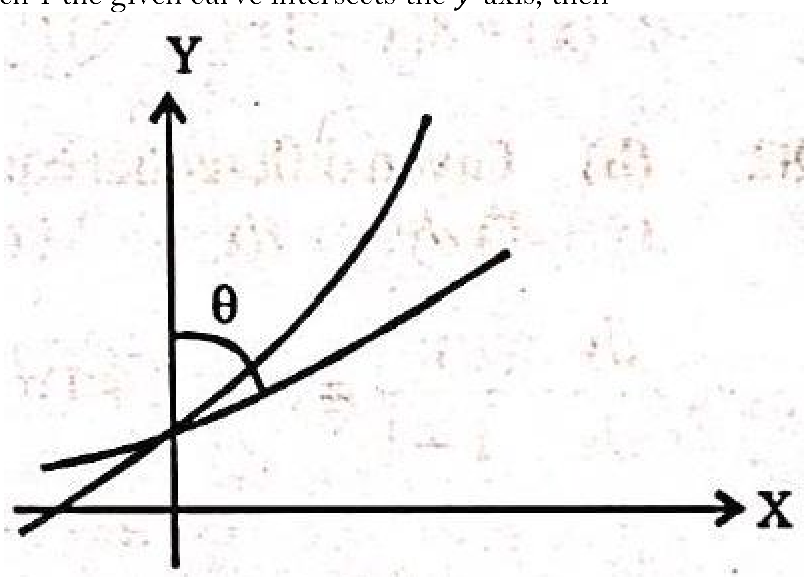
$y = x^2 + 4x + 4 - 4 - 17$
$y = (x + 2)^2 - 21 \Rightarrow$ Vertex is $(-2, -21)$
Also $y = x^2 + 4x - 17 \Rightarrow \frac{dy}{dx} = 2x + 4$
$\Rightarrow$ Slope of tangent at $\left(\frac{5}{2}, -\frac{3}{4}\right)$.
$m = \dfrac{dy}{dx} = 2 \times \dfrac{5}{2} + 4 = 9; \theta = \tan^{-1} 9$
$\therefore$ angle by y-axis $= \frac{\pi}{2} - \tan^{-1} 9 = \cot^{-1} 9$
67. (d) The equation of the given curve is $y = 4x^3 - 2x^5$
$\dfrac{dy}{dx} = 12x^2 - 10x^4$
Therefore, the slope of the tangent at point (x, y) is
$12\text{x}^2 - 10\text{x}^4$.
The equation of the tangent at (x, y) is given by
$Y - y = (12x^2 - 10x^4)(X - x)$
When, the tangent passes through the origin $(0,0)$, then $\text{X} = \text{Y} = 0$
Therefore, eq. (i) reduce to
$-y = (12x^2 - 10x^4)(-x) \Rightarrow y = 12x^3 - 10x^5$
Also, we have $y = 4x^3 - 2x^5$

$12x^3 - 10x^5 = 4x^3 - 2x^5 \therefore 12x^3 - 10x^5 = 4x^3 - 2x^5$
$\Rightarrow 8x^5 - 8x^3 = 0 \Rightarrow x^5 - x^3 = 0$
$\Rightarrow x^3(x^2 - 1) = 0 \Rightarrow x = 0, \pm 1$
When, $x = 0, y = 4(0)^3 - 2(0)^5 = 0$
When, $x = 1, y = 4(1)^3 - 2(1)^5 = 2$
When, $x = -1, y = 4(-1)^3 - 2(-1)^5 = -2$
Hence, the require points are $(0,0), (1,2)$ and $(-1,-2)$.

68. (b) Solving the given equations, we have, $y^2 = x$ and $x^2 = y \Rightarrow x^4 = x$.
or $x^4 - x = 0 \Rightarrow x(x^3 - 1) = 0 \Rightarrow x = 0, x = 1$
Therefore, $y = 0, y = 1$
i.e., points of intersection are $(0,0)$ and $(1,1)$.
Further $y^2 = x \Rightarrow 2y\frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dx} = \frac{1}{2y}$
and $x^2 = y \Rightarrow \frac{dy}{dx} = 2x$.
At $(0,0)$, the slope of the tangent to the curve $y^2 = x$ is parallel to Y-axis and the tangent to the curve $x^2 = y$ is parallel to X-axis.
$\Rightarrow$ Angle of intersection $= \frac{\pi}{2}$
At $(1,1)$ slope of the tangent to the curve $y^2 = x$ is equal to $\frac{1}{2}$ and that of $x^2 = y$ is 2 .

$$\tan\theta = \left|\frac{2 - \frac{1}{2}}{1 + 1}\right| = \frac{3}{4} \Rightarrow \theta = \tan^{-1}\left(\frac{3}{4}\right)$$

69. (b) Given $y = ke^{kx}$. The curve intersects the y-axis at $(0, k)$
So, $\left(\frac{dy}{dx}\right)_{(0,k)} = k^2$
If θ is the angle at which 1 the given curve intersects the y-axis, then



$$\tan\left(\frac{\pi}{2} - \theta\right) = \frac{k^2 - 0}{1 + 0.k^2} = k^2 \Rightarrow \theta = \cot^{-1}(k^2)$$

70. (b) Given curve is $x^{3/2} + y^{3/2} = 2a^{3/2}$

$\therefore \frac{3}{2}\sqrt{x}+\frac{3}{2}\sqrt{y}\frac{dy}{dx}=0$ or $\frac{dy}{dx}=-\frac{\sqrt{x}}{\sqrt{y}}$
Since the tangent is equally inclined to the axes,
$\frac{dy}{dx}=\pm 1$ $\therefore -\frac{\sqrt{x}}{\sqrt{y}}=\pm 1$ or $-\frac{\sqrt{x}}{\sqrt{y}}=-1$
$\therefore \sqrt{x}=\sqrt{y}[\because \sqrt{x}>0, \sqrt{y}>0]$
Putting $\sqrt{y}=\sqrt{x}$ in (1), we get
$2x^{3/2}=2a^{3/2}$ or $x^3=a^3$.
Therefore, $x=a$ and, so, $y=a$.

71. (c) Let r be the radius of the sphere and Δr be the error in measuring the radius. Then,
$\mathrm{r}=9\mathrm{cm}$ and $\Delta \mathrm{r}=0.03\mathrm{cm}$
Let V be the volume of the sphere. Then,
$$\mathrm{V}=\frac{4}{3}\pi \mathrm{r}^3 \Rightarrow \frac{\mathrm{dV}}{\mathrm{dr}}=4\pi \mathrm{r}^2 \Rightarrow \left(\frac{\mathrm{dV}}{\mathrm{dr}}\right)_{\mathrm{r}=9}=4\pi \times 9^2=324\pi$$
Let $\Delta \mathrm{V}$ be the error in V due to error $\Delta \mathrm{r}$ in r. Then,
$$\Delta \mathrm{V}=\frac{\mathrm{dV}}{\mathrm{dr}}\Delta \mathrm{r} \Rightarrow \Delta \mathrm{V}=324\pi \times 0.03=9.72\pi \mathrm{cm}^3$$
Thus, the approximate error in calculating the volume is $9.72\pi \mathrm{cm}^3$.

72.
(b) $\frac{\Delta \mathrm{r}}{\mathrm{r}}\times 100=\mathrm{k}$ (Given)
$$\mathrm{V}=\frac{4}{3}\pi \mathrm{r}^3 \Rightarrow \frac{\mathrm{dV}}{\mathrm{dr}}=4\pi \mathrm{r}^2$$
$$\Delta \mathrm{V}=\frac{\mathrm{dV}}{\mathrm{dr}}\times \Delta \mathrm{r} \Rightarrow \Delta \mathrm{V}=4\pi \mathrm{r}^2 \Delta \mathrm{r}$$
$$\Rightarrow \Delta \mathrm{V}=4\pi \mathrm{r}^2 \frac{\mathrm{kr}}{100} \Rightarrow \Delta \mathrm{V}=4\pi \mathrm{r}^3 \frac{\mathrm{k}}{100}$$
$$\Rightarrow \Delta \mathrm{V}\times 100=\frac{4\pi \mathrm{r}^3}{4/3\pi \mathrm{r}^3}\frac{\mathrm{k}}{100}\times 100=3\mathrm{k}\%$$

73. (a) Let $f(x)=x^{1/3} \Rightarrow f'(x)=\frac{1}{3}x^{-2/3}$
Now $f(x+\Delta x)-f(x)=f'(x)\cdot \Delta x=\frac{\Delta x}{3(x^{2/3})}$
We may write, $0.007=0.008-0.001$, taking. $x=0.008$
and $dx=-0.001$.
we have $f(0.007)-f(0.008)=-\frac{0.001}{3(0.008)^{2/3}}$
$$\Rightarrow f(0.007)-(0.008)^{1/3}=-\frac{0.001}{3(0.2)^2}$$
$$\Rightarrow f(0.007)=0.2-\frac{0.001}{3(0.04)}=0.2-\frac{1}{120}=\frac{23}{120}$$
Hence $(0.007)^{1/3}=\frac{23}{120}$

74. (b) Consider $f(x)=x^3-7x^2+15 \Rightarrow f'(x)=3x^2-14x$
Let $x=5$ and $\Delta x=0.001$
Also, $\mathrm{f}(\mathrm{x}+\Delta \mathrm{x}) \approx \mathrm{f}(\mathrm{x})+\Delta \mathrm{x}\mathrm{f}'(\mathrm{x})$
Therefore, $f(x+\Delta x)=(x^3-7x^2+15)+\Delta x(3x^2-14x)$

$\Rightarrow f(5.001) = (5^3 - 7 \times 5^2 + 15) + (3 \times 5^2 - 14 \times 5)(0.001)$
(as $x = 5, \Delta x_1 = 0.001$)
$= 125 - 175 + 15 + (75 - 70)(0.001) = -34.995$

75. (d) $\because A = \pi r^2 \Rightarrow \log A = \log \pi + 2\log r$
$\Rightarrow \frac{\Delta A}{A} \times 100 = 2 \times \frac{\Delta r}{r} \times 100 = 2 \times 0.05 = 0.1\%$

76. (d) Let Δx be the change in x and ΔV be the corresponding change in V.
It is given that $\frac{\Delta x}{x} \times 100 = 2$ $\therefore V = x^3 \Rightarrow \frac{dV}{dx} = 3x^2$
$\Delta V = \frac{dV}{dx} \times \Delta x \Rightarrow \Delta V = 3x^2 \Delta x \Rightarrow \Delta V = 3x^2 \times \frac{2x}{100}$
$\Rightarrow \Delta V = 0.06x^3$
Thus, the approximate change in volume is $0.06x^3 m^3$.

77. (a) Let $f(x, y) = (x^2 + 3y^4)^{1/6}$
Taking $x = 4, \Delta x = -0.08$ and $y = 2, \Delta y = 0.1$
Differentiating (1) w.r.t. x, treating y as constant,
$\therefore \frac{\Delta f}{\Delta x} = \frac{1}{6}(x^2 + 3y^4)^{-5/6}(2x)$
$= \frac{8}{6}(16 + 48)^{-5/6} = \frac{4}{3} \times 2^{-5} = \frac{1}{24}$ and differentiating (1) w.r.t. y treating x as constant,
$\therefore \frac{\Delta f}{\Delta y} = \frac{1}{6}(x^2 + 3y^4)^{-5/6}(12y^3)$
$= \frac{12(8)}{6}(64)^{-5/6} = 16(2)^{-5} = \frac{1}{2}$
$\therefore df = \frac{\Delta f}{\Delta x} \cdot dx + \frac{\Delta f}{\Delta y} dy = \frac{1}{24} \times -0.08 + \frac{1}{2} \times 0.1$
$= -\frac{0.01}{3} + \frac{0.1}{2} = 0.466$
$\therefore \{(3.92)^2 + 3(2.1)^4\}^{1/6} = f(4,2) + df = 2 + 0.466 = 2.466$

78. (c) Given error in diameter $= \pm 0.04$
$\therefore$ Error in radius, $\delta r = \pm 0.02$
$\therefore$ Percent error in the volume of sphere

$$= \frac{\delta r}{V} \times 100 = \frac{\delta\left(\frac{4}{3}\pi r^3\right)}{\frac{4}{3}\pi r^3} \times 100 = \frac{3\delta r}{r} \times 100$$

$$= \frac{3 \times (\pm 0.02)}{10} \times 100 = \pm 0.6$$

79. (c) Let radius of spherical balloon $= r$
After increasing 0.2%, radius $= r + r \times \frac{0.2}{100} = \frac{1002}{1000} r$
Original volume $= \frac{4}{3}\pi r^3$
and New volume $= \frac{4}{3}\pi \left(\frac{1002}{1000} r\right)^3$
$\therefore$ Increased volume $= \frac{4}{3}\pi \left(\frac{1002}{1000} r\right)^3 - \frac{4}{3}\pi r^3$

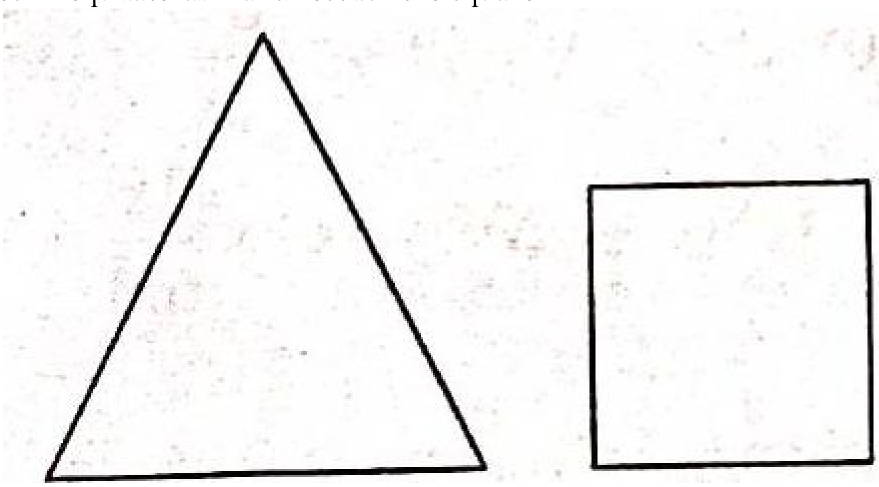
$$= \frac{4}{3}\pi r^3\left[\left(\frac{1002}{1000}\right)^3 - 1\right]$$

$\therefore$ % increased in volume $= \frac{\frac{4}{3}\pi r^3[(1.002)^3-1]}{\frac{4}{3}\pi r^3} \times 100$

$= (1.006 - 1) \times 100 = 0.006 \times 100 = 0.600 = 0.6\%$

80. (b) Since, length of the given wire $= 22\text{m}$

Let xm length is used in equilateral Δ and rest for the square



Let side of an equilateral $\Delta = a$ and side of a square $= b$

$\Rightarrow 3a = x;\ 4b = 22 - x;$

now, total area $= \frac{\sqrt{3}}{4}a^2 + b^2$

$$= \frac{\sqrt{3}}{4}x^2/9 + \frac{(22-x)^2}{16}$$

$$\frac{dA}{dx} = 0 \Rightarrow x\left(\frac{\sqrt{3}}{2 \times 9} + \frac{1}{8}\right) = \frac{22}{8}$$

$$\Rightarrow x\left(\frac{4\sqrt{3}+9}{36}\right) = \frac{11}{2}$$

$a = x/3$

$$a = \left(\frac{11/2}{\frac{4\sqrt{3}+9}{36}}\right)\left(\frac{1}{3}\right) = \frac{66}{4\sqrt{3}+9}$$

81. (c) $f'(x) = (x-3)^{n_1-1}(x-5)^{n_2-1}(n_1+n_2)\left(x - \frac{5n_1+3n_1}{n_1+n_2}\right)$

Option (c) is incorrect since

for $n_1 = 3, n_2 = 5$

$f'(x) = 8(x-3)^2(x-5)^4\left(x - \frac{30}{8}\right)$ minima at $x = \frac{30}{8}$

82. (b) Given differential equation is

$(x+1)dy - ydx = e^{3x}(x+1)^2$

$$\frac{dy}{dx} - \frac{y}{x+1} = e^{3x}(x+1)^2$$

I.F. $= e^{\int Pdx} = e^{-\int \frac{dx}{x+1}} = e^{-\log(x+1)} = \frac{1}{x+1}$

Solution is shown below

$y \times \mathrm{IF} = \int Q \times I \cdot F dx + c$

$\frac{y}{x+1} - \int \frac{e^{3x}(x+1)dx}{(x+1)}$

$\frac{y}{(x+1)} = \frac{e^{3x}}{3} + c$

Put $x = 0, y = \frac{1}{3}$, then, $c = 0$

So, $y = \frac{e^{3x}}{3}(x+1)$

Differentiate w.r.t. $x \frac{dy}{dx} e^{3x}(x+2)$

Put $\frac{dy}{dx} = 0$

$x = -2$

Again differentiate w.r.t. x

$\frac{d^2y}{dx^2} = e^{3x}(3x+7)$

$\frac{d^2y}{dx^2} = e^{-6} > 0$

83. (b) Here surface area of a cuboid is $= 2(lb + bh + lh)$ and surface area of a closed hemisphere is $3\pi \mathrm{r}^2$

Surface area $76x^2 + 3\pi r^2 =$ constant (K)$[76x^3 + 3\pi r^2 = \mathrm{K}] r^2 = \frac{\mathrm{K} - 76x^2}{3\pi}$

$$r = \left(\frac{\mathrm{K} - 76x^2}{3\pi}\right)^{\frac{1}{2}}$$

Volume of cuboid is l.b.h. and volume of hemisphere is $\frac{2}{3}\pi r^3$

$$\mathrm{V} = 40x^3 + \frac{2}{3}\pi r^3$$

$$\mathrm{V} = 40x^3 + \frac{2}{3}\pi\left(\frac{\mathrm{K} - 76x^2}{3\pi}\right)^{\frac{3}{2}}$$

$$\frac{d\mathrm{V}}{dx} = 120x^2 + \frac{2}{3}\pi \cdot \frac{3}{2}\left(\frac{\mathrm{K} - 76x^2}{3\pi}\right)^{\frac{1}{2}}\left(\frac{-76(2x)}{3\pi}\right)$$

Put $\frac{d\mathrm{V}}{dx} = 0 \Rightarrow 120x^2 + \frac{2}{3}\pi \cdot \frac{3}{2}\left(\frac{K - 76x^2}{3\pi}\right)^{\frac{1}{2}} \cdot \left(\frac{-76(2x)}{3\pi}\right) = 0$

$$\Rightarrow 120x^2 = \frac{152x}{3}\left(\frac{k - 76x^2}{3\pi}\right)^{\frac{1}{2}}$$

$$\Rightarrow \frac{45}{19}x^2 = x\left(\frac{k - 76x^2}{3\pi}\right)^{\frac{1}{2}}; x \neq 0$$

$$\Rightarrow \frac{45}{19}x = \left(\frac{k - 76x^2}{3\pi}\right)^{\frac{1}{2}} \Rightarrow \left(\frac{45}{19}\right)^2 x^2 = \frac{k - 76x^2}{3\pi}$$

$$\Rightarrow \left(\frac{45}{19}\right)^2 x^2 = r^2 \Rightarrow \frac{x^2}{r^2} = \left(\frac{19}{45}\right)^2$$

$\Rightarrow \frac{x}{r} = \frac{19}{45}$

84. (a) $f(x) = \begin{cases} x^2 - 4x - 2, & \forall x \in \left(-1, \frac{3-\sqrt{17}}{2}\right) \\ -x^2 + 2x + 2, & \forall x \in \left(\frac{3-\sqrt{17}}{2}, 2\right) \end{cases}$

$f'(x)$ when $x \in \left(-1, \frac{3-\sqrt{17}}{2}\right)$

$f'(x) = 2x - 4 = 0 \Rightarrow x = 2$

$f'(x) = 2(x-2) \Rightarrow f'(x)$ is always $\downarrow$

$f(2) = 2$

$f(-1) = 3$

$f\left(\frac{3-\sqrt{17}}{2}\right) = \frac{\sqrt{17}-3}{2} f'(x)$ when $x \in \left(\frac{3-\sqrt{17}}{2}, 2\right)$

$f'(x) = -2x + 2$

$f'(x) = -2(x-1)$

$f'(x) = 0$ when $x = 1$

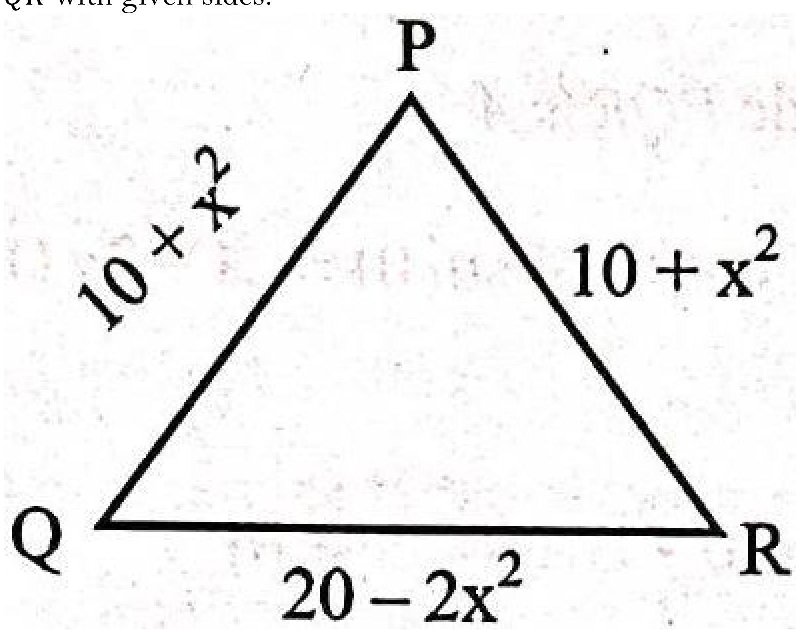
$f(1) = 3$

absoluteminimum value $= \frac{\sqrt{17}-3}{2}$

absolute maximum value $= 3$

Sum $= \frac{\sqrt{17}-3}{2} + 3 = \frac{\sqrt{17}+3}{2}$

85. (c) Required triangle PQR with given sides.



$a = 20 - 2x^2, b = 10 + x^2, c = 10 + x^2$

$S = \frac{a+b+c}{2} = 20.$

Use Heron's formula to find area of triangle.

$\Delta \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{20(2x^2)(10-x^2)(10-x^2)}$

$= 2\sqrt{10}\sqrt{x^2(10-x^2)^2} = 2\sqrt{10}|x(10-x^2)|$

$= 2\sqrt{10}|10x - x^3|$

Let $A = 2\sqrt{10}(10x - x^3)$

$\frac{dA}{dx} = 2\sqrt{10}(10 - 3x^2) \Rightarrow \frac{dA}{dx} = 0 \Rightarrow x^2 = \frac{10}{3}$

Therefore $3x^2 = 10$

86. (b) Given function is $f(x) = |2x^2 + 3x - 2| + \sin x\cos x$

$f(x) = |(2x-1)(x+2)| + \frac{\sin 2x}{2}$

Now, differentiate w.r.t 'x'

$$f'(x) = \begin{cases} 4x + 3 + \cos 2x & \frac{1}{2} \leq x < 1 \\ -(4x+3) + \cos 2x & 0 \leq x < \frac{1}{2} \end{cases}$$

In $0 \leq x < \frac{1}{2}$, Put $x = 0$ in $f'(x)$.

$\Rightarrow f'(x) < 0$

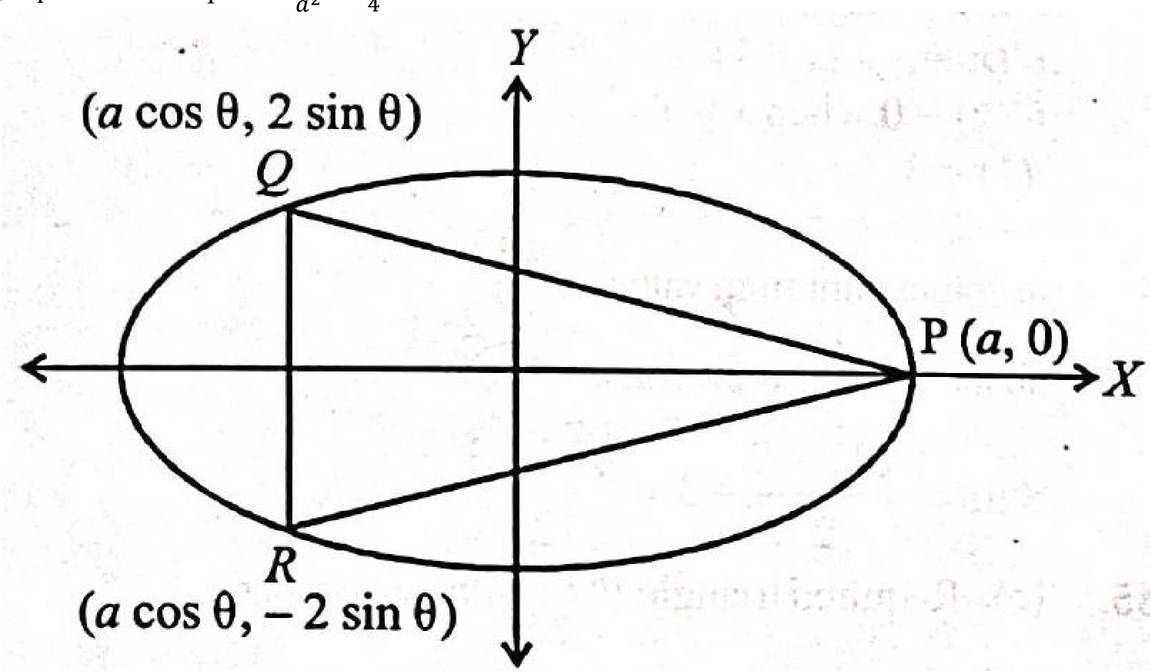
In $\frac{1}{2} \leq x \leq 1$, Put $x = \frac{1}{2}$ in $f'(x)$ then $\cos(1) > 0$.

$\Rightarrow f'(x) > 0$

So, f(x) local minima at $x = \frac{1}{2}$ and local maxima at $x = 1$.

Therefore, $f\left(\frac{1}{2}\right) = f(1) = 3 + \frac{1}{2}(1 + 2\cos 1)\sin 1$

87. (a) Given, equation of ellipse be $\frac{x^2}{a^2} + \frac{y^2}{4} = 1, a > 2$.

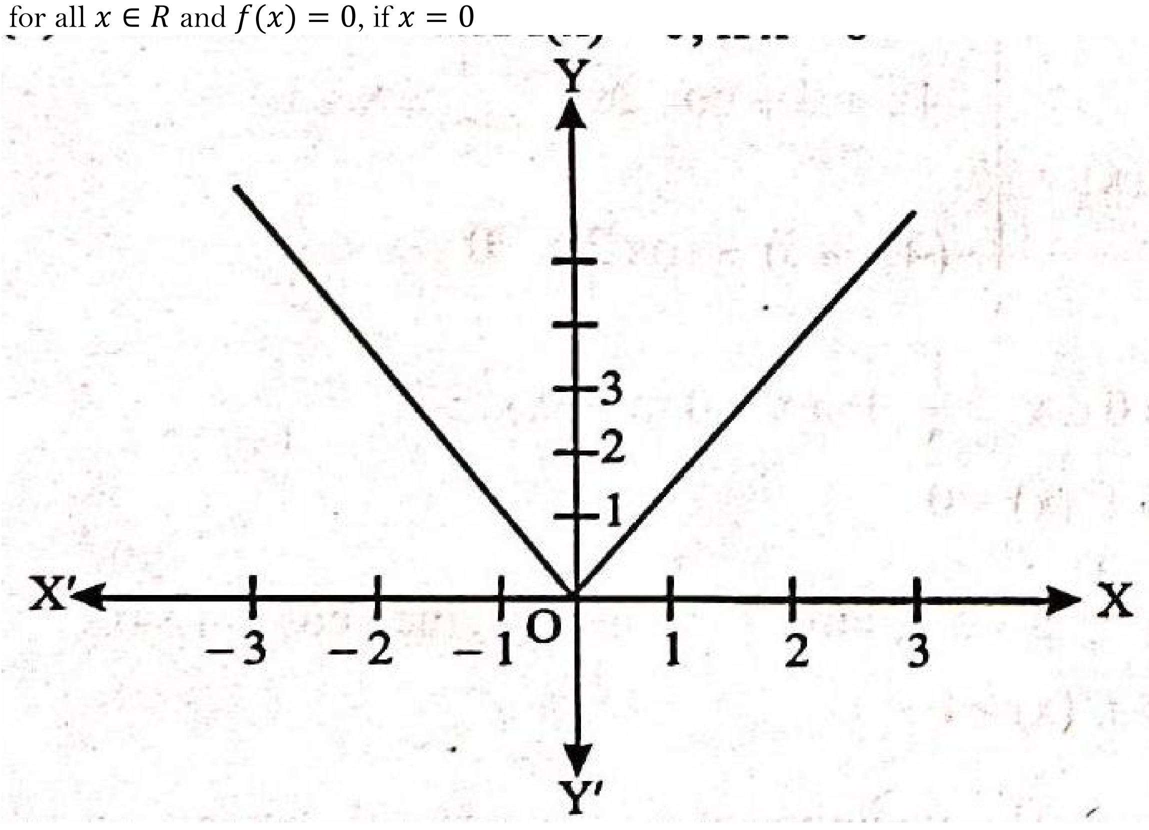


Area of triangle PQR is A.

$A = \frac{1}{2}a(1 - \cos\theta)(4\sin\theta) \Rightarrow A = 2a(1 - \cos\theta)\sin\theta$

Diff. w.r.t. 'θ'.

$\frac{dA}{d\theta} = 2a(\sin^2\ \theta + \cos\ \theta - \cos^2\ \theta)$
$\frac{dA}{d\theta} = 0 \Rightarrow 1 + \cos\ \theta - 2\cos^2\ \theta = 0$
$\cos\ \theta = 1$ which is not possible because of multiple of $n\pi$.
Now, $\cos\ \theta = \frac{-1}{2} \Rightarrow \theta = \frac{2\pi}{3}$
Again, differentiate w.r.t. θ.
$\frac{d^2A}{d\theta^2} = 2a(2\sin^2\ \theta - \sin\ \theta)$
$\frac{d^2A}{d\theta^2} < 0$ and maximum for $\theta = \frac{2\pi}{3}$
Now, put the value of θ in the value of A.
$A_{\max} = \frac{3\sqrt{3}}{2}a = 6\sqrt{3} \Rightarrow a = 4$
From equation ellipse $b = 2$.
Take, $e = \sqrt{1 - \frac{b^2}{a^2}} = \frac{\sqrt{3}}{2}$.
88. (b) From the graph of the given function, note that
$f(x) \geq 0$ for all $x \in R$ and $f(x) = 0$, if $x = 0$



Therefore, the function f has a minimum value 0 and the point of minimum value of f is $x = 0$. Also, the graph clearly shows that f has no maximum value in R and hence no point of maximum value in R
89. (a) For $0 < x \leq \frac{\pi}{2}$; $[\cos\ x] = 0$
Hence, $f(x) = 1$ for all $\left(0, \frac{\pi}{2}\right]$

Trivially $f(x)$ is continuous on $\left(0, \frac{\pi}{2}\right)$

This function is neither strictly increasing nor strictly decreasing and its global maximum is 1 .

90. (b) $f(x)=\sin\ 2x-x \Rightarrow f'(x)=2\cos\ 2x-1$

Therefore, $f'(x)=0 \Rightarrow \cos\ 2x=\frac{1}{2}$

$\Rightarrow 2x=\frac{\pi}{3}$ or $-\frac{\pi}{3} \Rightarrow x=-\frac{\pi}{6}$ or $\frac{\pi}{6}$

$\Rightarrow \mathrm{f}\left(-\frac{\pi}{2}\right)=\sin\ (-\pi)+\frac{\pi}{2}=\frac{\pi}{2}$

$\Rightarrow f\left(-\frac{\pi}{6}\right)=\sin\ \left(-\frac{2\pi}{6}\right)+\frac{\pi}{6}=-\frac{\sqrt{3}}{2}+\frac{\pi}{6}$

$\Rightarrow f\left(\frac{\pi}{6}\right)=\sin\ \left(\frac{2\pi}{6}\right)-\frac{\pi}{6}=\frac{\sqrt{3}}{2}-\frac{\pi}{6}.$

$\Rightarrow \mathrm{f}\left(\frac{\pi}{2}\right)=\sin\ (\pi)-\frac{\pi}{2}=-\frac{\pi}{2}$

Clearly, $\frac{\pi}{2}$ is the greatest value and $-\frac{\pi}{2}$ is the least.

Therefore, difference $=\frac{\pi}{2}+\frac{\pi}{2}=\pi$

91. (d) Let $y=\frac{\ln\ x}{x} \Rightarrow \frac{dy}{dx}=\frac{x \cdot \frac{1}{x}-\ln\ x \cdot 1}{x^2}=\frac{1-\log\ x}{x^2}$

For maxima, put $\frac{dy}{dx}=0$

$\Rightarrow \frac{1-\ln\ \mathrm{x}}{\mathrm{x}^2}=0 \Rightarrow \mathrm{x}=\mathrm{e}$

Now, $\frac{d^2y}{dx^2}=\frac{x^2\left(-\frac{1}{x}\right)-(1-\ln\ x)2x}{(x^2)^2}$

At $x=\mathrm{e}$ we have $\frac{d^2y}{dx^2}<0$

∴ The maximum value at $\mathrm{x}=\mathrm{e}$ is $\mathrm{y}=\frac{1}{\mathrm{e}}$

92. (d) Let $f(x)=x^4-62x^2+ax+9 \Rightarrow f'(x)=4x^3-124x+a$ It is given that function fattains its maximum value on the interval $[0,2]$ at $x=1$.

∴ $f'(1)=0 \Rightarrow 4\times 1^3-124\times 1+a=0$

$\Rightarrow 4-124+a=0 \Rightarrow a=120$

Hence, the value of a is 120 .

93. (c) It is a fundamental property.

94. (b) Let $\mathrm{y}=\mathrm{x}^{25}(1-\mathrm{x})^{75}$

$\Rightarrow \frac{dy}{dx}=25\mathrm{x}^{24}(1-\mathrm{x})^{74}(1-4\mathrm{x})$

For maximum value of $y, \frac{dy}{dx}=0$

$\Rightarrow \mathrm{x}=0,1,1/4 \Rightarrow x=1/4 \in (0,1)$

Also at $\mathrm{x}=0, \mathrm{y}=0$, at $\mathrm{x}=1, \mathrm{y}=0$, and at $\mathrm{x}=1/4, \mathrm{y}>0$

∴ Max. value of y occurs at $x=1/4$

95. (b) The diagonal $=\mathrm{R}$

Thus the area of rectangle

$=\frac{1}{2}\times \mathrm{R}\times \mathrm{R}=\frac{\mathrm{R}^2}{2}$

96. (a) Let x, y be two numbers such that $x+y=3 \Rightarrow y=3-x$ and let product $P=xy^2$ thus $P=x(3-x)^2=x^3-6x^2+9x$

For a maxima or minima $\frac{\mathrm{dP}}{\mathrm{dx}}=0$

Thus $\frac{dP}{dx}=3x^2-12x+9$ and $\frac{d^2P}{dx^2}=6x-12$

Now, $\frac{dP}{dx}=0 \Rightarrow 3x^2-12x+9=0 \Rightarrow x=1,3$.

Thus $\left(\frac{d^2P}{dx^2}\right)_{x=1}=-6$ and $\left(\frac{d^2P}{dx^2}\right)_{x=3}=6$

Thus P is maximum when $x=1 \Rightarrow y=2$

So, $\mathrm{P}=1.2^2=4$.

97. (a) Let the given points are

$(50+\alpha, 0)$ and $(0,50+\alpha)$

$\Rightarrow$ equation of line is

$$y-(50+\alpha)=\frac{((50+\alpha)-0)}{(0-(50+\alpha))}(x-(50+\alpha))$$

$\Rightarrow y-(50+\alpha)=(-1)(x-0) \Rightarrow x+y=50+\alpha$

Let $t=xy^4 \Rightarrow t=y^4(50+\alpha-y)$

$$\frac{dt}{dy}=4y^3(50+\alpha)-5y^4 \Rightarrow \frac{dt}{dy}=0 \Rightarrow y=\frac{(50+\alpha)4}{5}$$

$$\Rightarrow x=(50+\alpha)\cdot\frac{1}{5} \Rightarrow y=4x$$

98. (a) For real roots $(\alpha-6)^2-4\times 3(\alpha+3)\geq 0$

$\therefore \alpha \in(-\infty, 0] \cup[24, \infty)=\mathrm{A}$

We have $3x^2+(\alpha-6)x+(\alpha+3)=0$

Let roots are p and q

$\Rightarrow p+q=\frac{-(\alpha-6)}{3}$ and $p\cdot q=\frac{\alpha+3}{3}$

Now $F=p^2+q^2=(p+q)^2-2$ p.q

$$=\frac{\alpha^2-18\alpha+18}{9}$$

$$F'=\frac{1}{9}(2\alpha-18)=0=\alpha=9 \notin A$$

99. (a) $y(x)=ax^3+bx^2+cx+5$ is passing through

$(-2,0)$ then $8a-4b+2c=5$

$y'(x)=3ax^2+2bx+c$ touches x-axis at $(-2,0)$

$12a-4b+c=0$

acc. to question

$\mathrm{y}(-2)=0$

$y'(-2)=0$

$y'(0)=3$

again, for $x=0, y'(x)=3 \Rightarrow c=3$

Solving eqs (i), (ii) & (iii) $a=-\frac{1}{2}, b=-\frac{3}{4}$

$\Rightarrow \mathrm{y}'(\mathrm{x})=3\mathrm{ax}^2+2\mathrm{bx}+\mathrm{c}$

or $y'(x)=-\frac{3}{2}x^2-\frac{3}{2}x+3$

$y(x)$ has local maxima at $x=1 \Rightarrow y(1)=\frac{27}{4}$

100. (b) We have, $\mathrm{A}+\mathrm{B}=\frac{\pi}{3}$.

$$\therefore \mathrm{B}=\frac{\pi}{3}-\mathrm{A} \Rightarrow \tan \mathrm{B}=\frac{\sqrt{3}-\tan \mathrm{A}}{1+\sqrt{3} \tan \mathrm{A}}$$

Let $Z=\tan A \cdot \tan B$. Then,

$$Z=\tan A \cdot \frac{\sqrt{3}-\tan A}{1+\sqrt{3} \tan A}=\frac{\sqrt{3} \tan A-\tan ^{2} A}{1+\sqrt{3} \tan A}$$

$\Rightarrow Z=\frac{\sqrt{3} x-x^{2}}{1+\sqrt{3} x}$, where $x=\tan A$

$$\Rightarrow \frac{d Z}{d x}=-\frac{(x+\sqrt{3})(\sqrt{3} x-1)}{(1+\sqrt{3} x)^{2}}$$

For $\max Z, \frac{d Z}{d x}=0 \Rightarrow x=\frac{1}{\sqrt{3}},-\sqrt{3}$.

$\mathrm{x} \neq-\sqrt{3}$ because $\mathrm{A}+\mathrm{B}=\pi / 3$ which implies that $x=\tan A>0$. It can be easily checked that $\frac{\mathrm{d}^{2} \mathrm{Z}}{\mathrm{dx}^{2}}<0$ for $\mathrm{x}=\frac{1}{\sqrt{3}}$. Hence, Z is maximum for $x=\frac{1}{\sqrt{3}}$ i.e. $\tan A=\frac{1}{\sqrt{3}}$ or $A=\pi / 6$.

For this value of $\mathrm{x}, \mathrm{Z}=\frac{1}{3}$.

101. (d) Let $f(x)=4 x^{3}-11 x^{2}+8 x-5 \forall x \in R$

$\Rightarrow f'(x)=12 x^{2}-22 x+8$

$f'(x)=0$

$\Rightarrow 2(6 x^{2}-11 x+4)=0$

$\Rightarrow 6 x^{2}-8 x-3 x+4=0$

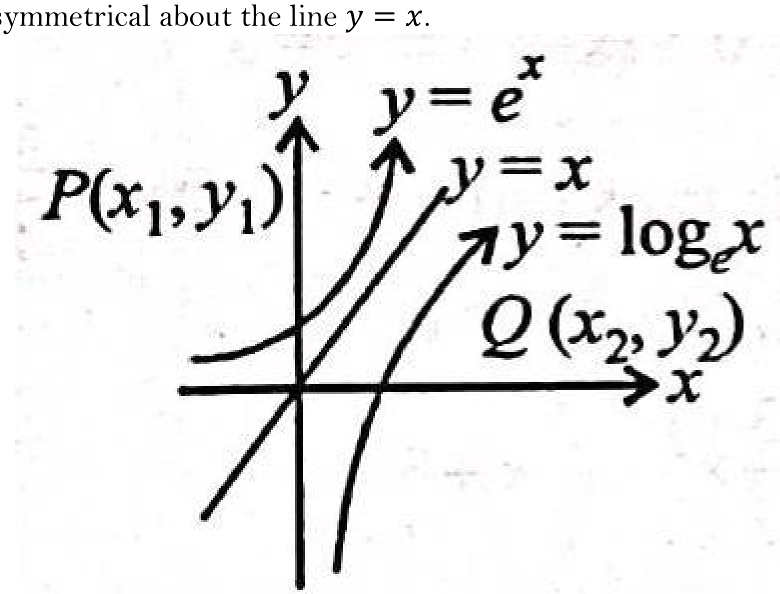
$\Rightarrow(2 x-1)(3 x-4)=0$

$\therefore$ function is decreasing in $\left(\frac{1}{2}, \frac{4}{3}\right)$

102. (d) Given an ellipse $x^{2}+a^{2} y^{2}=25 a^{2}$ & hyperbola $x^{2}-a^{2} y^{2}=5$.

Here, ' a ' is the minimum distance between the curves $y=e^{x}$ & $y=\log_{e} x$

These two curves are symmetrical about the line $y=x$.



Then, $m=1$.
Let point $P(x_1,y_1)$ on the curve $y=e^x \& Q(x_2,y_2)$ on the curve $y=\log x$. Take, $y=e^x$
Differentiate w.r.t. x,
$$\frac{dy}{dx}=e^x|_{(x_1,y_1)}=e^{x_1}$$
Put $e^{x_1}=1 \Rightarrow x_1=0$, then $y_1=1$
So, $y_2=0$, similarly, slope of curve $y=\log_e x$ is shown below
$$\frac{dy}{dx}=\frac{1}{x}\bigg|_{(x_2,y_2)}=\frac{1}{x_2}$$
Put $\frac{1}{x_2}=1, x_2=1$
Here, $P(x_1,y_2) \to (0,1) \& Q(x_2,y_2) \to (1,0)$.
Distance between $PQ=\sqrt{(0-1)^2+(1-0)^2}=\sqrt{1^2+1^2}$
$=a=\sqrt{2}$
Put $a=\sqrt{2}$ in the equation of ellipse.
$$x^2+a^2y^2=25a^2 \Rightarrow \frac{x^2}{a^2}+y^2=25 \Rightarrow \frac{x^2}{50}+\frac{y^2}{25}=1$$
Eccentricity, $e_1^2=\frac{c_1^2}{a_1^2}=\frac{a_1^2-b_1^2}{a_1^2}=1-\frac{25}{50}=\frac{25}{50}=\frac{1}{2}$
Equation of hyperbola is $x^2-a^2y^2=5$.
$$\Rightarrow x^2-2y^2=5 \Rightarrow \frac{x^2}{5}-\frac{y^2}{\left(\frac{5}{2}\right)}=1$$
Here, $a_2=\sqrt{5}, b_2=\sqrt{\frac{5}{2}}$
Eccentricity of hyperbola, $e_2=\frac{c_2}{a_2}$
$$e_2^2=\frac{a_2^2+b_2^2}{a_2^2}=\frac{5+\frac{5}{2}}{5}=\frac{3}{2}$$
According to question, $e_1^2=b^2e_2^2 \Rightarrow \frac{1}{2}=b^2\times\frac{3}{2} \Rightarrow b^2=\frac{1}{3}$
Required value $=a^2+\frac{i}{b^2}=2+3=5$

103. (b) $f(0)=\sin 0=0, f(0^+) \to 0^+$
$f(0^-)=\lim_{x\to 0^-}\sin(x^2-3x)=\lim_{h\to 0}\sin(h^2+3h) \to 0^+$
Thus, $f(0^+)>f(0)$ and $f(0^-)>f(0)$.
Hence, $x=0$ is a point of minima.

104. (d) Volume of cylinder, $(\mathrm{V})=\pi \mathrm{r}^2\mathrm{h}$;
Surface area, $(S)=2\pi\mathrm{rh}+\pi\mathrm{r}^2$
$$\Rightarrow \mathrm{h}=\frac{\mathrm{S}-\pi\mathrm{r}^2}{2\pi\mathrm{r}}$$
$$\therefore \mathrm{V}=\pi\mathrm{r}^2\left[\frac{\mathrm{S}-\pi\mathrm{r}^2}{2\pi\mathrm{r}}\right]=\frac{\mathrm{r}}{2}[\mathrm{S}-\pi\mathrm{r}^2]=\frac{1}{2}[\mathrm{Sr}-\pi\mathrm{r}^3]$$
Now, Differentiate both sides, w.r.t 'r'
$$\frac{\mathrm{dV}}{\mathrm{dr}}=\frac{1}{2}[\mathrm{S}-3\pi\mathrm{r}^2]$$

Now, circular cylinder will have the greatest volume, when

$$\frac{dV}{dr} = 0 \Rightarrow S = 3pr^2$$

$\Rightarrow 2\pi rh + \pi r^2 = 3\pi r^2 \Rightarrow 2\pi rh = 2\pi r^2 \Rightarrow r = h.$

105. (d) Let one side of quadrilateral be x and another side be y so, $2(x + y) = 34$

or, $(x + y) = 17$

We know from the basic principle that for a given perimeter square has the maximum area, so, $x = y$ and putting this value in equation (i)

$$x = y = \frac{17}{2}$$

Area $= x \cdot y = \frac{17}{2} \times \frac{17}{2} = \frac{289}{4} = 72.25$

106. (a)

$$\text{Let } f(x) = \frac{\sin\ 2x}{\sin\ \left(x + \frac{\pi}{4}\right)} = \sqrt{2}\left\{\frac{(\sin\ x + \cos)^2 - 1}{\sin\ x + \cos\ x}\right\}$$

$= \sqrt{2}\left(\frac{y^2-1}{y}\right)$, where $y = \sin\ x + \cos\ x$

Let $\phi(y) = \sqrt{2}\left(\frac{y^2-1}{y}\right)$, and $g(x) = \sin\ x + \cos\ x$

We have, $g'(x) = \cos\ x - \sin\ x$.

For max or min. $g'(x) = 0 \Rightarrow \tan\ x = 1$

$\Rightarrow x = \pi/4$. For this value of x.

$\overrightarrow{g''}(x) < 0$. Thus, $g(x)$ is max. at $x = \pi/4$ and hence the domain of $g(x)$ is $[1, \sqrt{2}]$ i.e. y lies between 1 and $\sqrt{2}$

Now, $\phi'(y) = \sqrt{2}\left(1 + \frac{1}{y^2}\right) > 0$ for all $y \in [1, \sqrt{2}]$.

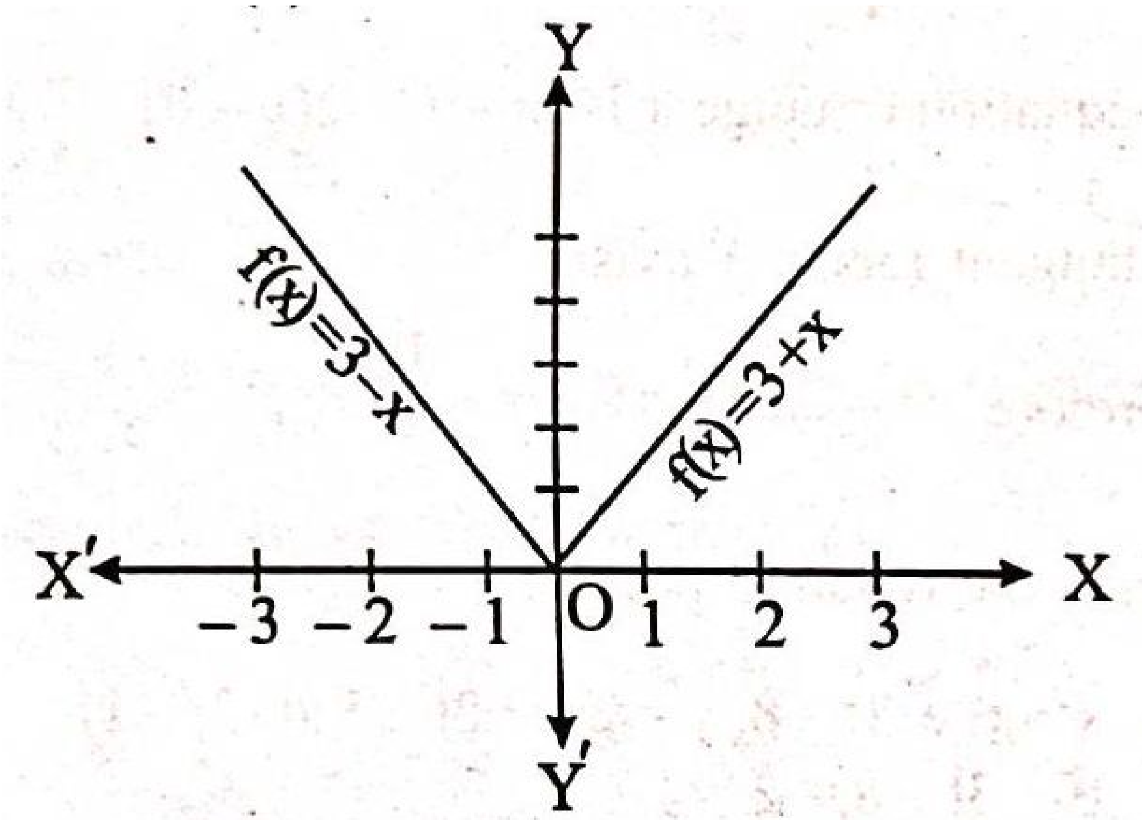
That is $\phi(y)$ is increasing for all $y \in [1, \sqrt{2}]$

Thus it attains the greatest value at $\sqrt{2}$ and is equal to $\sqrt{2}\left(\frac{(\sqrt{2})^2-1}{\sqrt{2}}\right) = 1$

Hence, greatest value of $f(x)$ on $[0, \pi/2]$ = greatest value of $\phi(y)$ on $[1, \sqrt{2}] = 1$.

107. (c) Note that the given function is not differentiable at $x = 0$. So, second derivative test fails. Let us try first derivative test.

Note that 0 is a critical point of f. Now to the left of 0 , $f(x) = 3 - x$ and so $f'(x) = -1 < 0$. Also to the right of 0 , $f(x) = 3 + x$ and so, $f'(x) = 1 > 0$. Therefore, by first derivative test, $x = 0$ is a point of local minima of f and local minimum value of f is $f(0) = 3$.



108. (a) $\frac{dy}{dx} = \frac{d}{dx}(x^4 - 2x^2 + 1) = 4x^3 - 4x = 4x(x^2 - 1)$

For max. or min, $\frac{dy}{dx} = 0 \Rightarrow 4x(x^2 - 1) = 0$

either $x = 0$ or $x = \pm 1$

$x = 0$ and $x = -1$ does not belong to $\left[\frac{1}{2}, 2\right]$.

$\frac{d^2y}{dx^2} = 12x^2 - 4 \Rightarrow \left(\frac{d^2y}{dx^2}\right)_{x=1} = 12(1)^2 - 4 = 8 > 0$

∴ there is minimum value of function at $x = 1$

∴ minimum value is

$y(1) = 1^4 - 2(1)^2 + 1 = 1 - 2 + 1 = 0$

109. (b) Let $f(x) = (x^2 - 2x + 7)e^{(4x^3 - 12x^2 - 180)}$

$f'(x) = e^{(4x^3 - 12x^2 - 180x + 31)}$

for $x \in [-3,0] \Rightarrow f(x) < 0$

$f(x)$ is decreasing function on $[-3,0]$

The absolute maximum value of the function f(x)

is at $x = -3 \Rightarrow \alpha = -3$

110. (a) $\because x^2 \geq 0 \therefore -x^2 \leq 0$

∴ the maximum value of $y = -x^2$ is 0 . This value is attained when $x = 0 \in [-1,1]$

EXERCISE - 2

1. (b) Let each side be x and area, $A=\frac{\sqrt{3}}{4}x^2$

Since, each side of an equilateral triangle expands at the rate of 2cm/s.

$\Rightarrow \left(\frac{dx}{dt}\right)=2\text{cm/s}$ and $A=\frac{\sqrt{3}}{4}x^2$

On differentiation, we get;

$$\frac{dA}{dt}=\frac{\sqrt{3}}{4}2x\frac{dx}{dt}, \text{ at } x=10$$
$$=\frac{\sqrt{3}}{4}\times 2\times 10\times 2=10\sqrt{3}\text{cm}^2/\text{s}$$

2. (b) Suppose the angle between floor and the ladder be θ. $PQ=y\text{cm}$ and $QR=z\text{cm}$

So, $\sin\theta=\frac{y}{500}\Rightarrow y=500\sin\theta$

and $\cos\theta=\frac{z}{500}\Rightarrow z_1=500\cos\theta$

Now $\frac{dy}{dt}=10\text{cm/s}$

$\therefore$ $500\cos\theta\frac{d\theta}{dt}=10$

Therefore, $\frac{d\theta}{dt}=\frac{1}{50\cos\theta}$

When $Z=2\text{m}=200\text{cm}$

$$\frac{d\theta}{dt}=\frac{1}{50\times\frac{z}{500}}=\frac{10}{x}=\frac{10}{200}=\frac{1}{20}\text{rad/s}$$

3. (b) Given $y=x^{\frac{1}{5}}$

$\Rightarrow \frac{dy}{dx}=\frac{1}{5}x^{-4/5}\Rightarrow\frac{dy}{dx}$ at $(0,0)=\frac{1}{5}(0)\Rightarrow\frac{dy}{dx}=0$

4. (c) Since, the equation of given curve is $3x^2-y^2=8$ and equation of given line is $x+3y=8$

$\Rightarrow y=-\frac{x}{3}+\frac{8}{3}$. So, slope of given line $=-\frac{1}{3}$.

Therefore, slope of the normal to the curve $=-\frac{1}{3}$

After differentiating curve equation $3x^2-y^2=8$ w.r.t. x, we have $6x-2y\frac{dy}{dx}=0$

Therefore, slope of normal to the curve $=\frac{1}{\left(\frac{dy}{dx}\right)}$

$$=-\frac{1}{\left(\frac{3x}{y}\right)}=-\frac{y}{3x}$$

Now, $-\left(\frac{y}{3x}\right)=-\frac{1}{3}\Rightarrow y=x$

After putting $y=x$ in $3x^2-y^2=8$, we get

$3x^2-x^2=8\Rightarrow x=\pm 2$

When $x=2, 3(2)^2-y^2=8\Rightarrow y=\pm 2$

When $x=-2, 3(-2)^2-y^2=8\Rightarrow y=\pm 2$

So, the equation of normal at $(\pm 2,\pm 2)$ is given by

$$y-(\pm 2)=-\frac{1}{3}[x-(\pm 2)]$$

Hence, $x + 3y \pm 8 = 0$

5. (d) Since, given curves are $C_1: ay + x^2 = 7$ and $C_2: x^3 = y$
After differentiating w.r.t. x in both curve equations, we have
$a\dfrac{dy}{dx} + 2x = 0 \Rightarrow \dfrac{dy}{dx} = -\dfrac{2x}{a} \Rightarrow \dfrac{dy}{dx} = 3x^2$
For C_1: $\left(\frac{dy}{dx}\right)_{(1,1)} = \frac{-2}{a} = m_1$
and for C_2: $\left(\frac{dy}{dx}\right)_{(1,1)} = 3 = m_2$
As, the curves cut orthogonally at $(1,1)$
Therefore $m_1 m_2 = -1 \Rightarrow \left(\frac{-2}{a}\right)3 = -1 \Rightarrow a = 6$

6. (a) Since, $y = x^4 - 10 \Rightarrow \frac{dy}{dx} = 4x^3$
Here, $\Delta x = 2.00 - 1.99 = 0.01$
So, $\Delta y = \frac{dy}{dx} \times \Delta x = 4x^3 \times \Delta x$
$= 4 \times 2^3 \times 0.01 = 32 \times 0.01 = 0.32$
Hence, the change in y is 0.32

7. (a) Since, equation of given curve is

$$y(1 + x^2) = 2 - x \qquad (i)$$

After differentiating w.r.t. x, we get
$y(0 + 2x) + (1 + x^2)\dfrac{dy}{dx} = 0 - 1$
$\Rightarrow 2xy + (1 + x^2)\dfrac{dy}{dx} = -1 \Rightarrow \dfrac{dy}{dx} = \dfrac{-(1 + 2xy)}{1 + x^2}$
As, the curve crosses x-axis
So, $0(1 + x^2) = 2 - x$
[From eq. (i)]
Now $\left(\frac{dy}{dx}\right)_{(2,0)} = \frac{-1-2\times 0}{1+2^2} = -\frac{1}{5}$
So, slope of tangent $= -\frac{1}{5}$
Hence, equation of tangent of the curve at $(2,0)$ is
given by $y - 0 = -\frac{1}{5}(x - 2)$
$\Rightarrow 5y + x = 2$

8. (d) Since, the equation of given curve is $y = x^3 - 12x + 18$
After differentiating w.r.t. $x\frac{dy}{dx} = 3x^2 - 12$
As, the tangent is parallel to the X-axis
So, $\left(\frac{dy}{dx}\right) = 0 \Rightarrow 3x^2 - 12 = 0 \Rightarrow x = \pm 2$
When $x = 2, y = 2^3 - 12 \times 2 + 18 = 2$
When $x = -2, y = (-2)^3 - 12(-2) + 18 = 34$
Hence, the required points are $(2,2)$ and $(-2,34)$.

9. (b) Since the equation of given curve is $y = e^{2x}$
$\Rightarrow \dfrac{dy}{dx} = 2e^{2x}$

Now, $\left(\frac{dy}{dx}\right)_{(0,1)} = 2e^0 = 2 =$ Slope of tangent

So, equation of tangent is: $y - 1 = 2(x - 0)$

$\therefore y = 2x + 1$

As tangent meets X-axis

Therefore, $0 = 2x + 1 \Rightarrow x = -\frac{1}{2}$

Hence, the required point is $\left(-\frac{1}{2}, 0\right)$

10. (b) $\mathrm{x} = \mathrm{t}^2 + 3\mathrm{t} - 8, \mathrm{y} = 2\mathrm{t}^2 - 2\mathrm{t} - 5$ at $(2, -1)$

$\therefore \mathrm{t}^2 + 3\mathrm{t} - 8 = 2$

$$2t^2 - 2t - 5 = -1 \qquad (i)$$

On solving eqs (i) and (ii) we get $t = 2$

Now $\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\mathrm{dy/dt}}{\mathrm{dx/dt}} = \frac{4\mathrm{t}-2}{2\mathrm{t}+3} \therefore \left[\frac{\mathrm{dy}}{\mathrm{dx}}\right]_{\mathrm{t}=2} = \frac{6}{7}$

11. (c) $x^3 - 3xy^2 + 2 = 0$

differentiating w.r.t. x :

$$3x^2 - 3x(2y)\frac{dy}{dx} - 3y^2 = 0$$

$\Rightarrow \frac{dy}{dx} = \frac{3x^2 - 3y^2}{6xy}$ and $3x^2y - y^3 - 2 = 0$

differentiating w.r.t. x

$$\Rightarrow 3x^2\frac{dy}{dx} + 6xy - 3y^2\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\left(\frac{6xy}{3x^2 - 3y^2}\right)$$

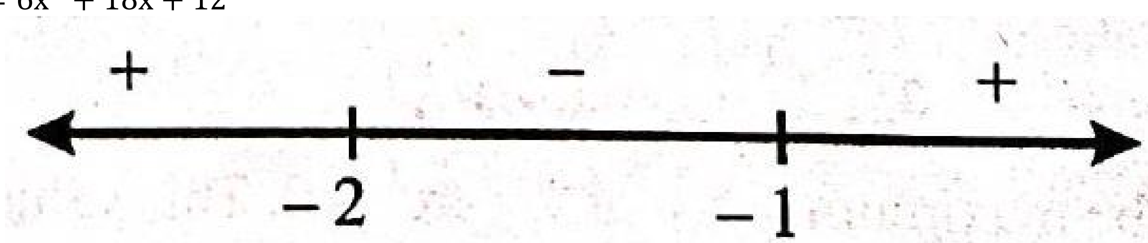
Now, product of slope $= \frac{3x^2-3y^2}{6xy} \times -\left(\frac{6xy}{3x^2-3y^2}\right) = -1$

$\therefore$ they are perpendicular.

Hence, angle $= \pi/2$

12. (b) $\because f(x) = 2x^3 + 9x^2 + 12x - 1$

$\therefore \mathrm{f}'(\mathrm{x}) = 6\mathrm{x}^2 + 18\mathrm{x} + 12$



$= 6(\mathrm{x}^2 + 3\mathrm{x} + 2)$

For decreasing function $\mathrm{f}'(\mathrm{x}) \leq 0$

$\Rightarrow 6(x + 1)(x + 2) \leq 0$

$\therefore$ f(x) is decreasing in $[-2, -1]$

13. (d) Since, $f(x) = 2x + \cos x$ So, $f'(x) = 2 - \sin x$

Therefore, $f'(x) > 0, \forall x [-1 \leq \sin x \leq 1]$

Hence, $f(x)$ is an increasing function.

14. (a) Since, $y = x(x - 3)^2$

After differentiating w.r.t. x, we get

$$\frac{dy}{dx} = x \cdot 2(x - 3) + (x - 3)^2$$

$= 3x^2 - 12x + 9 = 3(x-3)(x-1)$
Hence, $y = x(x-3)^2$ decreases for $1 < x < 3$.
15.
(b) Since, $f(x) = 4\sin^3 x - 6\sin^2 x + 12\sin x + 100$
So, $f'(x) = 12\sin^2 x\cos x - 12\sin x\cos x + 12\cos x$
$= 12\cos x(\sin^2 x - \sin x + 1)$
As $1 - \sin x > 0$ and $\sin^2 x > 0 \therefore \sin^2 x - \sin x + 1 > 0$
Therefore, $f'(x) > 0$, if $\cos x > 0 \Rightarrow x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$\therefore f(x)$ is increasing if $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $f'(x) < 0$,
if $\cos x < 0 \Rightarrow x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$
So, $f(x)$ is decreasing if $x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$.
As, $\left(\frac{\pi}{2}, \pi\right) \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$
Hence, $f(x)$ is strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$.
16. (c) $\because \mathrm{f(x)} = \cos \mathrm{x}$
$\Rightarrow f'(x) = -\sin x < 0$ for all $x \in \left(0, \frac{\pi}{2}\right)$
So, $f(x) = \cos x$ is decreasing in $\left(0, \frac{\pi}{2}\right)$
17. (a) Since, $f(x) = \tan x - x$
After differentiating w.r.t. x, we get
$f'(x) = \sec^2 x - 1$ So, $f'(x) > 0, \forall x \in R$
Hence, $f(x)$ is always increases
18. (c) Since, $f(x) = x^2 - 8x + 17$
After differentiating w.r.t. x, we get
$f'(x) = 2x - 8$ As $f'(x) = 0 \Rightarrow x = 4$
Here, $f''(x) = 2 > 0, \forall x$
Hence, $x = 4$ is point of local minnima and minimum value of $f(x)$
$f(4) = (4 \times 4) - (8 \times 4) + 17 = 1$
19. (b) $f(x) = x^3 - 18x^2 + 96x \Rightarrow f'(x) = 3x^2 - 36x + 96$
$\therefore f'(x) = 0 \Rightarrow x^2 - 12x + 32 = 0 \Rightarrow x = 8,4$
Now, $f(0) = 0, f(4) = 160, f(8) = 128, f(9) = 135$
So, smallest value of $f(x)$ is 0 at $x = 0$.
20. (c) $f(x) = 2x^3 - 3x^2 - 12x + 4$
$\Rightarrow f'(x) = 6x^2 - 6x - 12 = 6(x^2 - x - 2) = 6(x-2)(x+1)$
For maxima and minima $f'(x) = 0$
$\therefore 6(x-2)(x+1) = 0 \Rightarrow x = 2, -1$
Now, $\mathrm{f''(x)} = 12\mathrm{x} - 6$
At $\mathrm{x} = 2, \mathrm{f''(x)} = 24 - 6 = 18 > 0$
$\therefore$ $\mathrm{x} = 2$, local min. point
At $\mathrm{x} = -1, \mathrm{f''(x)} = 12(-1) - 6 = -18 < 0$
$\therefore \mathrm{x} = -1$ local max. point
21. (b) Since, $f(x) = \sin x \cdot \cos x = \frac{1}{2}\sin 2x$

After differentiating w.r.t. x, we get

$f'(x) = \frac{1}{2} \cdot \cos\ 2x.2 = \cos\ 2x$

As $f'(x) = 0 \Rightarrow \cos\ 2x = 0 \Rightarrow x = \frac{\pi}{4}$

Now, $f''(x) = \frac{d}{dx}\cos\ 2x = -2\sin\ 2x$

So, $[f''(x)]_{x=p/4} = -2\sin\ \left(2 \cdot \frac{\pi}{4}\right) = -2 < 0$

Hence, $\frac{\pi}{4}$ is point of maxima

and $f\left(\frac{\pi}{4}\right) = \frac{1}{2}\sin\ \left(2 \cdot \frac{\pi}{4}\right) = \frac{1}{2}$

22. (d) Since, $f(x) = 2\sin\ 3x + 3\cos\ 3x$

After differentiating w.r.t. x, we get

$f'(x) = 6\cos\ 3x - 9\sin\ 3x$

So $f''(x) = -18\sin\ 3x - 27\cos\ 3x$

$= -9(2\sin\ 3x + 3\cos\ 3x)$

As $f'\left(\frac{5\pi}{6}\right) = 6\cos\ \left(3 \times \frac{5\pi}{6}\right) - 9\sin\ \left(3 \times \frac{5\pi}{6}\right)$

$= 6\cos\ \frac{5\pi}{2} - 9\sin\ \frac{5\pi}{2} = 6\cos\ \left(2\pi + \frac{\pi}{2}\right) - 9\sin\ \left(2\pi + \frac{\pi}{2}\right)$

$= -9 \neq 0$

23. (b) Since, $y = -x^3 + 3x^2 + 9x - 27$

After differentiating w.r.t. x, we get

Slope of the curve $= \frac{dy}{dx} = -3x^2 + 6x + 9$

and $\frac{d^2y}{dx^2} = -6(x-1)$

Now, $\frac{d^2y}{dx^2} = 0 \Rightarrow -6(x-1) = 0 \Rightarrow x = 1 > 0$

As $\frac{d^3y}{dx^3} = -6 < 0$

Hence, the maximum slope of curve is given by

$\left(\frac{dy}{dx}\right)_{(x=1)} = -3(1^2) + 6(1) + 9 = 12$

24. (b) Since, $f(x) = x^x$

Suppose $y = x^x \therefore \log\ y = x\log\ x$

After differentiating w.r.t. x, we get

$\frac{1}{y}\frac{dy}{dx} = x\left(\frac{1}{x}\right) + \log\ x$ So, $\frac{dy}{dx} = (1 + \log\ x)x^x$

Now, $\frac{dy}{dx} = 0 \Rightarrow (1 + \log\ x) \cdot x^x = 0$

$\Rightarrow \log\ x = -1 \Rightarrow x = e^{-1} = \frac{1}{e}$.

Hence, $f(x)$ has a stationary point at $x = \frac{1}{e}$

25. (c) Let $y = \left(\frac{1}{x}\right)^x \Rightarrow \log\ y = x\log\ \left(\frac{1}{x}\right) \Rightarrow \log\ y = -\log\ x$ differentiating w.r.t.x

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = -(1 + \log\ x) \Rightarrow \frac{dy}{dx} = -y(1 + \log\ x)$

$$\Rightarrow \frac{d^2y}{dx^2} = -\frac{dy}{dx}(1+\log\,x) - \frac{y}{x}$$
$$\Rightarrow \frac{d^2y}{dx^2} = y(1+\log\,x)^2 - \frac{y}{x}$$
$$\Rightarrow \frac{d^2y}{dx^2} = \left(\frac{1}{x}\right)^x (1+\log\,x)^2 - \frac{1}{x(x+1)}$$
For maximum value $\frac{dy}{dx} = 0 \Rightarrow -y(1+\log\,x) = 0$

$\Rightarrow 1+\log\,x = 0 (\because y \neq 0) \Rightarrow \log\,x = -1$

$\Rightarrow \mathrm{x} = \mathrm{e} - 1 \Rightarrow \mathrm{x} = 1/\mathrm{e}$

Also $\left[\frac{d^2y}{dx^2}\right]_{x=\frac{1}{e}} = e^{1/e}\left(1+\log\frac{1}{e}\right)^2 - e^{(1/e+1)}$

$= \mathrm{e}^{1/\mathrm{e}}(1-\log\,\mathrm{e})^2 - \mathrm{e}^{1/\mathrm{e}+1} = -\mathrm{e}^{1/\mathrm{e}+1} < 0$

So, $x = 1/\mathrm{e}$ is a point of local maxima.

Hence, local maximum value $y = (e)^{1/e}$.

26. (d) $f(x) = \tan^{-1}\left(\sqrt{\frac{1+\sin\,x}{1-\sin\,x}}\right) = \tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)$

$$\Rightarrow y = \frac{\pi}{4}+\frac{x}{2} \Rightarrow \frac{dy}{dx} = \frac{1}{2}$$
Slope of normal $= \frac{-1}{\left(\frac{dy}{dx}\right)} = -2$

At $\left(\frac{\pi}{6}, \frac{\pi}{4}+\frac{\pi}{12}\right)$; $y - \left(\frac{\pi}{4}+\frac{\pi}{12}\right) = -2\left(x-\frac{\pi}{6}\right)$

$$\Rightarrow y - \frac{4\pi}{12} = -2x + \frac{2\pi}{6} \Rightarrow y = -2x + \frac{2\pi}{3}$$
This equation is satisfied only by the point $\left(0, \frac{2\pi}{3}\right)$

27. (a) $4\mathrm{x} + 2\pi\mathrm{r} = 2\square \Rightarrow 2\mathrm{x} + \pi\mathrm{r} = 1$

$$S = x^2 + \pi r^2 \Rightarrow S = \left(\frac{1-\pi r}{2}\right)^2 + \pi r^2$$
$$\Rightarrow \frac{\mathrm{dS}}{\mathrm{dr}} = 2\left(\frac{1-\pi\mathrm{r}}{2}\right)\left(\frac{-\pi}{2}\right) + 2\pi\mathrm{r} \Rightarrow \frac{-\pi}{2} + \frac{\pi^2\mathrm{r}}{2} + 2\pi\mathrm{r} = 0$$
$$\Rightarrow r = \frac{1}{\pi+4} \Rightarrow x = \frac{2}{\pi+4} \Rightarrow x = 2r$$
28. (c) We have $y = \frac{x+6}{(x-2)(x-3)}$

At y-axis, $x = 0 \Rightarrow y = 1$

On differentiating, we get
$$\frac{dy}{dx} = \frac{(x^2-5x+6)(1)-(x+6)(2x-5)}{(x^2-5x+6)^2}$$
$\frac{\mathrm{dy}}{\mathrm{dx}} = 1$ at point $(0,1)$ $\therefore$ Slope of normal $= -1$

Now equation of normal is $y - 1 = -1(x-0)$

$\Rightarrow \mathrm{y} - 1 = -\mathrm{x} \Rightarrow \mathrm{x}+\mathrm{y} = 1 \therefore \left(\frac{1}{2}, \frac{1}{2}\right)$ satisfy it. 1

29. (d) We have

Total length $= r + r + r\theta = 20$

$\Rightarrow 2\mathrm{r} + \mathrm{r}\theta = 20 \Rightarrow \theta = \dfrac{20 - 2\mathrm{r}}{\mathrm{r}}$

$A = \text{Area} = \frac{\theta}{2\pi} \times \pi r^2 = \frac{1}{2} r^2 \theta = \frac{1}{2} r^2 \left(\frac{20-2r}{r}\right)$

$A = 10r - r^2$

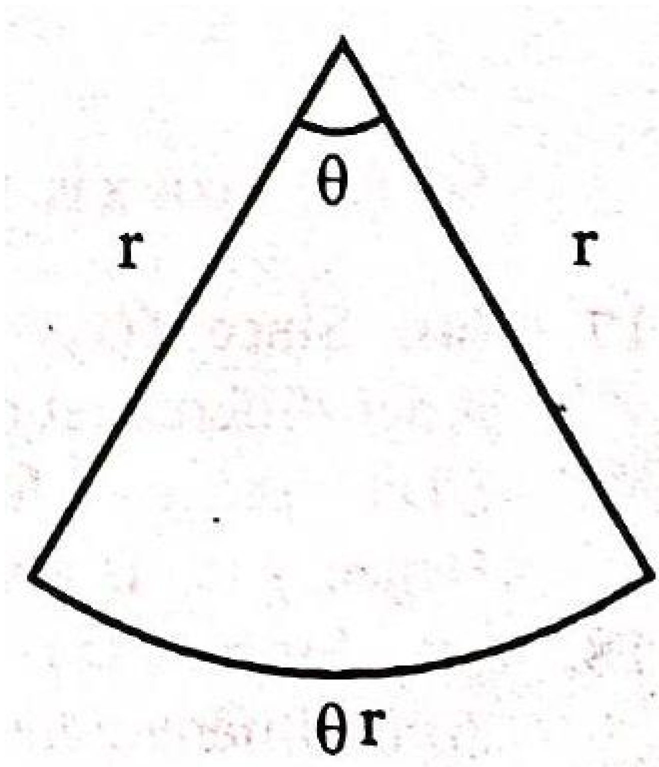
For A to be maximum

$\dfrac{\mathrm{dA}}{\mathrm{dr}} = 0 \Rightarrow 10 - 2\mathrm{r} = 0 \Rightarrow \mathrm{r} = 5$

$\dfrac{\mathrm{d}^2\mathrm{A}}{\mathrm{dr}^2} = -2 < 0$

$\therefore$ For $r = 5$ A is maximum

From (i)



$\theta = \frac{20-2(5)}{5} = \frac{10}{5} = 2 \Rightarrow A = \frac{2}{2\pi} \times \pi(5)^2 = 25$ sq. m

30. (c) Eccentricity of ellipse $= \frac{1}{2}$

Now, $-\frac{a}{e} = -4 \Rightarrow a = 4 \times \frac{1}{2} = 2 \Rightarrow a = 2$

We have $b^2 = a^2(1 - e^2) = a^2\left(1 - \frac{1}{4}\right) = 4 \times \frac{3}{4} = 3$

$\therefore$ Equation of ellipse is : $\frac{\mathrm{x}^2}{4} + \frac{\mathrm{y}^2}{3} = 1$

Now differentiating, we get $\Rightarrow \frac{x}{2} + \frac{2y}{3} \times y' = 0$

$\Rightarrow y' = -\dfrac{3x}{4y}$

$y'|_{(1,3/2)} = -\dfrac{3}{4} \times \dfrac{2}{3} = -\dfrac{1}{2}$

Slope of normal $= 2$ ∴ Equation of normal at $\left(1, \frac{3}{2}\right)$ is

$y - \frac{3}{2} = 2(x - 1) \Rightarrow 2y - 3 = 4x - 4$ ∴ $4x - 2y = 1$

31. (c) Here, $h(x) = \frac{x^2 + \frac{1}{x^2}}{x - \frac{1}{x}} = \left(x - \frac{1}{x}\right) + \frac{2}{x - \frac{1}{x}}$

when $x - \frac{1}{x} < 0$ ∴ $x - \frac{1}{x} + \frac{2}{x - \frac{1}{x}} \leq -2\sqrt{2}$

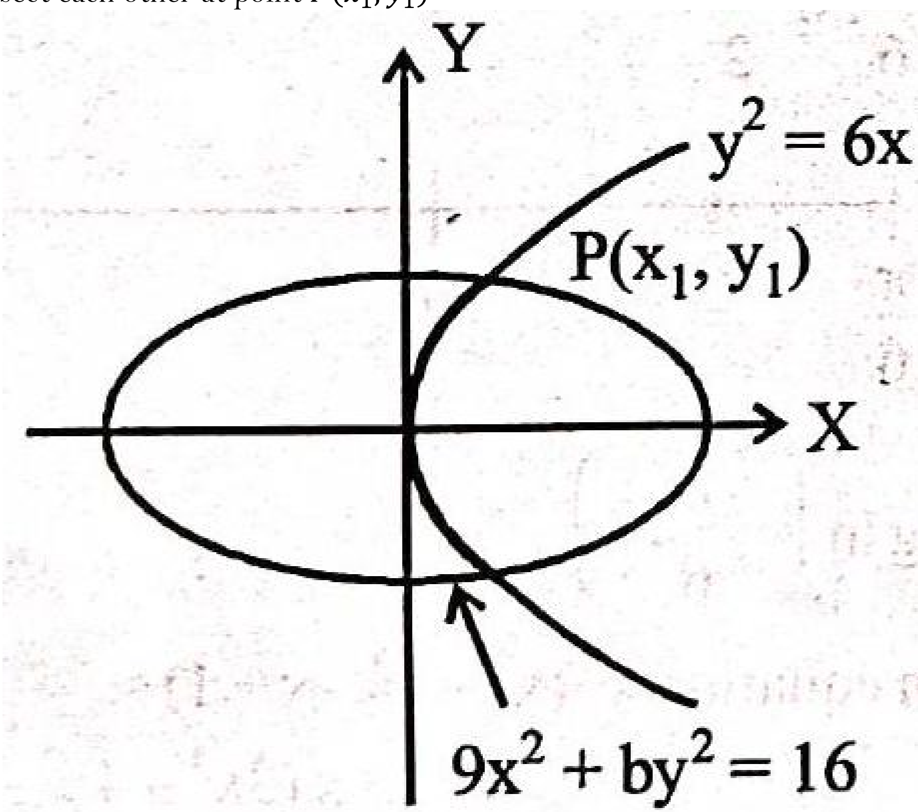
Hence, $-2\sqrt{2}$ will be local maximum value of $h(x)$.

when $x - \frac{1}{x} > 0$

∴ $x - \frac{1}{x} + \frac{2}{x - \frac{1}{x}} \geq 2\sqrt{2}$

Hence, $2\sqrt{2}$ will be local minimum value of $h(x)$.

32. (c) Let curve intersect each other at point $P(x_1, y_1)$



Since, point of intersection is on both the curves, then

$y_1^2 = 6x_1$

and $9x_1^2 + by_1^2 = 16$

Now, find the slope of tangent to both the curves at the point of intersection $P(x_1, y_1)$

For slope of curves:

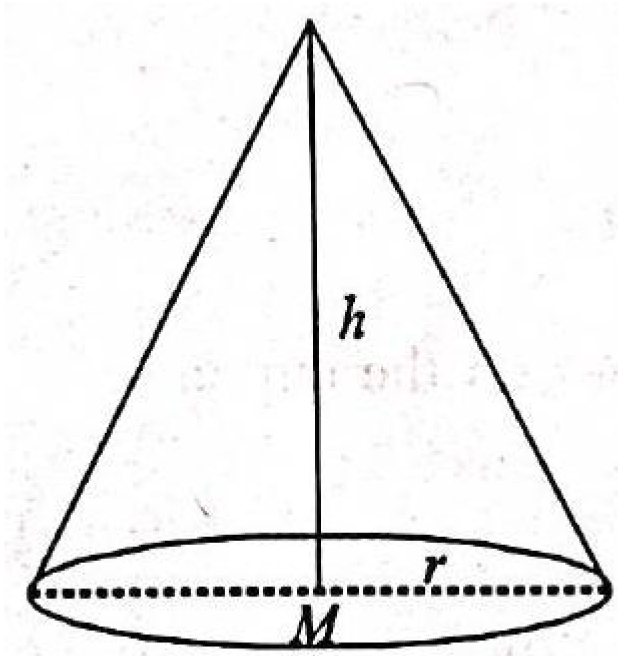
curve (i): $\left(\frac{dy}{dx}\right)_{(x_1, y_1)} = m_1 = \frac{3}{y_1}$ curve (ii): $\left(\frac{dy}{dx}\right)_{(x_1, y_1)} = m_2 = -\frac{9x_1}{by_1}$

Since, both the curves intersect each other at right angle then,

$m_1m_2 = -1 \Rightarrow \frac{27x_1}{by_1^2} = 1 \Rightarrow b = 27\frac{x_1}{y_1^2}$

∴ from equation (i), $b = 27 \times \frac{1}{6} = \frac{9}{2}$

33. (d)



$$h^2 + r^2 = l^2 = 9 \qquad (i)$$

Volume of cone

$$V = \frac{1}{3}\pi r^2 h \qquad (ii)$$

From (i) and (ii),

$\Rightarrow V = \frac{1}{3}\pi(9 - h^2)h$

$\Rightarrow V = \frac{1}{3}\pi(9h - h^3) \Rightarrow \frac{dv}{dh} = \frac{1}{3}\pi(9 - 3h^2)$

For maxima/minima,

$\frac{dV}{dh} = 0 \Rightarrow \frac{1}{3}\pi(9 - 3h^2) = 0$

$\Rightarrow h = \pm\sqrt{3} \Rightarrow h = \sqrt{3}$

Now; $\frac{d^2V}{dh^2} = \frac{1}{3}\pi(-6h)$

Here, $\left(\frac{d^2V}{dh^2}\right)_{\text{at } h=\sqrt{3}} < 0$

Then, $h = \sqrt{3}$ is point of maxima

Hence, the required maximum volume is,

$V = \frac{1}{3}\pi(9 - 3)\sqrt{3} = 2\sqrt{3}\pi$

34. (b) Since, the equation of curves are

$$y = 10 - x^2 \qquad (i)$$

$$y = 2 + x^2 \qquad (ii)$$

Adding eqs. (i) and (ii), we get

$$2y = 12 \Rightarrow y = 6$$

Then, from eqn (i)

$x = \pm 2$

Differentiate equation (i) with respect to x

$\frac{dy}{dx} = -2x \Rightarrow \left(\frac{dy}{dx}\right)_{(2,6)} = -4$ and $\left(\frac{dy}{dx}\right)_{(-2,6)} = 4$

Differentiate equation (ii) with respect to x

$$\frac{dy}{dx} = 2x \Rightarrow \left(\frac{dy}{dx}\right)_{(2,6)} = 4 \text{ and } \left(\frac{dy}{dx}\right)_{(-2,6)} = -4$$

At $(2,6) \tan\ \theta = \left(\frac{(-4)-(4)}{1+(-4)\times(4)}\right) = \frac{-8}{15}$

At $(-2,6), \tan\ \theta = \frac{(4)-(-4)}{1+(4)(-4)} = \frac{8}{-15}$

$\therefore |\tan\ \theta| = \frac{8}{15}$

35. (d) $y = x^3 + ax - b$

Since, the point $(1,-5)$ lies on the curve.

$\Rightarrow 1 + a - b = -5$

$\Rightarrow a - b = -6$

Now, $\frac{dy}{dx} = 3x^2 + a \Rightarrow \left(\frac{dy}{dx}\right)_{\text{at } x=1} = 3 + a$.

Since, required line is perpendicular to $y = x - 4$, then slope of tangent at the point $P(1,-5) = -1$

$\therefore 3 + \text{a} = -1$

$\Rightarrow \text{a} = -4$

$\Rightarrow \text{b} = 2$

$\therefore$ the equation of the curve is $y = x^3 - 4x - 2$

$\Rightarrow (2,-2)$ lies on the curve

36. (d) $y = f(x) = x^3 - x^2 - 2x$

$\Rightarrow \frac{dy}{dx} = 3x^2 - 2x - 2$

$f(1) = 1 - 1 - 2 = -2, f(-1) = -1 - 1 + 2 = 0$

Since the tangent to the curve is parallel to the line segment joining the points $(1,-2)$ and $(-1,0)$

And their slopes are equal.

$\Rightarrow 3x^2 - 2x - 2 = \frac{-2-0}{2}$

$\Rightarrow x = 1, \frac{-1}{3}$

Hence, the required set $S = \left\{\frac{-1}{3}, 1\right\}$

37. (c) $f(x) = ax^5 + bx^4 + cx^3$

$$\lim_{x \to 0}\left(2 + \frac{ax^5 + bx^4 + cx^3}{x^3}\right) = 4$$

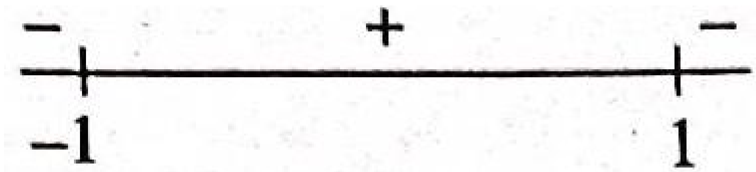
$$\Rightarrow 2 + c = 4 \Rightarrow c = 2$$

$$f'(x) = 5ax^4 + 4bx^3 + 6x^2$$

$$= x^2(5ax^2 + 4bx + 6)$$

Since, $x = \pm 1$ are the critical points,

$\therefore f'(1) = 0 \Rightarrow 5a + 4b + 6 = 0$
$f'(-1) = 0 \Rightarrow 5a - 4b + 6 = 0$
From eqns. (i) and (ii),
$b = 0$ and $a = -\frac{6}{5} f(x) = \frac{-6}{5}x^5 + 2x^3$
$f'(x) = -6x^4 + 6x^2 = 6x^2(-x^2 + 1)$
$= -6x^2(x + 1)(x - 1)$

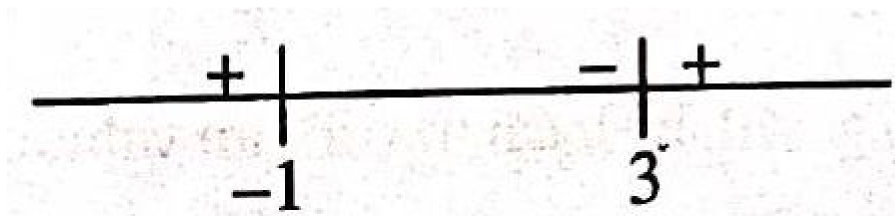


$\therefore f(x)$ has minima at $x = -1$ and maxima at $x = 1$

38. (3) Let $f(x) = ax^3 + bx^2 + cx + d$

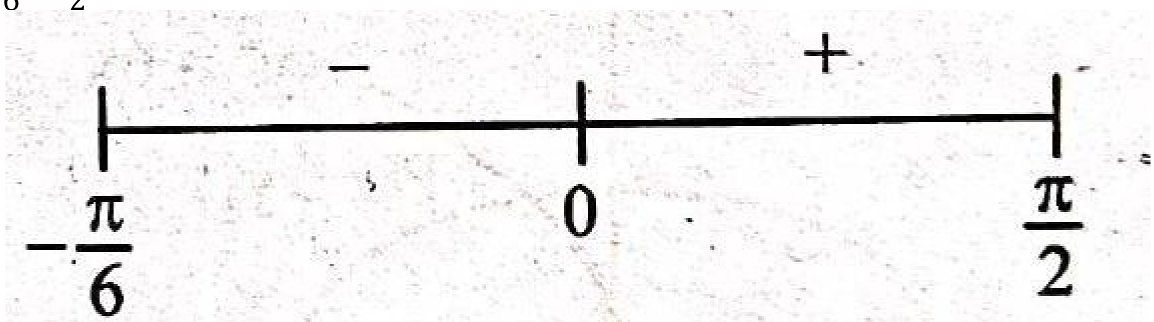
$f(-1) = 10$ and $f(1) = -6$

Solving equations (i) and (ii), we get
$a = \frac{1}{4}, d = \frac{35}{4}$
$b = \frac{-3}{4}, c = -\frac{9}{4}$
$\Rightarrow f(x) = a(x^3 - 3x^2 - 9x) + d$
$f'(x) = \frac{3}{4}(x^2 - 2x - 3) = 0$
$\Rightarrow x = 3, -1$



Local minima exist at $x = 3$

39. (d) Given that
$f(x) = 3\sin^4 x + 10\sin^3 x + 6\sin^2 x - 3, x \in \left[-\frac{\pi}{6}, \frac{\pi}{2}\right]$
$f'(x) = 12\sin^3 x\cos x + 30\sin^2 x\cos x + 12\sin x\cos x$
$= 6\sin x\cos x(2\sin^2 x + 5\sin x + 2)$
$= 6\sin x\cos x(2\sin x + 1)(\sin x + 2) = 0$
$\Rightarrow x = -\frac{\pi}{6}, 0, \frac{\pi}{2}$



Decreasing in $\left(-\frac{\pi}{6}, 0\right)$

40. (3) Given equation $x^5(x^3 - x^2 - x + 1) + x(3x^3 - 4x^2 - 2x + 4) - 1 = 0$

$\Rightarrow x^5(x^2 - 1)(x - 1) + (x^2 - 1)(x - 1)(3x - 1) = 0$

$\Rightarrow (x^2 - 1)(x - 1)(x^5 + 3x - 1) = 0$

$\Rightarrow (x - 1)^2(x + 1)(x^5 + 3x - 1) = 0$

Take, $(x - 1)^2 = 0, x = 1, x + 1 = 0, x = -1,$

$f(x) = x^5 + 3x - 1$

$f'(x) = 5x^4 + 3 = 0$

$x^4 = \frac{-3}{5},$ ⬚ so, it is imaginary root.

Therefore, three distinct roots are 1, 1, and -1.

41. (45)Given function $f(x) = 2x^2 - \log_e x$.

differentiate w.r.t.x.

$$f'(x) = 4x - \frac{1}{x}$$

Take $f'(x) = 0$

$\Rightarrow 4x - \frac{1}{x} = 0 \Rightarrow 4x^2 - 1 = 0 \Rightarrow x = \pm\frac{1}{2}$.

According to given interval $(0, a)$ and $(a, 4)$, a must be positive.

Then, $x = \frac{1}{2}$. So, $a = \frac{1}{2}$.

Let point $p(h, k)$ on parabola $y^2 = 4ax$.

point $(8a, 8a - 1) \to \left(8 \times \frac{1}{2}, 8 \times \frac{1}{2} - 1\right) \to (4,3)$

$\frac{1}{k} = \frac{3 - k}{4 - h} \Rightarrow k^2 - 6k + 8 = 0$

$k = 2,4$

satisfy point $p(h, k)$ on equation of parabola $k^2 = 2h$

So, $p(h, k) \to (8,4)$

Equation of normal at $p(y - 4) = -4(x - 8)$

$\Rightarrow y - 4 = -4x + 32 \Rightarrow 4x + y = 36$

$\Rightarrow \frac{x}{9} + \frac{y}{36} = 1$

Here, $\alpha = 9, \beta = 36$

So, $\alpha + \beta = 9 + 36 = 45$

42. (c) Given function $f(x) = \begin{cases} x^3 - x^2 + 10x - 7, & x \leq 1 \\ -2x + \log_2(b^2 - 4), & x > 1 \end{cases}$

Put $x = 1$, then $f(x) = 1 - 1 + 10 - 7 = 3$

For $x < 1, f(x) = x^3 - x^2 + 10x - 7$

$f'(x) = 3x^2 - 2x + 10$

When put any value then $f'(x) > 0$ and it is increasing similarly, for $x > 1, f(x) = -2x + \log_2(b^2 - 4)$

$f'(x) = -2 < 0$

it is decreasing

so, $-2 + \log_2(b^2 - 4) \leq 3$

$\log_2(b^2 - 4) \leq 5$

$b^2 - 4 \leq 32$

$b^2 \leq 36$
$\mathrm{b} \leq \pm 6$
Required interval of b is $[(-6,-2) \cup (2,6)]$.
43. (11) Let $f(x)=(x+1)(ax+b)$
$1=2a+2b$
$f'(x)=(ax+b)+a(x+1)$
$1=(3a+b)$
$\Rightarrow b=1/4, a=1/4$
$f(x)=\dfrac{(x+1)^2}{4} \Rightarrow f'(x)=\dfrac{x}{2}+\dfrac{1}{2}$
At $(\alpha, \alpha+1)$,
$\alpha+1=\dfrac{(\alpha+1)^2}{4}, \alpha>-1$
$\alpha+1=4 \Rightarrow \alpha=3$
normal at $(3,4), \mathrm{m}=2y-4=-\frac{1}{2}(x-3) \Rightarrow 2y+x=11$
at $\mathrm{y}=0, \mathrm{x}=$ intercept $=11$
44. (a) Let $f(x)=x-\sin\ 2x+\frac{1}{3}\sin\ 3x$
For max; $f'(x)=0$
$f'(x)=1-2\cos\ 2x+\cos\ 3x=0 \Rightarrow x=\dfrac{5\pi}{6}, \dfrac{\pi}{6}$
$\therefore f''(x)=4\sin\ 2x-3\sin\ 3x$
$f''\left(\frac{5\pi}{6}\right)<0 \Rightarrow \left(\frac{5\pi}{6}\right)$ is point of maxima
And $f''\left(\frac{\pi}{6}\right)>0$. So $\frac{\pi}{6}$ is point of minima
Now, $f\left(\frac{5\pi}{6}\right)=\frac{5\pi}{6}+\frac{\sqrt{3}}{2}+\frac{1}{3}$

EXERCISE - 3

1. (c) Given that $g(u)=2\tan^{-1}(e^u)-\frac{\pi}{2}$
$\therefore g(-u)=2\tan^{-1}(e^{-u})-\dfrac{\pi}{2}=2\tan^{-1}\left(\dfrac{1}{e^u}\right)-\dfrac{\pi}{2}$
$=2\cot^{-1}(e^u)-\dfrac{\pi}{2}=2\left[\dfrac{\pi}{2}-\tan^{-1}(e^u)\right]-\dfrac{\pi}{2}$
$=\pi-2\tan^{-1}(e^u)-\dfrac{\pi}{2}=\dfrac{\pi}{2}-2\tan^{-1}(e^u)=-g(u)$
$\therefore g$ is an odd function.
Also $g'(u)=\frac{2e^u}{1+e^{2u}}>0, \forall u \in(-\infty, \infty)$
$\therefore g$ is strictly increasing on $(-\infty, \infty)$.
2. (c) Let, $f(x)=\sum_{i=1}^{n}(x-a_i)^2$
$\Rightarrow f'(x)=\sum_{i=1}^{n} 2(x-a_i)=2nx-(a_1+a_2 \ldots \ldots+a_n)$
$=2nx-2nA$ where $\mathrm{A}=\frac{a_1+a_2+\cdots+a_n}{n}$
$f'(x)=0 \Rightarrow x=\mathrm{A}$. Again $f''(x)=2n>0$

3. (b) Obviosuly, f is increasing and g is decreasing in R. Hence,
$f\left(g(\alpha^2-2\alpha)\right) > f(g(3\alpha-4))$
or $g(\alpha^2-2\alpha) > g(3\alpha-4)$
($\because f$ is increasing)
or $\alpha^2-2\alpha < 3\alpha-4$ (As g is decreasing)
or $\alpha^2-5\alpha+4<0$ or $(\alpha-1)(\alpha-4)<0$ or $\alpha\in(1,4)$

4. (b) Let $f(x)=ax^3+bx^2+cx+d$
Then, $f(2)=18 \Rightarrow 8a+4b+2c+d=18$
$f(1)=-1 \Rightarrow a+b+c+d=-1$
$f(x)$ has local max. at $x=-1$
$\Rightarrow 3a-2b+c=0$
$f'(x)$ has local min. at $x=0 \Rightarrow b=0$
Solving (i), (ii), (iii) and (iv), we get
$$f(x)=\frac{1}{4}(19x^3-57x+34) \Rightarrow f(0)=\frac{17}{2}$$
Also $f'(x)=\frac{57}{4}(x^2-1)>0, \forall x>1$
Also $f'(x)=0 \Rightarrow x=1,-1$
$f''(-1)<0, f''(1)>0 \Rightarrow x=-1$ is a point of local max. and $x=1$ is a point of local min. Distance between $(-1,2)$ and $(1,f(1))$, i.e. $(1,-1)$ is $\sqrt{13}\neq 2\sqrt{5}$

5. (c) $\frac{dy}{dx}=x^2$.
Equation of tangent at $P_1(x_1,y_1)$ is $y-y_1=x_1^2(x_2-x_1)$
$\Rightarrow x_2^3-x_1^3=3x_1^2(x_2-x_1) \Rightarrow x_2^2+x_1^2+x_1x_2=3x_1^2$
$\Rightarrow 2x_1^2-x_1x_2-x_2^2=0 \Rightarrow x_2=-2x_1$
Similarly, $x_3=-2x_2=4x_1$
$x_4=-2x_3=-8x_1$
$x_{2n}=-2^{2n-1}x_1$
$3(y_1+y_2+y_3 \ldots\ldots..+y_{2n})$
$=(x_1^3+x_2^3+x_3^3+\cdots\ldots..+x_{2n}{}^3)+2n$
$$=x_1^3\frac{((-8)^{2n}-1)}{-8-1}+2n=-\frac{x_1^3}{9}(8^{2n}-1)+2n$$
i.e. $y_1+y_2+y_3+\cdots.+y_{2n}=-\left(\frac{x_1}{3}\right)^3(2^{6n}-1)+\left(\frac{2n}{3}\right)$,
Now put $n=30$ and $x_1=2$ then
$$y_1+y_2+y_3+\cdots..+y_{60}=-\left(\frac{2}{3}\right)^3(2^{180}-1)+20$$
$\Rightarrow S=-\frac{2^{183}-8}{27}+20$ or $S+\left(\frac{2^{183}-8}{27}\right)=20$
$\Rightarrow 5\mathrm{k}=20 \Rightarrow \mathrm{k}=4$

6. (c) Given, $y=3\sin\,\theta\cdot\cos\,\theta$
$$\frac{dy}{d\theta}=3[\sin\,\theta(-\sin\,\theta)+\cos\,\theta(\cos\,\theta)]$$
$$\frac{dy}{d\theta}=3[\cos^2\,\theta-\sin^2\,\theta]=3\cos\,2\theta$$
and $x=\mathrm{e}^{\theta}\sin\,\theta \Rightarrow \frac{dx}{d\theta}=e^{\theta}\cos\,\theta+\sin\,\theta e^{\theta}$

$$\frac{dx}{d\theta}=e^{\theta}(\sin\ \theta+\cos\ \theta)$$

Dividing (i) by (ii) $\frac{dy}{dx}=\frac{3\cos\ 2\theta}{e^{\theta}(\sin\ \theta+\cos\ \theta)}=\frac{3(\cos^2\ \theta-\sin^2\ \theta)}{e^{\theta}(\sin\ \theta+\cos\ \theta)}$

$$\frac{dy}{dx}=\frac{3(\cos\ \theta+\sin\ \theta)(\cos\ \theta-\sin\ \theta)}{e^{\theta}(\sin\ \theta+\cos\ \theta)}$$

$$\frac{dy^2}{dx}=\frac{3(\cos\ \theta-\sin\ \theta)}{e^{\theta}}$$

Given tangent is parallel to x-axis then $\frac{dy}{dx}=0$

$$0=\frac{3(\cos\ \theta-\sin\ \theta)}{e^{\theta}}$$

or $\cos\ \theta-\sin\ \theta=0\Rightarrow\cos\ \theta=\sin\ \theta$

$$\Rightarrow\tan\ \theta=1\Rightarrow\tan\ \theta=\frac{\tan\ \pi}{4}\Rightarrow\theta=\frac{\pi}{4}$$

7. (c) $f(x)=(\sin^2\ x-1)^n(2+\cos^2\ x)$

$f'(x)=n(\sin^2\ x-1)^{n-1}\cdot\sin\ 2x(2+\cos^2\ x)$

$+(\sin^2\ x-1)^n(-\sin\ 2x)$

$=(\sin^2\ x-1)^{n-1}\cdot\sin\ 2x[n(2+\cos^2\ x)-\sin^2\ x+1]$

$=(\sin^2\ x-1)^{n-1}\cdot\sin\ 2x[2n+(n+1)\cos^2\ x]$

Now $\sin^2\ x-1\leq 0\forall x$ and $2n+(n+1)\cos^2\ x>0\forall x$

$\therefore f'\left(\frac{\pi}{2}-h\right)>0$ and $f'\left(\frac{\pi}{2}+h\right)<0$ if $n-1$ is even

$\Rightarrow f(x)$ has local maxima at $x=\frac{\pi}{2}$ if n is odd

Again $f'\left(\frac{\pi}{2}-h\right)<0$ and $f'\left(\frac{\pi}{2}+h\right)>0$ if $n-1$ is odd $\Rightarrow f(x)$ has local minima at $x=\frac{\pi}{2}$ if n is even

8. (b) Consider the function $f(x)=\frac{x^2}{x^3+200}$ in the interval $[1,\infty)$.

Since the derivative $f'(x)=\frac{x(400-x^3)}{(x^3+200)^2}$ is positive at $0<x\sqrt[3]{400}$ and negative at $x>\sqrt[3]{400}$, the function $f(x)$ increases at $0<x<\sqrt[3]{400}<8$ it follows that the largest term in the sequence can be either a_7 or a_8. Since $a_7=49/534>a_8=8/89$, the largest term in the given sequence is $a_7=\frac{49}{543}$.

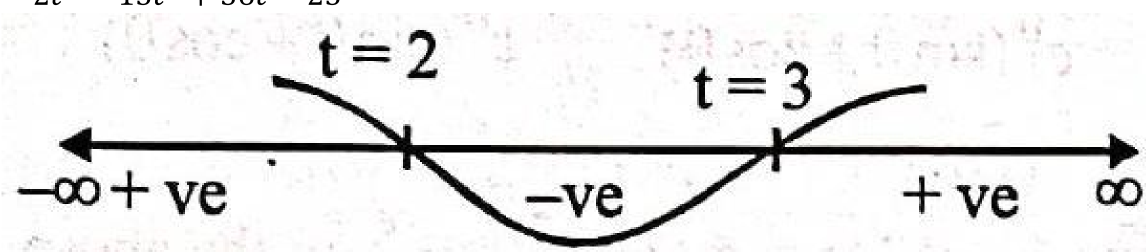
9. (c) $\lim_{x\to 0}\frac{f(x^2)-f(x)}{f(x)-f(0)}$

Using L.H. Rule

$$=\lim_{x\to 0}\frac{f'(x^2)\cdot 2x-f'(x)}{f'(x)}=\lim_{x\to 0}\frac{2xf'(x^2)}{f'(x)}-1=0-1=-1$$

10. (d)

Let $g(t)=2t^3-15t^2+36t-25$



$$g'(t) = 6t^2 - 30t + 36 = 6(t^2 - 5t + 6)$$
$$= 6(t-2)(t-3) = 0 \Rightarrow t = 2,3$$

For $2 \leq t \leq 4$

$$g(t)_{\min} = g(3) = 2 \times 27 - 15 \times 9 + 36 \times 3 - 25 = 2$$

Also $2 + |\sin\ t| \geq 2$

Henceminimum $\phi(t) = 2$

$\therefore 3^x + 3^{f(x)} = 2 \Rightarrow 3^{f(x)} = 2 - 3^x$

$\Rightarrow 3^{f(x)} > 0 \Rightarrow 2 - 3^x > 0 \Rightarrow 3^x < 2$

$\Rightarrow x < \log_3\ 2 \therefore x \in (-\infty, \log_3\ 2)$

11. (b) If $f(x) = x^{1/x}$, then $f'(x) = \frac{1}{x^2}\left[x^{1/x}(1 - \ln\ x)\right]$

f is decreasing if $x > e$ and f is increasing if $x < e$.

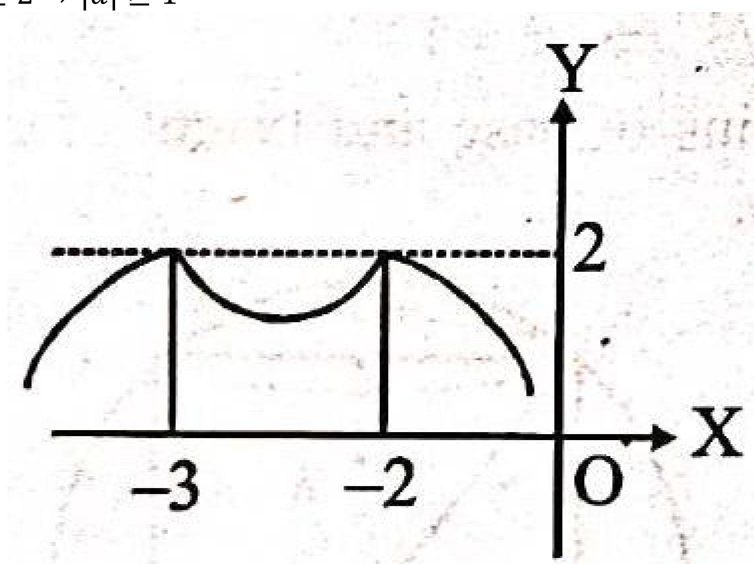
As $e < 3 < 4 < 5 < 6 < 7$

$\therefore \text{Max}\left\{3^{1/3}, 4^{1/4}, 5^{1/5}, 6^{1/6}, 7^{1/7}\right\} = 3^{1/3}$

Also $1 < 2 < e \therefore \text{Max}\left\{1, 2^{1/2}\right\} = 2^{1/2}$

But $2^{1/2} = 4^{1/4} < 3^{1/3} \therefore$ the greatest number is $3^{1/3}$

12. (a) The graph of $y = 2 - |x^2 + 5x + 6|$ is drawn in the adjacent figure. Clearly $f(x)$ will have maxima at $x = -2$ only if $a^2 + 1 \geq 2 \Rightarrow |a| \geq 1$



13. (b) $f'(x) > 0$ if $x \geq 0$ and $g'(x) < 0$ if $x \geq 0$

Let $h(x) = f(g(x))$ then $h'(x) = f'(g(x)) \cdot g'(x) < 0$ if $x \geq 0$

$\therefore$ h(x) is decreasing function

$\therefore \text{h(x)} \leq \text{h(0)}$ if $\text{x} \geq 0$

$\therefore \text{f(g(x))} \leq \text{f(g(0))} = 0$

But codomain of each function is $[0, \infty)$

$\therefore \text{f(g(x))} = 0$ for all $\text{x} \geq 0$

$\therefore \text{f(g(x))} = 0$

Also $\text{g(f(x)} \leq \text{g(f(0))}$ [as above]

14. (d) r must be an even integer because two decreasing are required to make it increasing function.

Let $y = r(n - r)$

When n is odd, $r = \frac{n-1}{2}$ or $\frac{n+1}{2}$ for maximum value of y When n is even, $r = \frac{n}{2}$ for maximum value of y

$\therefore$ Maximum $(y) = \frac{n^2-1}{4}$ when n is odd and $\frac{n^2}{4}$ when n is even.

15. (b) $f(x) = (a^2 - 3a + 2)(\cos^2\, x/4 - \sin^2\, x/4) + (a-1)x + \sin\, 1$

$\Rightarrow f(x) = (a-1)(a-2)\cos\, x/2 + (a-1)x + \sin\, 1$

$\Rightarrow f'(x) = -\frac{1}{2}(a-1)(a-2)\sin\frac{x}{2} + (a-1)$

$\Rightarrow f'(x) = (a-1)\left[1 - \frac{(a-2)}{2}\sin\frac{x}{2}\right]$

If $f(x)$ does not possess critical points, then $f'(x) \neq 0$ for any $x \in \mathbf{R}$

$\Rightarrow (a-1)\left[1 - \frac{(a-2)}{2}\sin\frac{x}{2}\right] \neq 0$ for any $x \in \mathbf{R}$

$\Rightarrow a \neq 1$ and $1 - \left(\frac{a-2}{2}\right)\sin\frac{x}{2} = 0$

must not have any solution in $\mathbf{R}$.

$\Rightarrow a \neq 1$ and $\sin\frac{x}{2} = \frac{2}{a-2}$ is not solvable in R

$\Rightarrow a \neq 1$ and $\left|\frac{2}{a-2}\right| > 1 \begin{bmatrix} \text{For } a = 2, f(x) = x + \sin\, 1 \\ \therefore f'(x) = 1 \neq 0 \end{bmatrix}$

$\Rightarrow a \neq 1$ and $|a-2| < 2 \Rightarrow a \neq 1$ and $-2 < a - 2 < 2$

$\Rightarrow a \neq 1$ and $0 < a < 4 \Rightarrow a \in (0,1) \cup (1,4)$.

EXERCISE - 4

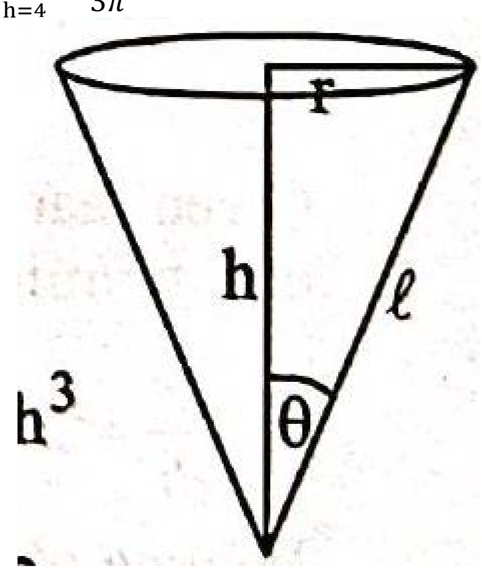
1. (5) Given a cone with angle $\tan^{-1}\frac{3}{4}$

$\tan\, \theta = \frac{3}{4} = \frac{r}{h}$

$\frac{dV}{dt} = 6$

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi h^3 \tan^2\, \theta = \frac{9\pi}{48}h^3 = \frac{3\pi}{16}h^3$

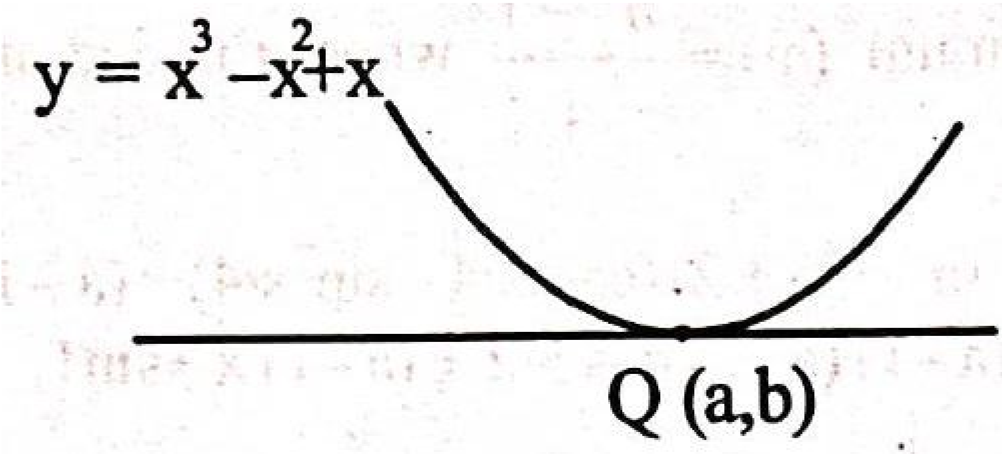
$\Rightarrow \frac{dv}{dt} = \frac{3\pi}{16} \cdot 3h^2 \cdot \frac{dh}{dt} = 6 \Rightarrow \left(\frac{dh}{dt}\right)_{h=4} = \frac{2}{3\pi}$ m/hr



Now, $S = \pi r\ell = \frac{15}{16}\pi h^2$
Differentiate w.r.t. both sides.

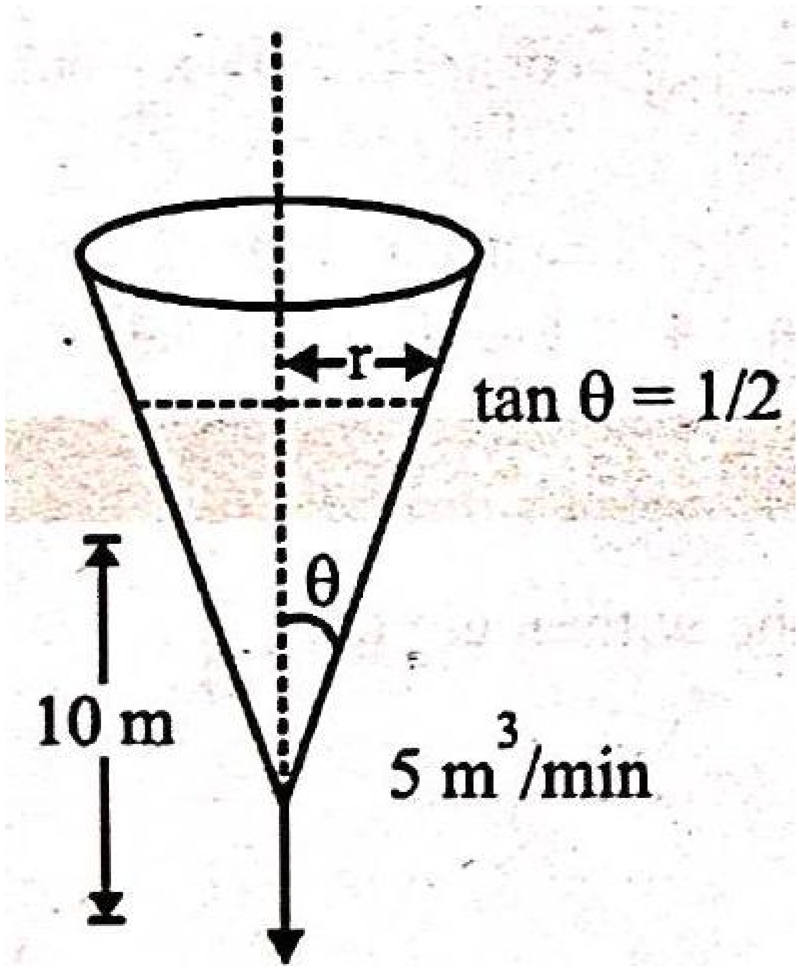
$$\Rightarrow \frac{dS}{dt} = \frac{15\pi}{16} \cdot 2h\frac{dh}{dt}$$
$$\Rightarrow \left(\frac{dS}{dt}\right)_{h=4} = 5m^2/hr$$

2. (195) Given function is
$y = 5x^2 + 2x - 25$
Diff. w.r.t. x both sides, $y' = 10x + 2$
Satisfy point $P(2, -1)$
$y_p' = 22$
Therefore, tangent to curve at P
$y + 1 = 22(x - 2)$
$y = 22x - 45$



$\left.\frac{dy}{dx}\right|_{C_2} = 3x^2 - 2x + 1$
$\left.\frac{dy}{dx}\right|_{Q} = 3a^2 - 2a + 1$
Hence, $3a^2 - 2a + 1 = 22$
Hence, $3a^2 - 2a - 21 = 0$
$3a^2 - 9a + 7a - 21 = 0$
$(3a + 7)(a - 3) = 0$
Here, $a = \frac{-7}{3}, 3$
Here, curve is $b = a^3 - a^2 + a$
$a = 3, b = 21$
$|2a + 9b| = 195$
at $a = -7/3$ tangent will be parallel
Therefore, it is rejected.

3. $(\mathbf{0.2})$



Given that water is poured into the tank at a constant rate of $5\mathrm{m}^3/$ minute.

$\therefore \dfrac{dv}{dt} = 5\mathrm{m}^3/\mathrm{min}$

Volume of the tank is,

$V = \dfrac{1}{3}\pi r^2 h$

where r is radius and h is height at any time.

By the diagram,

$$\tan\ \theta = \frac{r}{h} = \frac{1}{2}$$

Differentiate eq. (i) w.r.t. ' t ', we get

$\dfrac{dV}{dt} = \dfrac{1}{3}\left(\pi 2r\dfrac{dr}{dt}h + \pi r^2\dfrac{dh}{dt}\right)$

Putting $h = 10, r = 5$ and $\frac{dV}{dt} = 5$ in the above equation.

$5 = \frac{75\pi}{3}\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{1}{5\pi}\mathrm{m/min}.$

$\Rightarrow \dfrac{k}{\pi} = \dfrac{1}{5\pi} \Rightarrow k = \dfrac{1}{5} = 0.2$

4. (1) $\because$ Slope of the tangent $= \frac{x^2-2y}{x}$

$\because \frac{dy}{dx} = \frac{x^2 - 2y}{x}$

$\frac{dy}{dx} + \frac{2}{x}y = x$

I.F. $= e^{\int \frac{2}{x}dx} = e^{2\ln x} = x^2$

Solution of equation $y \cdot x^2 = \int x \cdot x^2 dx$

$x^2 y = \frac{x^4}{4} + C$

$\therefore$ ⬚ curve passes through point $(1, -2)$

$$(1)^2(-2) = \frac{1^4}{4} + C \Rightarrow C = \frac{-9}{4}$$

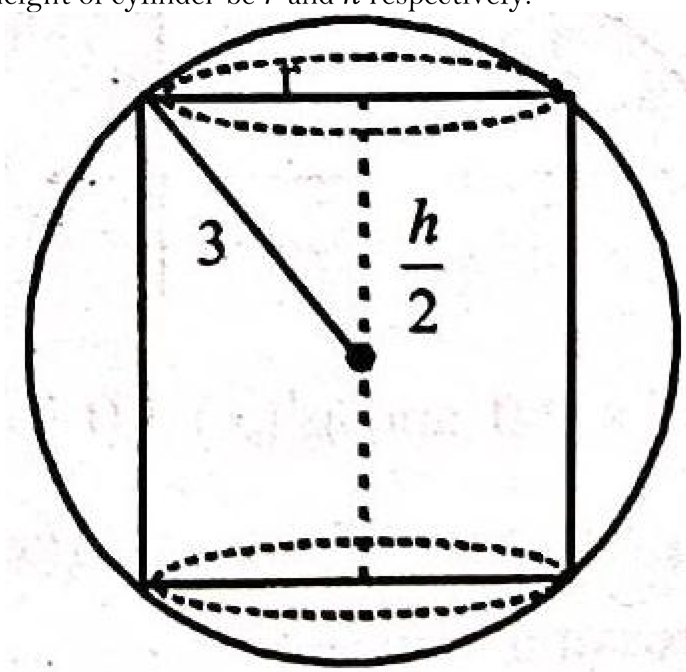
Then, equation of curve $y = \frac{x^2}{4} - \frac{9}{4x^2}$

Since, above curve satisfies the point.

Hence, the curve passes through $(\sqrt{3}, 0)$.

$\Rightarrow \sqrt{3}a = \sqrt{3} \Rightarrow a = 1$

5. (2) Let radius of base and height of cylinder be r and h respectively.



$\therefore r^2 + \frac{h^2}{4} = 9$

Now, volume of cylinder, $V = \pi r^2 h$

Substitute the value of r^2 from equation (i),

$V = \pi h \left(9 - \frac{h^2}{4}\right) \Rightarrow V = 9\pi h - \frac{\pi}{4}h^3$

Differentiating w.r.t. h,

$\frac{dV}{dh} = 9\pi - \frac{3}{4}\pi h^2$

For maxima/minima,

$\frac{dV}{dh} = 0 \Rightarrow h = \sqrt{12}$

and $\frac{d^2V}{dh^2} = -\frac{3}{2}\pi h$

$\therefore \left(\frac{d^2V}{dh^2}\right)_{h=\sqrt{12}} < 0$

$\Rightarrow$ Volume is maximum when $h = 2\sqrt{3}$

$\Rightarrow m\sqrt{3} = 2\sqrt{3} \Rightarrow m = 2$

6. (2) $\frac{x}{2} + \frac{2}{x}$ is of the form $y + \frac{1}{y}$ where $y + \frac{1}{y} \geq 2$ and equality holds for $y = 1$

$\therefore$ Min value of function occurs at $\frac{x}{2} = 1$ i.e., at $x = 2$

7. (19) Given curve is, $y = \frac{x}{x^2-3}$

$\Rightarrow \frac{dy}{dx} = \frac{(x^2-3) - x(2x)}{(x^2-3)^2} = \frac{-x^2-3}{(x^2-3)^2}$

$\left.\frac{dy}{dx}\right|_{(\alpha,\beta)} = \frac{\alpha^2-3}{(\alpha^2-3)^2} = -\frac{2}{6} = -\frac{1}{3}$

$3(\alpha^2+3) = (\alpha^2-3)^2 \Rightarrow \alpha^2 = 9.$

And, $\beta = \frac{x}{\alpha^2-3} \Rightarrow \alpha^2 - 3 = \frac{\alpha}{\beta} \Rightarrow \frac{\alpha}{\beta} = 6$

$\Rightarrow a = \pm 3, \beta = \pm\frac{1}{2}$

8. (2) We have

$$y^5 - 9xy + 2x = 0$$

now $y' = 5y^4\frac{dy}{x} - 9x\frac{dy}{dx} - 9y + 2 = 0$

or $\frac{dy}{dx}(5y^4 - 9x) = 9y - 2$

$\frac{dy}{dx} = \frac{9y-2}{5y^4-9x} = 0$ (for horizontal tangent)

$y = \frac{2}{9} \Rightarrow$ Which does not satisfy the original equation $\mathrm{M} = \mathbf{0}$.

Now $5y^4 - 9x = 0$ (for vertical tangent)

$5y^4(9y-2) - 9y^5 = 0$

$y^4[45y - 10 - 9y] = 0$

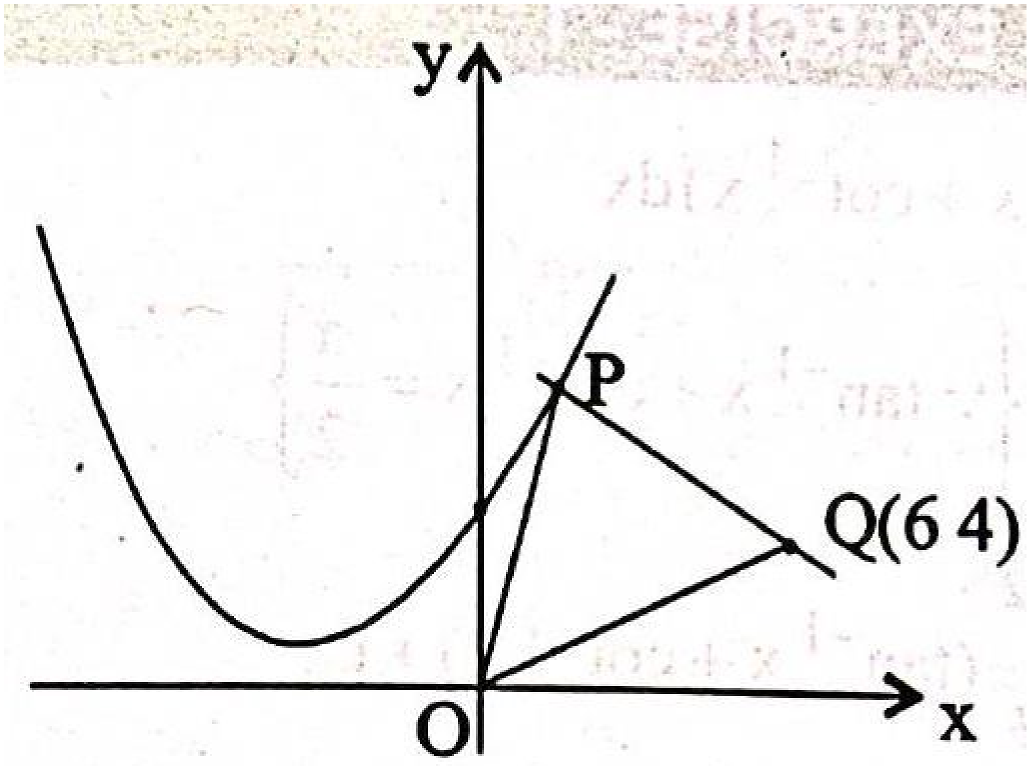
$\mathrm{y} = 0$ (or) $36\mathrm{y} = 10y = \frac{5}{18}$

$y = 0 \Rightarrow x = 0 \& y = \frac{5}{18} \Rightarrow (0,0)$ and $\left(x, \frac{5}{18}\right)$

$\mathrm{N} = 2$ and $\mathrm{M} + \mathrm{N} = 2$

9. (13) Given curve is

$y = 2x^2 + x + 2$



Differentiate w.r.t.x,

$$\frac{dy}{dx} = 4x + 1$$

Assume, loordinate P(h, k)

Equation of line PQ is,

$$y - k = -\frac{1}{4h+1}(x - h)$$

Satisfy, $Q(6,4)$ in above equation.

$$\therefore 4 - k = -\frac{1}{4h+1}(6 - h)$$

$$\Rightarrow (4h+1)(4-k) + 6 - 4 = 0$$

Given curve also passes through point P(h, k).

Now, satisfy point P. Then $k = 2h^2 + h + 2$

Put the value of k in eq. (i)

$$(4h+1)(4 - 2h^2 - h - 2) + 6 + \text{h} = 0$$

$$\Rightarrow 4h^3 - 3h^2 + 3h - 8 = 0$$

$$\Rightarrow \text{h} = 1, k = 5$$

Here, Point P is $(1,5)$

10.

Now area of $\triangle$ OPQ will be $= \frac{1}{2}\begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 5 \\ 1 & 6 & 4 \end{vmatrix} = 13$

(4) $\because y^3 = 8x \Rightarrow 3y^2\frac{dy}{dx} = 8 \Rightarrow \frac{dy}{dx} = \frac{8}{3y^2}$

Also, $y^2 = 12 - \frac{12x^2}{a^2} \Rightarrow 2y\frac{dy}{dx} = \frac{-24x}{a^2}$

$$\Rightarrow \frac{dy}{dx} = \frac{-12x}{a^2 y}$$

$\therefore \left(\frac{-12x}{a^2 y}\right)\left(\frac{8}{3y^2}\right) = -1$ [∵ Intersect at right angles]

$\Rightarrow a^2 y^3 = 32x \Rightarrow a^2 = 4$ [$\because y^3 = 8x$]