CHAPTER

8

BINOMIAL THEOREM

Exercise 1: NCERT Based Topic-wise MCQs

8.1 INTRODUCTION

1. Binomial theorem is useful for giving an easier way to expand of NCERT Page-160/N-126
(a) $(a+b)^{n}$
(b) $(1+x)^{n}$
(c) $(1-x)^{n}$
(d) All of the above

8.2 BINOMIAL THEOREM FOR POSITIVE INTEGRAL INDICES

2. The remainder when 3^{2022} is divided by 5 is NCERT Page-164/N-131
(a) 1
(b) 2
(c) 3
(d) 4

3. The number of terms in the expansion of $(1+x)^{101}(1+x^{2}-x)^{100}$ in powers of x is: NCERT Page-164/N-131
(a) 302
(b) 301
(c) 202
(d) 101

4. If $\left(2+\frac{x}{3}\right)^{55}$ is expanded in the ascending powers of x and the coefficients of powers of x in two consecutive terms of the expansion are equal, then these terms are: NCERT Page-168
(a) 7^{th} and 8^{th}
(b) 8^{th} and 9^{th}
(c) 28^{th} and 29^{th}
(d) 27^{th} and 28^{th}

5. The remainder when $(2021)^{2023}$ is divided by 7 is : NCERT Page-164/N-131
(a) 1
(b) 2
(c) 5
(d) 6

6. If $1 + x^4 + x^5 = \sum_{i=0}^{5} a_i (1 + x)^i$, for all x in R, then a_2 is: NCERT Page-176/N-133
(a) -4
(b) 6
(c) -8
(d) 10

7. The smallest natural number n, such that the coefficient of x in the expansion of $\left(x^2 + \frac{1}{x^3}\right)^n$ is ${}^nC_{23}$, is : NCERT Page-170
(a) 38
(b) 58
(c) 23
(d) 35

8. The remainder when $(11)^{1011} + (1011)^{11}$ is divided by 9 is NCERT Page-164/N-131
(a) 1
(b) 4
(c) 6
(d) 8

9. If the sum of the coefficients in the expansion of $(a + b)^n$ is 4096, then the greatest coefficient in the expansion is NCERT Page-168
(a) 1594
(b) 792
(c) 924
(d) 2924

10. If $(1 + x)^n = \sum_{r=0}^{n} a_r \cdot x^r$ and $b_r = 1 + \frac{a_r}{a_{r-1}}$ and $\prod_{r=1}^{n} b_r = \frac{(101)^{100}}{100!}$, then n is NCERT Page-164
(a) 99
(b) 100
(c) 101
(d) None of these

11. $\sqrt{5}\left[(\sqrt{5} + 1)^{50} - (\sqrt{5} - 1)^{50}\right]$ is NCERT Page-169/N-133
(a) an irrational number
(b) 0
(c) a natural number
(d) None of these

12. Expand by using binomial and find the degree of polynomial $\left(x + \sqrt{x^3 - 1}\right)^5 + \left(x - \sqrt{x^3 - 1}\right)^5$ is NCERT Page-168/N-133
(a) 7
(b) 6
(c) 5
(d) 4

13. The remainder when $(2021)^{2022}+(2022)^{2021}$ is divided by 7 is NCERT Page-164/N-132
(a) 0
(b) 1
(c) 2
(d) 6

14. Number of terms involving x^6 in the expansion of $\left(2x^2-\frac{3}{x}\right)^{11}, r \neq 0$, is NCERT Page-169
(a) 1
(b) 2
(c) 6
(d) 0

15. If the second, third and fourth terms in the expansion of $(a+b)^n$ are 135,30 and $10/3$ respectively, then the value of n is NCERT Page-169
(a) 6
(b) 5
(c) 4
(d) None of these

16. If ' n ' is positive integer and three consecutive coefficient in the expansion of $(1+x)^n$ are in the ratio $6:33:110$, then n is equal to : NCERT Page-168
(a) 9
(b) 6
(c) 12
(d) 16

17. The remainder when $7^{2022}+3^{2022}$ is divided by 5 is: NCERT Page-164/N-132
(a) 0
(b) 2
(c) 3
(d) 4

18. In the binomial expansion of $(a-b)^n, n \geq 5$ the sum of the 5 th and 6 th terms is zero.
Then a/b equals : NCERT Page-168
(a) $\frac{n-5}{6}$
(b) $\frac{n-4}{5}$
(c) $\frac{5}{n-4}$
(d) $\frac{6}{n-5}$

19. The coefficient of x^n in expansion of $(1+x)(1-x)^n$ is NCERT Page-169
(a) $(-1)^{n-1}n$
(b) $(-1)^n(1-n)$
(c) $(-1)^{n-1}(n-1)^2$
(d) $(n-1)$

20. If number of terms in the expansion of $(x-2y+3z)^n$ is 45 then $\mathrm{n}=$
(a) 7
(b) 8
(c) 9
(d) 6^{10}

21. If 7^9+9^7 is divided by 64 then the remainder is
(a) 0
(b) 1
(c) 2
(d) 63

22. The total number of terms in the expansion of $(x+a)^{51}-(x-a)^{51}$ after simplification is
NCERT Page-164/N-131
(a) 102
(b) 25
(c) 26
(d) None of these

23. The formula $(a+b)^m=a^m+ma^{m-1}b+\frac{m(m-1)}{1.2}a^{m-2}b^2+\cdots$ holds when NCERT Page-163/N-131
(a) b < a
(b) a < b
(c) $|a|<|b|$
(d) | b | < | a |

24. If $x=99^{50}+100^{50}$ and $y=(101)^{50}$ then NCERT Page-165/N-131
(a) $x=y$
(b) $x<y$
(c) $x>y$
(d) None of these

25. How many terms are present in the expansion of $\left(x^2+\frac{2}{x^2}\right)^{11}$? NCERT Page-164/N-131
(a) 11
(b) 12
(c) 10
(d) 11!

26. The minimum positive integral value of m such that $(1073)^{71}-\mathrm{m}$ may be divisible by 10 , is
NCERT Page-166/N-131
(a) 1
(b) 3
(c) 7
(d) 9

27. If ' n ' is positive integer and three consecutive coefficient in the expansion of $(1+x)^{\mathrm{n}}$ are in the ratio $6:33:110$, then n is equal to : NCERT Page-168

(a) 9
(b) 6
(c) 12
(d) 16

28. The last two digits of the number 3^{400} are NCERT Page-165/N-131
(a) 00
(b) 01
(c) 21
(d) 81

29. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \cdots .. + C_n x^n$, then value of $\frac{(C_0+C_1)(C_1+C_2)....(C_{n-1}+C_n)}{C_0 C_1 C_2 .... C_{n-1}}$ isNCERT Page-164
(a) $\frac{(n+3)^3}{(2n)!}$
(b) $\frac{(n+1)^n}{n!}$
(c) $\frac{(2n)!}{(n+1)!}$
(d) $\frac{(n-1)^n}{n!}$

30. The number of dissimilar terms in the expansion of $(a+b)^n$ is $n+1$, therefore number of dissimilar terms in the expansion of $(a+b+c)^{12}$ is NCERT Page-165
(a) 13
(b) 39
(c) 78
(d) 91

31. The approximation of $(0.99)^5$ using the first three terms of its expansion is NCERT Page-164/N-131
(a) 0.851
(b) 0.751
(c) 0.951
(d) None of these

32. The number of zero terms in the expansion of $(1+3\sqrt{2}x)^9 + (1-3\sqrt{2}x)^9$ is NCERT Page-163/N-133
(a) 2
(b) 3
(c) 4
(d) 5

33. Number of terms in the expansion of $(1+5\sqrt{2}x)^9 + (1-5\sqrt{2}x)^9$ is NCERT Page-163/N-133
(a) 2
(b) 3
(c) 4
(d) 5

34. After simplification, what is the number of terms in the expansion of $[(3x+y)^5]^4-[(3x-y)^4]^5$? NCERT Page-164/N-133
(a) 4
(b) 5
(c) 10
(d) 11

35. The last digit in 7^{300} is : NCERT Page-164/N-133
(a) 7
(b) 9
(c) 1
(d) 3

8.3 GENERAL AND MIDDLE TERMS

36. The coefficient of x^{101} in the expression $(5+x)^{500}+x(5+x)^{499}+x^2(5+x)^{498}+\cdots x^{500}, x>0$, is NCERT Page-168
(a) ${}^{501}\dot{C}_{101}(5)^{399}$
(c) ${}^{501}C_{100}(5)^{400}$
(b) ${}^{501}C_{101}(5)^{400}$
(d) ${}^{500}C_{101}(5)^{399}$

37. For two positive real numbers a and b such that $\frac{1}{a^2}+\frac{1}{b^3}=4$, the minimum value of the constant term in the expansion of $\left(ax^{\frac{1}{8}}+bx^{-\frac{1}{12}}\right)^{10}$ is : NCERT Page-168
(a) $\frac{105}{2}$
(b) $\frac{105}{4}$
(c) $\frac{105}{8}$
(d) $\frac{105}{16}$

38. $\sum_{t=1}^{20}(r^2+1)(r!)$ is equal to : NCERT Page-170
(a) $22!-21!$
(b) $22!-2(21!)$
(c) $21!-2(20!)$
(d) $21!-20!$

39. If the constant term in the expansion of $\left(3x^3-2x^2+\frac{5}{x^5}\right)^{10}$ is $2^k\cdot l$, where l is an odd integer, then the value of k is equal to: NCERT Page-169
(a) 6
(b) 7
(c) 8
(d) 9

40. Let $n \geq 5$ bean integer. If $9^n - 8^n - 1 = 64\alpha$ and $6^n - 5^n - 1 = 25\beta$, then $\alpha - \beta$ is equal to
(a) $1 + {}^nC_2(8-5) + {}^nC_3(8^2 - 5^2) + \cdots + {}^nC_n(8^{n-1} - 5^{n-1})$ NCERT Page-170
(b) $1 + {}^nC_3(8-5) + {}^nC_4(8^2 - 5^2) + \cdots + {}^nC_n(8^{n-2} - 5^{n-2})$
(c) ${}^nC_3(8-5) + {}^nC_4(8^2 - 5^2) + \cdots + {}^nC_n(8^{n-2} - 5^{n-2})$
(d) ${}^nC_4(8-5) + {}^nC_5(8^2 - 5^2) + \cdots + {}^nC_n^n(8^{n-3} - 5^{n-3})$

41. If $\sum_{k=1}^{31} ({}^{31}C_k)({}^{31}C_{k-1}) - \sum_{k=1}^{30} ({}^{30}C_k)({}^{30}C_{k-1})$
$= \frac{\alpha(60!)}{(30!)(31!)}$, where $\alpha \in \mathbf{R}$, then the value of 16α is equal to NCERT Page-171
(a) 1411
(b) 1320
(c) 1615
(d) 1855

42. The number of ways to distribute 30 identical candies among four children C_1, C_2, C_3 and C_4 so that C_2 receives atleast 4 and atmost 7 candies, C_3 receives atleast 2 and atmost 6 candies, is equal to
(a) 205 NCERT Page-168
(b) 615
(c) 510
(d) 430

43. The term independent of x in the expression of $(1 - x^2 + 3x^3)\left(\frac{5}{2}x^3 - \frac{1}{5x^2}\right)^{11}, x \neq 0$ is NCERT Page-171
(a) $\frac{7}{40}$
(b) $\frac{33}{200}$
(c) $\frac{39}{200}$
(d) $\frac{11}{50}$

44. If the middle term in the expansion of $\left(\frac{1}{x} + x\sin x\right)^{10}$ equals to $7\frac{7}{8}$ then x is equal to $(n \in I$
(a) $2n\pi \pm \frac{\pi}{6}$ NCERT Page-168
(b) $n\pi + \frac{\pi}{6}$
(c) $n\pi + (-1)^n \frac{\pi}{6}$
(d) $n\pi + (-1)^n \frac{5\pi}{6}$

45. The middle term in the expansion of $\left(\frac{10}{x} + \frac{x}{10}\right)^{10}$ is NCERT Page-168
(a) ${}^{10}C_5$
(b) ${}^{10}C_6$
(c) ${}^{10}C_5 \frac{1}{x^{10}}$
(d) ${}^{10}C_5 x^{10}$

46. What is the middle term in the expansion of $\left(\frac{x\sqrt{y}}{3}-\frac{3}{y\sqrt{x}}\right)^{12}$? NCERT Page-169

(a) $C(12,7)x^{3}y^{-3}$
(c) $C(12,7)x^{-3}y^{3}$
(b) $C(12,6)x^{-3}y^{3}$
(d) $\mathrm{C}(12,6)\mathrm{x}^{3}\mathrm{y}^{-3}$

47. If x^{18} occurs in the rth term in the expansion of $\left(x^{4}+\frac{1}{x^{3}}\right)^{15}$,then what is the value of r? NCERT Page-168

(a) 3
(b) 5
(c) 7
(d) 9

48. If the coefficients of r^{th} and $(\mathrm{r}+1)^{\text{th}}$ terms in the expansion of $(3+7x)^{29}$ are equal, then the value of r is NCERT Page-169

(a) 31
(b) 11
(c) 18
(d) 21

49. If x^{4} occurs in the t^{th} term in the expansion of $\left(x^{4}+\frac{1}{x^{3}}\right)^{15}$, then the value of t is equal to NCERT Page-169

(a) 7
(b) 8
(c) 9
(d) 10

50. In the expansion of $(1+x)^{18}$, if the coefficients of $(2r+4)^{\text{th}}$ and $(r-2)^{\text{th}}$ terms are equal, then the value of r is : NCERT Page-168

(a) 12
(b) 10
(c) 8
(d) 6

51. Value of ' a ', if 17^{th} and 18^{th} terms in the expansion of $(2+a)^{50}$ are equal, is NCERT Page-168

(a) 1
(b) 2
(c) 3
(d) 4

52. One value of α for which the coefficients of the middle terms in the expansion of $(1+\alpha x)^{4}$ and $(1-\alpha x)^{6}$ are equal, is $\frac{-3}{10}$. Other value of ' α ' is NCERT Page-168

(a) 0
(b) 1
(c) 2
(d) 3

53. What is the coefficient of x^3 in $\frac{(3-2x)}{(1+3x)^3}$? NCERT Page-168

(a) -272
(b) -540
(c) -870
(d) -918

54. If the coefficients of x^7 and x^8 in $\left(2+\frac{x}{3}\right)^n$ are equal, then

(a) 56
(b) 55
(c) 45
(d) 15

55. The coefficient of the term independent of x in the expansion of $\left(\sqrt{\frac{x}{3}}+\frac{3}{2x^2}\right)^{10}$ is NCERT Page-169

(a) $5/4$
(c) $9/4$
(b) $7/4$
(d) None of these

56. In the expansion of $\left(x+\frac{2}{x^2}\right)^{15}$, the term independent of x is : NCERT Page-171

(a) $^{15}C_6 . 26$
(b) $^{15}C_5 \cdot 2^5$
(c) $^{15}C_4 \cdot 2^4$
(d) None of these

57. If in the binomial expansion of $(1+x)^n$ where n is a natural number, the coefficients of the 5th, 6th and 7th terms are in A.P., then n is equal to: NCERT Page-168

(a) 7 or 13
(b) 7 or 14
(c) 7 or 15
(d) 7 or 17

58. What is the coefficient of x^3y^4 in $(2x+3y^2)^5$? NCERT Page-168

(a) 240
(b) 360
(c) 720
(d) 1080

59. What are the values of k if the term independent of x in the expansion of $\left(\sqrt{x}+\frac{k}{x^2}\right)^{10}$ is 405 ?

(a) ± 3 NCERT Page-171
(b) ± 6
(c) ± 5
(d) ± 4

60. The term independent of x in the expansion of $\left(\sqrt[6]{x}-\frac{1}{\sqrt[3]{x}}\right)^{9}$ is NCERT Page-171

(a) $-{}^{9}C_3$
(b) $-{}^{9}C_4$
(c) $-{}^{9}C_5$
(d) $-{}^{8}C_3$

61. The number of real negative terms in the binomial expansion of $(1+ix)^{4n-2}, n \in N, x>0$ is

(a) n
(b) $n+1$
(c) $n-1$
(d) $2n$

62. Coefficient of x^{13} in the expansion of $(1-x)^5(1+x+x^2+x^3)^4$ is NCERT Page-173

(a) 4
(b) 6
(c) 32
(d) 5

63. A positive value of m for which the coefficient of x^2 in the expansion $(1+x)^m$ is 6 , is

(a) 3
(c) 0
(b) 4
(d) None of these

64. If the coefficient of x in $(x^2+k/x)^5$ is 270 , then the value of k is NCERT Page-173

(a) 2
(b) 3
(c) 4
(d) 5

65. If $(1+x)^{2n}=a_0+a_1x+a_2x^2+\cdots..+a_{2n}x^{2n}$, then NCERT Page-173

(a) $a_0+a_2+a_4+\cdots.=\frac{1}{2}(a_0+a_1+a_2+a_3+\cdots.)$
(b) $a_{n+1}<a_n$
(c) $a_{n-3}=a_{n+3}$
(d) All of these

66. If the fourth term in the expansion of $\left(ax+\frac{1}{x}\right)^{n}$ is $\frac{5}{2}$, then the value of $a\times n$ is NCERT Page-173

(a) 2
(b) 6
(c) 3
(d) 4

67. The term independent of x in the expansion of $\left(9x-\frac{1}{3\sqrt{x}}\right)^{18}, x>0$, is ' a ' times the corresponding binomial coefficient. Then ' a ' is NCERT Page-171

(a) 3
(b) $1/3$
(c) $-1/3$
(d) None of these

68. The coefficient of x^5 in $(1+2x+3x^2+\cdots)^{-7/2}$ is

(a) 15
(b) 21
(c) 12
(d) 30

69. If x is positive, the first negative term in the expansion of $(1+x)^{27/5}$ is NCERT Page-173

(a) 6th term
(b) 7 th term
(c) 5 th term
(d) 8th term

70. If $a_n=2n+1$ and $C_r={}^nC_r$ then $a_0C_0^2+a_1C_1^2+a_2C_2^2+\cdots\ldots\ldots\ldots a_nC_n^2=$ NCERT Page-168

(a) $(n-1)({}^{2n}C_n)$
(b) $n({}^{2n}C_n)$
(c) $(n+1)({}^{2n}C_n)$
(d) $(n+1)\left({}^nC_{n/2}\right)$

71. The value of ${}^{50}C_4+\sum_{r=1}^{6}{}^{56-r}C_3$ is NCERT Page-172

(a) ${}^{55}C_4$
(b) ${}^{55}C_3$
(c) ${}^{56}C_3$
(d) ${}^{56}C_4$

72. In the expansion of $(1+x)^{50}$, the sum of the coefficients of odd powers of x is : NCERT Page-164

(a) 0
(b) 2^{49}
(c) 2^{50}
(d) 2^{51}

73. Value of $\sum_{r=1}^{10} r\cdot\frac{{}^nC_r}{{}^nC_{r-1}}$ is NCERT Page-168

(a) $10n-45$ (b) $10n+45$ (c) $10n-35$ (d) $10n^2-35$

Exercise 2 : NCERT Exemplar & JEE Main

NCERT EXEMPLAR QUESTIONS

1. The total number of temrs in the expansion of $(x+a)^{100}+(x-a)^{100}$ after simplification is
(a) 50 NCERT Page-171/N-133
(b) 202
(c) 51
(d) None of these

2. If the integers $r>1, n>2$ and coefficients of $(3r)^{\text{th}}$ and $(\mathrm{r}+2)^{\text{nd}}$ terms in binomial expansion of $(1+\mathrm{x})^{2\mathrm{n}}$ are equal then NCERT Page-164
(a) $\mathrm{n}=2\mathrm{r}$
(b) $n=3\mathrm{r}$
(c) $\mathrm{n}=2\mathrm{r}+1$
(d) none of these

3. The two successive temrs in the expansion of $(1+x)^{24}$ whose coefficients are in the ratio $1:4$ are
(a) 3^{rd} and 4^{th} NCERT Page-168
(b) 4^{th} and 5^{th}
(c) 5^{th} and 6^{th}
(d) 6^{th} and 7^{th}

4. The coefficients of x^n in the expansions of $(1+x)^{2n}$ and $(1+x)^{2n-1}$ are in the ratio NCERT Page-173
(a) $1:2$
(b) $1:3$
(c) $3:1$
(d) $2:1$

5. If the coefficients of 2^{nd}, 3^{rd} and the 4^{th} terms in the expansion of $(1+x)^n$ are in A.P., then value of n is NCERT Page-173
(a) 3
(b) 7
(c) 11
(d) 14

6. If A and B are coefficients of x^n in the expansion of $(1+x)^{2n}$ and $(1+x)^{2n-1}$ respectively, then $\frac{A}{B}$ equals NCERT Page-171
(a) 1
(b) 2

(c) $\frac{1}{2}$
(d) $\frac{1}{n}$

JEE MAIN

7. If the number of terms in the expansion of $\left(1-\frac{2}{x}+\frac{4}{x^2}\right)^n$, $x \neq 0$, is 28 , then the sum of the coefficients of all the terms in this expansion, is NCERT Page-173
(a) 243
(b) 729
(c) 64
(d) 2187

8. The value of $({}^{21}C_1 - {}^{10}C_1) + ({}^{21}C_2 - {}^{10}C_2) + ({}^{21}C_3 - {}^{10}C_3) + ({}^{21}C_4 - {}^{10}C_4) + \cdots . + ({}^{21}C_{10} - {}^{10}C_{10})$ is : NCERT Page-164
(a) $2^{20} - 2^{10}$
(b) $2^{21} - 2^{11}$
(c) $2^{21} - 2^{10}$
(d) $2^{20} - 2^{9}$

9. The sum of the co-efficients of all odd degree terms in the expansion of $\left(x+\sqrt{x^3-1}\right)^5 + \left(x-\sqrt{x^3-1}\right)^5, (x>1)$ is : NCERT & Page-165/N-133
(a) 0
(b) 1
(c) 2
(d) -1

10. The coefficient of t^4 in the expansion of $\left(\frac{1-t^6}{1-t}\right)^3$ NCERT Page-168
(a) 14
(b) 15
(c) 10
(d) 12

11. In the expansion of $\left(\frac{x}{\cos\theta}+\frac{1}{x\sin\theta}\right)^{16}$ if l_1 is the least value of the term independent of x when $\frac{\pi}{8} \leq \theta \leq \frac{\pi}{4}$ and l_2 is the least value of the term independent of x when $\frac{\pi}{16} \leq \theta \leq \frac{\pi}{8}$ then the ratio $l_2 : l_1$ is equal to :
NCERT Page-173
(a) $1:8$
(b) $16:1$
(c) $8:1$
(d) $1:16$

12. If $(2021)^{3762}$ is divided by 17 , then the remainder is NCERT Page-164/N-132

13.
$\sum_{\substack{i,j=0 \\ i\neq j}}^{n} {}^{n}C_i \, {}^{n}C_j$ is equal to
(a) $2^{2n} - {}^{2n}C_n$
(b) $2^{2n-1} - {}^{2n-1}C_{n-1}$
(c) $2^{2n} - \frac{1}{2}\,{}^{2n}C_n$
(d) $2^{2n-1} + {}^{2n-1}C_n$

14. If the coefficients of x and x^2 in the expansion of $(1+x)^p(1-x)^q$, $p, q \leq 15$, are -3 and -5 respectively, then the coefficient of x^3 is equal to NCERT Page-173

15. If the 1011^{th} term from the end in the binomial expansion of $\left(\frac{4x}{5} - \frac{5}{2x}\right)^{2022}$ is 1024 times 1011^{th} term from the beginning, then $|x|$ is equal to NCERT Page-169 |
(a) 12
(b) 8
(c) 10
(d) · 15

16. The value of
$\frac{1}{1!50!} + \frac{1}{3!48!} + \frac{1}{5!46!} + \cdots + \frac{1}{49!2!} + \frac{1}{51!1!}$ is :
(a) $\frac{2^{51}}{50!}$
(b) $\frac{2^{51}}{51!}$
(c) $\frac{2^{50}}{51!}$
(d) $\frac{2^{50}}{50!}$

Exercise 3 : Skill Enhancer MCQs

1. Let $N = 2^{1224} - 1, \alpha = 2^{153} + 2^{77} + 1$ and $\beta = 2^{408} - 2^{204} + 1$. Then which of the following statement is correct ?
(a) α divides N but β does not
(b) β divides N but α does not
(c) α and β both divides N
(d) neither α nor β divides N

2. If n is a positive integer and k is a positive integer not exceeding n, then
$\sum_{k=1}^{n} k^3 \left(\frac{C_k}{C_{k-1}}\right)^2$, where $C_k = {}^{n}C_k$, is
(a) $\frac{n(n+1)(n+2)}{12}$
(b) $\frac{n(n+1)^2(n+2)}{12}$

(c) $\frac{n(n+1)^2(n+2)}{6}$
(d) None of these

3. If I is integral part of $(2+\sqrt{3})^n$ and f is its fractional part. Then $(I+f)(1-f)$ is
(a) $I+1$
(b) 1
(c) n
(d) 2^n

4. If $x+\frac{1}{x}=1$ and $p=x^{1000}+\frac{1}{x^{1000}}$ and q be the digit at unit place in the number $2^{2^n}+1, n \in N$ and $n>1$, then $p+q=$
(a) 8
(b) 6
(c) 7
(d) 0

5. $\frac{\left[{}^nC_r+4.{}^nC_{r+1}+6.{}^nC_{r+2}+4.{}^nC_{r+3}+{}^nC_{r+4}\right]}{\left[{}^nC_r+3.{}^nC_{r+1}+3.{}^nC_{r+2}+{}^nC_{r+3}\right]}=\frac{n+\lambda}{r+\lambda}$ the value of λ is.
(a) 1
(b) 2
(c) 3
(d) 4

6. In the expansion of $\left(\frac{x}{\cos\theta}+\frac{1}{x\sin\theta}\right)^{16}$, if l_1 is the least value of the term independent of x when $\frac{\pi}{8}\le\theta\le\frac{\pi}{4}$ and l_2 is the least value of the term independent of x when $\frac{\pi}{16}\le\theta\le\frac{\pi}{8}$, then the value of $\frac{l_2}{l_1}$ is
(a) 8
(b) 32
(c) 16
(d) 64

7. Let $f(x)=a_0+a_1x+a_2x^2+\cdots+a_nx^n+\cdots$
and $\frac{f(x)}{1-x}=b_0+b_1x+b_2x^2+\cdots+b_nx^n+\cdots$.
If $a_0=1$ and $b_1=3$, then find the unit digit of $b_{10}=$:
Given that $\frac{a_0}{a_1}=\frac{a_1}{a_2}=\cdots..=$ Constant
(a) 5
(b) 6
(c) 7
(d) 8

8. If $\frac{3}{4!}+\frac{4}{5!}+\frac{5}{6!}+\cdots..+50$ term $=\frac{1}{3!}-\frac{1}{(k+3)!}$, then sum of coefficients in the expansion

$(1+2x_1+3x_2+\cdots..+100x_{100})^k$ is :
(where $x_1, x_2, x_3, \ldots.., x_{100}$ are independent variable)
(a) $(5050)^{49}$
(c) $(5050)^{52}$
(b) $(5050)^{51}$
(d) $(5050)^{50}$

9. If $(1+x)^n = C_0 + C_1x + C_2x^2 + \cdots \ldots + C_nX^n$, where $C_0, C_1, C_2, \ldots..$ are binomial coefficients and $C_r = {}^nC_r$. Then $2(C_0+C_3+C_6+\cdots..+C_n)+(C_1+C_4+C_7+\cdots\ldots+C_{n-2})(1+\omega)+(C_2+C_5+C_8+\cdots..C_{n-1})(1+\omega^2)$.
(where ω is the non real complex cube root of unity and n is an odd multiple of 3), is equal to
(a) 2^n+1
(b) $2^{n-1}+1$
(c) $2^{n+1}-1$
(d) 2^n-1

10. Let $(1+X)^{15} = a_0X^{P_0} + a_1X^{P_1} + a_2X^{P1} + a_2X^{P_2} + \cdots..+a_{15}X^{R_5}$ where $a_0 \geq a_1 \geq a_2 \geq a_3 \ldots \geq a_{15} > 0$ and $\begin{cases} P_{i+1} > P_i, \text{ if } a_i = a_{i+1} = 2n+1, n \in I, i \in \{0,1,2,\ldots,14\} \\ P_{i+1} < P_i, \text{ if } a_i = a_{i+1} = 2n, n \in I, i \in \{0,1,2,\ldots,14\} \end{cases}$
then the value of $\frac{a_5+P_5+a_{11}+P_{11}+869}{a_9+P_9+P_{14}+a_{14}+32}$ is
(a) 5
(b) 6
(c) 7
(d) 8

11. If $a_n = (\log_e 3)^n \sum_{k=1}^{n} \frac{k^2}{k!(n-k)!}$ then $a_1+a_2+a_3+\cdots..\infty$ is equal to
(a) $3\log_e 9$
(c) $9\log_e 3(\log_e 3+1)$
(b) $9\log_e 3$
(d) $(\log_e 9)^2$

12. Which of the following is NOT CORRECT?
(a) The greatest integer less than or equal to $(\sqrt{2}+1)^6$ is 197 .
(b) The integer next above $(\sqrt{3}+1)^{2\mathrm{m}}$ contains $2^{\mathrm{m}+1}$ as factor
(c) The greatest integer less than or equal to the number $(7+4\sqrt{3})^{\mathrm{m}}$ is a multiple of 2 .
(d) If $R = (6\sqrt{6}+14)^{2n+1}$ and $f = R-[R]$ where $[R]$ is integer and $0 \leq \mathrm{f} < 1$ then $\mathrm{Rf} = 20^{2\mathrm{n}+1}$.

13. If k be positive integer and $S_k = 1k+2k+\cdots+n^k$, then find the value of $\sum_{r=1}^{m} {}^{m+1}C_r s_r$ if $n=10, \mathrm{m}=11$.
(a) $11^{12}-11$
(b) $10^{11}-10$
(c) $10^{12}-11$
(d) None of these

14. Which of the following is the greatest?

(a) $\square^{31}C_0\square^3 - \square^{31}C_1\square^2 + \square^{31}C_2\square^2 - \cdots - \square^{31}C_{31}\square^2$

(b) $\square^{32}C_0^2 - \square^{32}C_1^2 + \square^{32}C_1^2 - \cdots + \square^{32}C_{32}\square^2$

(c) $\square^{32}C_0\square^2 + \square^{32}C_1\square^2 + \square^{32}C_2\square^2 - \cdots + \square^{32}C_{32}\square^2$

(d) $\square^{34}C_0\square^2 - \square^{34}C_1\square^2 + \square^{34}C_2\square^2 - \cdots + \square^{34}C_{32}\square^2$

15. The interval in which x must lies so that the numerically greatest term in the expansion of $(1-x)^{21}$ has the greatest coefficient is, $(\mathrm{x} > 0)$.

(a) $\left[\frac{5}{6}, \frac{6}{5}\right]$

(b) $\left(\frac{5}{6}, \frac{6}{5}\right)$

(c) $\left(\frac{4}{5}, \frac{5}{4}\right)$

(d) $\left[\frac{4}{5}, \frac{5}{4}\right]$

Exercise 4: Numeric Value Answer Questions

1. If $\sum_{\mathrm{r}=0}^{\mathrm{n}} \frac{\mathrm{r}+2}{\mathrm{r}+1} \square^{\mathrm{n}}C_{\mathrm{r}} = \frac{2^8-1}{6}$, then $\mathrm{n} =$

2. Let $(x+10)^{50} + (x-10)^{50} = a_0 + a_1x + a_2x^2 + \cdots. + a_{50}x^{50}$, for all $x \in \mathbf{R}$; then $\frac{\mathrm{a}_2}{\mathrm{a}_0}$ is equal to

3. The sum of the co-efficients of all even degree terms in x in the expansion of $\left(x+\sqrt{x^3-1}\right)^6 + \left(x-\sqrt{x^3-1}\right)^6, (x>1)$ is equal to.:

4. If the fourth term in the binomial expansion of $\left(\sqrt{\frac{1}{x^{1+\log_{10} x}}} + x^{\frac{1}{12}}\right)^6$ is equal to 200 , and $x > 1$, then the value of x is

5. The expression $\left[x+(x^3-1)^{1/2}\right]^5 + \left[x-(x^3-1)^{1/2}\right]^5$ is a polynomial of degree

6. The remainder when 27^{40} is divided by 12 is

7. If the ratio of the coefficient of third and fourth term in the expansion of $\left(x-\frac{1}{2x}\right)^n$ is $1:2$, then the value of $-n$ will be

8. If the middle term in the expansion of $\left(x^2+\frac{1}{x}\right)^n$ is $924x^6$, then find n.

9. For natural numbers m, n if $(1-y)^m(1+y)^n$.

$= 1 + a_1 y + a_2 y^2 + \cdots \ldots$ and $a_1 = a_2 = 10$, then $(m+n)$ is

10. The coefficient of the middle term in the binomial expansion in powers of x of $(1+\alpha x)^4$ and of $(1-\alpha x)^6$ is the same, then $-\alpha$ equals to

11. The number of integral terms in the expansion of $(\sqrt{3}+\sqrt[8]{5})^{256}$ is

12. $\left(\sqrt[6]{3}\sqrt{2}+\frac{1}{\sqrt[3]{3}}\right)^n, \frac{t_7 \text{ from the } 1^{\text{st}}}{t_7 \text{ from the last}} = \frac{1}{6}$, then the value of n is

13. Let for the 9^{th} term in the binomial expansion of $(3+6x)^{\mathrm{n}}$, in the increasing powers of $6x$, to be the greatest for $x = \frac{3}{2}$, the least value of n is n_0. If k is the ratio of the coefficient of x^6 to the coefficient of x^3, then $k + n_0$ is equal to

14. Let the coefficients of x^{-1} and x^{-3} in the expansion of $\left(2x^{\frac{1}{5}} - \frac{1}{x^{\frac{1}{5}}}\right)^{15}, x > 0$, be m and n respectively. If r is a positive integer such $mn^2 = {}^{15}C_r \cdot 2^r$, then the value of r is equal to

15. If the coefficient of x^{10} in the binomial expansion of $\left(\frac{\sqrt{x}}{5^{1/4}} + \frac{\sqrt{5}}{x^{1/3}}\right)^{60}$. is $5^k l$, where $l, \mathrm{k} \in \mathrm{N}$ and l is coprime to 5 , then k is equal to

16. If the sum of the coefficients of all the positive even powers of x in the binomial expansion of $\left(2x^3 + \frac{3}{x}\right)^{10}$ is $5^{10} - \beta . 3^9$, then β is equal to

17. Let the ratio of the fifth term from the beginning to the fifth term from the end in the binomial expansion of $\left(\sqrt[4]{2}+\frac{1}{\sqrt[4]{3}}\right)^n$, in the increasing powers of $\frac{1}{\sqrt[4]{3}}$ be $\sqrt[4]{6}: 1$. If the sixth term from the beginning is $\frac{\alpha}{\sqrt[4]{3}}$, then α is equal to

18. Let the coefficients of the middle terms in the expansion of $\left(\frac{1}{\sqrt{6}}+\beta x\right)^4, (1-3\beta x)^2$ and $\left(1-\frac{\beta}{2}x\right)^6, \beta > 0$, respectively from the first three terms of an A. P. If d is the common difference of this A.P., then $50 - \frac{2d}{\beta^2}$ is equal to

19. If the maximum value of the term independent of t in the expansion of $\left(t^2 x^{\frac{1}{5}} + \frac{(1-x)^{\frac{1}{10}}}{t}\right)^{15}, x \geq 0$, is K, then $8K$ is equal to

20. The number of positive integers k such that the constant term in the binomial expansion of $\left(2x^3 + \frac{3}{x^k}\right)^{12}, x \neq 0$ is $2^8 . l$, where l is an odd integer, is

21. If $\sum_{k=1}^{10} K^2 ({}^{10}C_K)2 = 22000L$, then L is equal to

22. If $1 + (2 + {}^{49}C_1 + {}^{49}C_2 + \cdots + {}^{49}C_{49})({}^{50}C_2 + {}^{50}C_4 + \cdots + {}^{50}C_{50})$ is equal to $2^n \cdot m$, where m is odd, then $n + m$ is equal to

23. If the sum of the coefficients of all the positive powers of x, in the binomial expansion of $\left(x^n + \frac{2}{x^5}\right)^7$ is 939 , then the sum of all the possible integral values of n is

24. If $({}^{40}C_0) + ({}^{41}C_1) + ({}^{42}C_2) + \cdots + ({}^{60}C_{20}) = \frac{m}{n} {}^{60} \cdot C_{20}$, m and n are coprime, then $m + n$ is equal to

25. Let C_r denote the binomial coefficient of x^r in the expansion of $(1 + x)^{10}$. If $\alpha, \beta \in R. C_1 + 3 \cdot 2C_2 + 5 \cdot 3C_3 + \cdots$ upto 10 terms $= \frac{\alpha \times 2^{11}}{2^\beta - 1}\left(C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \cdots\right.$ upto 10 terms $\left.\right)$ then the value of $\alpha + \beta$ is equal to

Answer Keys

EXCERSISE-1

1	(d)	9	(c)	17	(c)	25	(b)	33	(d)	41	(a)	49	(c)	57	(b)	65	(d)	73	(a)
2	(d)	10	(b)	18	(b)	26	(c)	34	(c)	42	(d)	50	(d)	58	(c)	66	(c)		
3	(c)	11	(c)	19	(b)	27	(c)	35	(c)	43	(b)	51	(a)	59	(a)	67	(d)		
4	(a)	12	(a)	20	(b)	28	(b)	36	(a)	44	(c)	52	(a)	60	(a)	68	(b)		
5	(c)	13	(a)	21	(a)	29	(b)	37	(c)	45	(a)	53	(d)	61	(a)	69	(d)		
6	(a)	14	(d)	22	(c)	30	(d)	38	(b)	46	(d)	54	(b)	62	(a)	70	(c)		
7	(a)	15	(b)	23	(d)	31	(c)	39	(d)	47	(c)	55	(a)	63	(a)	71	(d)		
8	(d)	16	(c)	24	(b)	32	(d)	40	(c)	48	(d)	56	(b)	64	(b)	72	(b)		

EXCERSISE-2

1	(c)	3	(c)	5	(b)	7	(b)	9	(c)	11	(b)	13	(a)	15	(Bonus)				
2	(a)	4	(d)	6	(b)	8	(a)	10	(b)	12	(4)	14	(23)	16	(c)				

EXCERSISE-3

1	(c)	3	(b)	5	(d)	7	(c)	9	(d)	11	(c)	13	(a)	15	(b)				
2	(b)	4	(b)	6	(c)	8	(d)	10	(d)	12	(c)	14	(c)						

EXCERSISE-4

1	(5)	4	(10)	7	(10)	10	(0.3)	13	(24)	16	(83)	19	(6006)	22	(99)	25	(280)		
2	(12.25)	5	(7)	8	(12)	11	(33)	14	(5)	17	(84)	20	(2)	23	(57)				
3	(24)	6	(9)	9.	(80)	12	(10)	15	(5)	18	(57)	21	(221)	24	(102)				

HINTS AND SOLUTIONS

EXERCISE - 1

1. (d)
2. (d) Given: $3^{2022} = (3^2)^{1011} = 9^{1011} = (10-1)^{1011}$
$= 10\text{r} - 1 = 10\text{r} - 5 + 4 = 5(2\text{r} - 1) + 4$(r is integer)
Remainder $= 4$
3. (c) Given expansion is
$(1+x)^{101}(1-x+x^2)^{100}$
$= (1+x)(1+x)^{100}(1-x+x^2)^{100}$
$= (1+x)[(1+x)(1-x+x^2)]^{100}$
$= (1+x)[(1-x^3)^{100}]$
Expansion $(1-x^3)^{100}$ will have $100+1 = 101$ terms
So, $(1+x)(1-x^3)^{100}$ will have $2 \times 101 = 202$ terms
4. (a) Let r^{th} and $(r+1)^{\text{th}}$ term has equal coefficient
$\left(2+\frac{x}{3}\right)^{55} = 2^{55}\left(1+\frac{x}{6}\right)^{55}$
r^{th} term $= 2^{55}\,{}^{55}C_r\left(\frac{x}{6}\right)^r$
Coefficient of x^r is $2^{55}\,{}^{55}C_r\frac{1}{6^r}$
$(r+1)^{\text{th}}$ term $= 2^{55}\,{}^{55}C_{r+1}\left(\frac{x}{6}\right)^{r+1}$
Coefficient of x^{r+1} is $2^{55}\,{}^{55}C_{r+1}\cdot\frac{1}{6^{r+1}}$
Both coefficients are equal
$2^{55}\,{}^{55}C_r\frac{1}{6^r} = 2^{55}\,{}^{55}C_{r+1}\frac{1}{6^{r+1}}$
$\frac{1}{|r|55-r} = \frac{1}{|r+1|54-r}\cdot\frac{1}{6}$
$6(r+1) = 55-r$
$6r+6 = 55-r$
$7r = 49$
$r = 7$
$(r+1) = 8$
Coefficient of 7^{th} and 8^{th} terms are equal.
5. (c) $(2021)^{2023} = (7\text{K}-2)^{2023}$

$= {}^{2023}C_0(7\lambda)^{2023} - \cdots .. {}^{2023}C_{2023}2^{2023} = 7\mu - 2^{2023}$
$\therefore -2^{2023} = -2 \times 2^{2022}$
$= -2 \times (2^3)^{674} = -2(1+7\gamma)^{674} = -(7\beta + 2)$
$\Rightarrow$ remainder $= -2$ or $+5$

Binomial Theorem

6.

(a) $1 + x^4 + x^5 = \sum_{i=0}^{5} a_i(1+x)^i$
$= a_0 + a_1(1+x)^1 + a_2(1+x)^2 + a_3(1+x)^3$
$+ a_4(1+x)^4 + a_5(1+x)^5$
$\Rightarrow 1 + x^4 + x^5$
$= a_0 + a_1(1+x) + a_2(1+2x+x^2) + a_3(1+3x+3x^2+x^3)$
$+ a_4(1+4x+6x^2+4x^3+x^4) + a_5(1+5x+10x^2+10x^3+5x^4+x^5)$
$\Rightarrow 1 + x^4 + x^5$
$= a_0 + a_1 + a_1x + a_2 + 2a_2x + a_2x^2 + a_3 + 3a_3x$
$+ 3a_3x^2 + a_3x^3 + a_4 + 4a_4x + 6a_4x^2 + 4a_4x^3 + a_4x^4 + a_5$
$+ 5a_5x + 10a_5x^2 + 10a_5x^3 + 5a_5x^4 + a_5x^5$
$\Rightarrow 1 + x^4 + x^5$
$= (a_0 + a_1 + a_2 + a_3 + a_4 + a_5) + x(a_1 + 2a_2 + 3a_3 + 4a_4 + 5a_5)$
$+ x^2(a_2 + 3a_3 + 6a_4 + 10a_5) + x^3(a_3 + 4a_4 + 10a_5)$
$+ x^4(a_4 + 5a_5) + x^5(a_5)$

On comparing the like coefficients, we get

$a_5 = 1$ ... (i) , m; $a_4 + 5a_5 = 1$ (ii)

; $a_3 + 4a_4 + 10a_5 = 0$
and $a_2 + 3a_3 + 6a_4 + 10a_5 = 0$
from (i) & (ii), we get
$a_4 = -4$...(v) from (i), (iii) & (v), we get
$a_3 = +6$
Now, from (i), (v) and (vi), we get $a_2 = -4$

7. (a) $\left(x^2 + \frac{1}{x^2}\right)^n$

General term $T_{r+1} = {}^nC_r(x^2)^{n-r}\left(\frac{1}{x^3}\right)^r = {}^nC_r \cdot x^{2n-5r}$
To find coefficient of $x, 2n - 5r = 1$
Given ${}^nC_r = {}^nC_{23} \Rightarrow r = 23$ or $n - r = 23$
$\therefore n = 58$ or $n = 38$
Minimum value is $n = 38$

8. (d) Given that $(11)^{1011} + (1011)^{11}$
$= (9+2)^{1011} + (1008+3)^{11}$
$\Rightarrow 9\lambda + 2^{1011} + 9\mu + 3^{11} \Rightarrow (2^3)^{337} + 3^2 \times 3^9 = 8^{337}$
$\Rightarrow (9-1)^{337} = 9\mathrm{K} - 1^{337}$
$\therefore$ Remainder $= 9 - 1 = 8$.

9. (c) We have $2^n = 4096 = 2^{12} \Rightarrow n = 12$;

the greatest coeff = coeff of middle term.
So, middle term = t_7
Coeff of $t_7 = {}^{12}C_6 = \frac{12!}{6!6!} = 924$.
10. (b)
11. (c) $\sqrt{5}[(\sqrt{5}+1)^{50} - (\sqrt{5}-1)^{50}]$
$= 2\sqrt{5}[{}^{50}C_1(\sqrt{5})^{49} + {}^{50}C_3(\sqrt{5})^{47} + \cdots..]$
$= 2[{}^{50}C_1(\sqrt{5})^{50} + {}^{50}C_3(\sqrt{5})^{48} + \cdots..]$ = a natural number
12. (a) $\left(x+\sqrt{x^3-1}\right)^5 + \left(x-\sqrt{x^3-1}\right)^5$
$= 2[x^5 + {}^5C_2x^3(x^3-1) + {}^5C_4x(x^3-1)^2]$
$= 2[x^5 + 10x^3(x^3-1) + 5x(x^6-2x^3+1]$
$= 10x^7 + 20x^6 + 2x^5 - 20x^4 - 20x^3 + 10x$
∴ polynomial has degree 7 .
13. (a) $(2021)^{2022} + (2022)^{2021}$
$= (2023-2)^{2022} + (2023-1)^{2021}$
$= 7k_1 + 2^{2022} + 7k_2 - 1 = 7(k_1+k_2) + 8^{674} - 1$
$= 7(k_1+k_2) + (7-1)^{674} - 1 = 7(k_1+k_2) + 7k_3 + 1 - 1$
$= k_1 + k_2 + k_3$
∴ Given number is divisible by 7 hence remainder is zero.
14. (d) Suppose x^6 occurs in $(r+1)^{\text{th}}$ term in the expansion of $\left(2x^2 - \frac{3}{x}\right)^{11}$
Now, $T_{r+1} = {}^{11}C_r(2x^2)^{11-r}\left(-\frac{3}{x}\right)^r$
$= {}^{11}C_r(-1)^r 2^{11-r} 3^r x^{22-3r}$
For this term to contain x^6, we must have
$22 - 3r = 6 \Rightarrow r = \frac{16}{3}$, which is a fraction.
But, r is a natural number. Hence, there is no term containing x^6.
15. (b) $T_2 = {}^nC_1ab^{n-1} = 135$
$T_3 = {}^nC_2a^2b^{n-2} = 30$
$T_4 = {}^nC_3a^3b^{n-3} = \frac{10}{3}$
Dividing (i) by (ii): $\frac{{}^nC_1ab^{n-1}}{{}^nC_2a^2b^{n-2}} = \frac{135}{30}$

$$\frac{n}{\frac{n}{2}(n-1)}\frac{b}{a} = \frac{9}{2}\frac{b}{a} = \frac{9}{4}(n-1)$$

Dividing (ii) by (iii): $\frac{\frac{n(n-1)}{2}}{\frac{n(n-1)(n-2)}{3.2}} \cdot \frac{b}{a} = 9$
Eliminating a and b from (v) and (vi), we get $n = 5$
16. (c) Let the consecutive coefficient of
$(1+x)^n$ are ${}^nC_{r-1}, {}^nC_r, {}^nC_{r+1}$
From the given condition, ${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 6:33:110$
Now ${}^nC_{r-1} : {}^nC_r = 6:33 \Rightarrow \frac{n!}{(r-1)!(n-r+1)!} \times \frac{r!(n-r)!}{n!} = \frac{6}{33}$
$\Rightarrow \frac{r}{n-r+1} = \frac{2}{11} \Rightarrow 11r = 2n - 2r + 2$

$\Rightarrow 2n - 13r + 2 = 0$

and ${}^{n}C_r : {}^{n}C_{r+1} = 33:110$

$\Rightarrow \frac{n!}{r!(n-r)!} \times \frac{(r+1)!(n-r-1)!}{n!} = \frac{33}{110} = \frac{3}{10}$

$\Rightarrow \frac{(r+1)}{n-r} = \frac{3}{10} \Rightarrow 3n - 13r - 10 = 0$

Solving (i) & (ii), we get $n = 12$

17. (c) We are given that $7^{2022} + 3^{2022}$

$= (49)^{1011} + (9)^{1011} = (50-1)^{1011} + (10-1)^{1011}$

$= 5\lambda - 1 + 5K - 1 = 5m - 2$

$\Rightarrow$ Remainder $= 5 - 2 = 3$

18. (b) Given, $T_5 + T_6 = 0 \Rightarrow {}^{n}C_4 a^{n-4} b^4 - {}^{n}C_5 a^{n-5} b^5 = 0$

$\Rightarrow {}^{n}C_4 a^{n-4} b^4 = {}^{n}C_5 a^{n-5} b^5 \Rightarrow \frac{a}{b} = \frac{{}^{n}C_5}{{}^{n}C_4} = \frac{n-4}{5}$

19. (b) Coeff. of x^n in $(1+x)(1-x)^n$ = coeff. of x^n in

$(1+x)(1 - {}^{n}C_1 x + {}^{n}C_2 x^2 - \cdots . + (-1)^n {}^{n}C_n x^n)$

$= (-1)^n {}^{n}C_n + (-1)^{n-1} {}^{n}C_{n-1} = (-1)^n + (-1)^{n-1} \cdot n$

$= (-1)^n (1-n)$

20. (b) No. of terms in the expansion $= {}^{n+3-1}C_{3-1}$

$\therefore {}^{n+2}C_2 = 45 \Rightarrow n = 8$

21. (a) We have

$$7^9 + 9^7 = (8-1)^9 + (8+1)^7 = (1+8)^7 - (1-8)^9$$
$$= [1 + {}^{7}C_1 8 + {}^{7}C_2 8^2 + \cdots .. + {}^{7}C_7 8^7]$$
$$- [1 - {}^{9}C_1 8 + {}^{9}C_2 8^2 - \cdots .. {}^{9}C_9 8^9]$$
$$= {}^{7}C_1 8 + {}^{9}C_1 8 + [{}^{7}C_2 + {}^{7}C_3 . 8 + \cdots . - {}^{9}C_2 + {}^{9}C_3 . 8 - \cdots .] 8^2$$
$$= 8(7+9) + 64k = 8..16 + 64k = 64q,$$

where $q = k + 2$. Thus, $7^9 + 9^7$ is divisible by 64 .

22. (c) Since the total number of terms are 52 of which 26 terms get cancelled.

23. (d) The expression can be written as $a^m \left\{ \left(1 + \frac{b}{a}\right)^m \right\}$

24. (b) $(101)^{50} - (99)^{50} = (100+1)^{50} - (100-1)^{50}$

$= 2[{}^{50}C_1 (100)^{49} + {}^{50}C_3 (100)^{47} + \cdots .... + {}^{50}C_{49} (100)]$

$> 2. {}^{50}C_1 \cdot (100)^{49} = 2 \times 50 (100)^{49} = (100)^{50}$

$\Rightarrow (101)^{50} > (99)^{50} + (100)^{50} \Rightarrow y > x \Rightarrow x < y.$

25. (b) 12 terms. [$\because$ No. of terms in $(x+a)^n = n+1$)]

26. (c) $(1073)^{71} - m = (73 + 1000)^{71} - m$

$= {}^{71}C_0 (73)^{71} + {}^{71}C_1 (73)^{70} (1000) + {}^{71}C_2 (73)^{69} (1000)^2$

$+ \cdots .... + {}^{71}C_{71} (1000)^{71} - m$

Above will be divisible by 10 if ${}^{71}C_0 (73)^{71}$ is divisible by 10

Now ${}^{71}C_0 (73)^{71} = (73)^{70} \cdot 73 = (73^2)^{35} \cdot 73$

The last digit of 73^2 is 9 , so the last digit of $(73^2)^{35}$ is 9 .

$\therefore$ Last digit of $(73^2)^{35} . 73$ is 7

Hence, the minimum positive integral value of m is 7 , so that it is divisible by 10 .

27. (c) Let the consecutive coefficient of

$(1+x)^n$ are ${}^nC_{r-1}, {}^nC_r, {}^nC_{r+1}$
From the given condition,
${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 6:33:110$
Now ${}^nC_{r-1} : {}^nC_r = 6:33$

$$\Rightarrow \frac{n!}{(r-1)!\,(n-r+1)!} \times \frac{r!\,(n-r)!}{n!} = \frac{6}{33}$$

$$\Rightarrow \frac{\mathrm{r}}{\mathrm{n}-\mathrm{r}+1} = \frac{2}{11}$$

$$\Rightarrow 11\mathrm{r} = 2\mathrm{n} - 2\mathrm{r} + 2 \Rightarrow 2\mathrm{n} - 13\mathrm{r} + 2 = 0$$

and ${}^nC_r : {}^nC_{r+1} = 33:110$

$$\Rightarrow \frac{\mathrm{n}!}{\mathrm{r}!\,(\mathrm{n}-\mathrm{r})!} \times \frac{(\mathrm{r}+1)!\,(\mathrm{n}-\mathrm{r}-1)!}{\mathrm{n}!} = \frac{33}{110} = \frac{3}{10}$$

$$\Rightarrow \frac{(\mathrm{r}+1)}{\mathrm{n}-\mathrm{r}} = \frac{3}{10} \Rightarrow 3\mathrm{n} - 13\mathrm{r} - 10 = 0$$

Solving (i) & (ii), we get $\mathrm{n} = 12$

28. (b) $3^{400} = 9^{200} = (10-1)^{200}$

$$= {}^{200}C_0 10^{200} - {}^{200}\dot{C}_1 10^{199} + {}^{200}C_2 10^{198} - \cdots ..$$

$$+ {}^{200}C_{198} 10^2 - {}^{200}C_{199} 10 + 1$$

$$= 100k + \frac{200 \times 198}{2} \times 100 - 200 \times 10 + 1$$

$$= 100\mathrm{k} + 2000(990-1) + 1 = 100\mathrm{k} + 2000 \times 989 + 1 = 100\mathrm{q} + 1$$

Thus the last two digits are 01 .

29. (b) The given expression,

$$\left(1+\frac{C_1}{C_0}\right)\left(1+\frac{C_2}{C_1}\right)\left(1+\frac{C_3}{C_2}\right)\ldots\left(1+\frac{C_n}{C_{n-1}}\right)$$

$$= \left(1+\frac{n}{1}\right)\left(1+\frac{n-1}{2}\right)\left(1+\frac{n-2}{3}\right)\ldots\left(1+\frac{1}{n}\right) = \frac{(n+1)^n}{n!}$$

30. (d) $(a+b+c)^{12} = [(a+b)+c]^{12}$

$$= {}^{12}\mathrm{C}_0(\mathrm{a}+\mathrm{b})^{12} + {}^{12}\mathrm{C}_1(\mathrm{a}+\mathrm{b})^{11}\mathrm{c} + \cdots + {}^{12}\mathrm{C}_{12}\mathrm{c}^{12}$$

The R.H.S. contains, $13+12+11+\cdots.+1$ terms

$= \frac{13(13+1)}{2} = 91$ terms

Also no. of term in the expansion of $(a+b+c)^{\mathrm{n}}$ is given by ${}^{n+2}C_2$.

Thus for $\mathrm{n} = 12$; ${}^{\mathrm{n}+2}\mathrm{C}_2 = {}^{14}\mathrm{C}_2 = \frac{14\times13}{2} = 91$.

31. (c) Now, $(0.99)^5 = (1-0.01)^5$

$$= {}^5\mathrm{C}_0(1)^5 - {}^5\mathrm{C}_1(1)^4(0.01) + {}^5\mathrm{C}_2(1)^3(0.01)^2$$

(ignore the other terms).

$$= 1 - 5 \times 1 \times 0.01 + \frac{5\times4}{2} \times 1 \times 0.01 \times 0.01$$

$$= 1 - 0.05 + 10 \times 0.0001 = 1 - 0.05 + 0.001$$

$$= 1.001 - 0.05 = 0.951$$

32. (d) Given expression

$$= 2\big[1 + {}^9C_2(3\sqrt{2}x)^2 + {}^9C_4(3\sqrt{2}x)^4$$

$$+ {}^9C_6(3\sqrt{2}x)^6 + {}^9C_8(3\sqrt{2}x)^8\big]$$

$\therefore$ the number of non-zero terms is 5 .

33. (d) If n is odd, then the expansion of $(x+a)^n+(x-a)^n$ contains $\left(\frac{n+1}{2}\right)$ terms. So, the expansion of $(1+5\sqrt{2}x)^9+(1-5\sqrt{2}x)^9$ has $\left(\frac{9+1}{2}\right)=5$ terms.

34. (c) Given expression is :

$$[(3x+y)^5]^4-[(3x-y)^4]^5=[(3x+y)]^{20}-[(3x-y)]^{20}$$

First and second expansion will have 21 terms each but odd terms in second expansion be Ist, 3rd, 5th,.....21st will be equal and opposite to those of first expansion.

Thus, the number of terms in the expansion of above expression is 10 .

35. (c) We have, $7^1=7, 7^2=7\times 7=49$

$$7^3=7\times 7\times 7=343; 7^4=7\times 7\times 7\times 7=2401$$

$$7^5=7\times 7\times 7\times 7\times 7=16807$$

Last digit of $7^1=7, 7^2=9, 7^3=3, 7^4=1$ and $7^5=7$ thus cycle of last digit repeats at 7^5. $\therefore$ Last digit of $7^{300}=1$

36. (a) Given expression is

$$(5+x)^{500}+x(5+x)^{499}+x^2(5+x)^{498}+\cdots+x^{500}$$

Take $(5+x)^{500}$ common, then it forms G.P.

$$\left[(5+x)^{500}\left(\left(\frac{x}{5+x}\right)^{501}-1\right)\right]\left(\frac{x}{5+x}\right)-1$$

$$=\frac{(5+x)^{501}-x^{501}}{5}$$

$\Rightarrow$ Coefficient x^{101} in given expression is calculated by

$$\left(\frac{1}{5}(x+5)^{501}\right) \text{ only } =\frac{{}^{501}C_{101}5^{400}}{5}={}^{501}C_{101}5^{399}$$

37. (c) General term of $\left(ax^{1/8}+bx^{-1/12}\right)^{10}$ is

$$={}^{10}C_r\left(ax^{1/8}\right)^{10-r}\left(bx^{-1/12}\right)^r$$

On solving $r=6$

$$\Rightarrow (a^4b^6)^{\frac{1}{4}}\geq\frac{4}{2\left(\frac{1}{a^2}+\frac{1}{b^2}\right)}\Rightarrow a^4b^6\geq\left(\frac{1}{2}\right)^4$$

$$\Rightarrow {}^{10}C_6\left(\frac{1}{2}\right)^4=\frac{210}{16}=\frac{105}{8}$$

38. (b) Given expression is $\sum_{x=1}^{20}(r^2+1)r$!

$$\Rightarrow \sum_{x=1}^{20}((r+1)^2-2r)r!$$

$$\Rightarrow \sum_{x=1}^{20}((\mathrm{r}+1)(\mathrm{r}+1)!-\mathrm{r}.\mathrm{r}!)-\sum_{r=1}^{20}r.r!$$

$$\Rightarrow \sum_{x=1}^{20}((\mathrm{r}+1)(\mathrm{r}+1)!-\mathrm{r}\cdot\mathrm{r}!)-\sum_{r=1}^{20}((r+1)!-r!)$$

$$=(21.|21-1)-(|21-1)$$

$$=20.21!=22!-2.21\,!$$

39. (d) Constant term in

$\left(3x^3-2x^2+\frac{5}{x^5}\right)^{10}$ to make one term without x

Let, $x^{50}(3x^8-2x^7+5)^{10}$

how general term of the expression is

$$\frac{10!}{p!\,q!\,r!}(3x^8)^p(-2x^7)^q(5)^r$$

Here $8p+7q=50$ and $p+q+r=10$

$\Rightarrow p = 1, q = 6, r = 3$ is only valid sol.

$\therefore \frac{10!}{1!\,6!\,r!} 3^1 2^6 \cdot 5^3 = 2^K . l \Rightarrow K = 9$

40. (c) $\alpha = \frac{(1+8)^n - 8n - 1}{64} = {}^nC_2 + {}^nC_3 8 + {}^nC_4 8^2 + \cdots$

$\beta = {}^nC_2 + {}^nC_3 5 + {}^nC_4 5^2 + \cdots$

option (c) will be the answer.

41. (a) Given expression is $\sum_{K=1}^{31} {}^{31}C_K \, {}^{31}C_{K-1}$

$= {}^{31}C_1 \cdot {}^{31}C_0 + {}^{31}C_2 \cdot {}^{31}C_1 + \cdots . + {}^{31}C_{31} \cdot {}^{31}C_{30}$

Here, ${}^nL_r = {}^nL_{n-r_1} \, {}^{31}C_0 \cdot {}^{31}C_{30} + {}^{+31}C_1 \cdot {}^{31}C_{29} + \cdots . + {}^{31}C_{30} \cdot {}^{31}C_0$

$= {}^{62}C_{30}.$

Similarly

$$\sum_{K=1}^{30} ({}^{30}C_K \cdot {}^{30}C_{K-1}) = {}^{60}C_{29} = 1\,{}^{62}C_{30} - {}^{60}C_{29} = \frac{62!}{30!\,32!} - \frac{60!}{29!\,31!}$$

$$= \frac{60!}{29!\,31!}\left\{\frac{62.61}{30.32} - 1\right\} = \frac{60!}{30!\,31!}\left(\frac{2822}{32}\right)$$

compare above equation with $\frac{\alpha(60!)}{(30!)(31!)}$

So, $\alpha = \frac{2822}{32} \; \therefore 16\alpha = 16 \times \frac{2822}{32} = 1411$

42. (d) We are given that $t_1 + t_2 + t_3 + t_4 = 30$

Now coefficient of x^{30} in $(1 + x + x^2 + \cdots + x^{30})^2$

$(x^4 + x^5 + x^6 + x^7)(x^2 + x^3 + x^4 + x^5 + x^6)$

$x^6 \left(\frac{1 - x^{31}}{1 - x}\right)^2 (1 + x + x^2 + x^3)(1 + x + x^2 + x^3 + x^4)$

$x^6 (1 - x^{31})^2 (1 - x^4)(1 - x^5)(1 - x)^4$

$x^6 (1 - x^4 - x^5 + x^9)(1 + x^{62} - 2x^{31})(1 - x)^{-4}$

$x^6 (1 - x^4 - x^5 + x^9)(1 - x)^{-4}$

Coefficient of x^n in $(1 - x)^{-r}$ is ${}^{n+r-1}C_{r-1}$

$\Rightarrow {}^{27}C_3 - {}^{23}C_3 - {}^{22}C_3 - {}^{18}C_3$

$2925 - 1771 - 1540 + 816 = 430$

Alternate

$x_2 \in [4,7], x_3 \in [2,6]$

$\Rightarrow t_1 + t_2 + t_3 + t_4 = 24$

total ways $=$

${}^{24+4-1}C_{4-1} - {}^{20+4-1}C_{4-1} - {}^{19+4-1}C_{4-1} + {}^{15+4-1}C_{4-1}$

$= {}^{27}C_3 - {}^{23}C_3 - {}^{22}C_3 + {}^{18}C_3 = 430$

43. (b) We are given that the expression is

$(1 - x^2 + 3x^3)\left(\frac{5}{2}x^3 - \frac{1}{5x^2}\right)^{11}; x \neq 0$

$\because$ General term of $\left(\frac{5}{2}x^3 - \frac{1}{5x^2}\right)^{11}$

${}^{11}C_r \left(\frac{5}{2}x^3\right)^{11-r} \left(-\frac{1}{5x^2}\right)^r$

[$\because$ General term of $(x + y)^n$ is ${}^nC_r (x)^{n-r} \cdot y^r$]

Now general term of ${}^{11}C_r \left(\frac{5}{2}\right)^{11-r} \left(-\frac{1}{5}\right)^r x^{33-5r}$

∴ Term independent of x is

$1 \times$ coefficient of x^0 in $\left(\frac{5}{2}x^3 - \frac{1}{5x^2}\right)^{11}$ +

$-1 \times$ coefficient of x^{-2} in $\left(\frac{5}{2}x^3 - \frac{1}{5x^2}\right)^{11}$ +

$3 \times$ coefficient of x^{-3} in $\left(\frac{5}{2}x^3 - \frac{1}{5x^2}\right)^{11}$

for coefficient of x^0 $33 - 5r = 0$ not possible

for coefficient of x^{-2} $33 - 5r = -2$

$\Rightarrow 35 = 5r \Rightarrow r = 7$

for coefficient of x^{-3} $33 - 5r = -3 \Rightarrow 36 = 5r$ not possible

So term independent of x is

$$(-1)^{11}C_7\left(\frac{5}{2}\right)^4 \cdot \left(-\frac{1}{5}\right)^7 = \frac{33}{200}$$

44. (c) Middle term in the expansion is $\left(\frac{10}{2}+1\right)^{\text{th}}$ i.e., 6^{th} term.

Thus $T_6 = 7\frac{7}{8} \Rightarrow {}^{10}C_5 \frac{1}{x^5} \cdot x^5 \sin^5 x = \frac{63}{8}$

$$\Rightarrow 252 \cdot \sin^5 x = \frac{63}{8} \Rightarrow \sin^5 x = \frac{1}{32} \Rightarrow \sin x = \frac{1}{2}$$

$$\therefore x = n\pi + (-1)^n \frac{\pi}{6}$$

45. (a) General term $= \mathrm{T}_{r+1} = {}^{10}\mathrm{C}_r \left(\frac{10}{x}\right)^{10-r}\left(\frac{x}{10}\right)^r$

Here $n = 10$, which is an even number.

Now, $\left[\frac{10}{2}+1\right]^{\text{th}}$ term i.e. 6^{th} term is the middle term.

Hence, middle term $= \mathrm{T}_6$

$\mathrm{T}_{5+1} = {}^{10}\mathrm{C}_5 \left(\frac{10}{x}\right)^{10-5}\left(\frac{x}{10}\right)^5 = {}^{10}\mathrm{C}_5 \left(\frac{10}{x}\right)^5\left(\frac{x}{10}\right)^5 = {}^{10}\mathrm{C}_5$.

46. (d) In the expansion of $\left(\frac{x\sqrt{y}}{3} - \frac{3}{y\sqrt{x}}\right)^{12}$, $n = 12$ (even)

then middle term is $\frac{12}{2} + 1 = 7^{\text{th}}$ term.

$(\mathrm{r}+1)^{\text{th}}$ term; $T_{r+1} = {}^{12}C_r\left[\frac{x\sqrt{y}}{3}\right]^{12-r} \cdot \left(-\frac{3}{y\sqrt{x}}\right)^r$

$$\therefore \mathrm{T}_7 = \mathrm{T}_{6+1} = {}^{12}\mathrm{C}_6\left(\frac{\mathrm{x}\sqrt{\mathrm{y}}}{3}\right)^6\left(-\frac{3}{\mathrm{y}\sqrt{\mathrm{x}}}\right)^6$$

$$= {}^{12}C_6\frac{x^6y^3}{y^6x^3} = {}^{12}C_6 x^3 y^{-3} = C(12,6)x^3y^{-3}$$

47. (c) In the expansion of $\left(x^4 + \frac{1}{x^3}\right)^{15}$, let T_r is the r^{th} term

$$\mathrm{T}_\mathrm{r} = 15_{\mathrm{C}_{\mathrm{r}-1}}(\mathrm{x}^4)^{15-\mathrm{r}+1}\left(\frac{1}{\mathrm{x}^3}\right)^{\mathrm{r}-1}$$

$$= 15_{C_{r-1}}x^{64-4r-3r+3} = 15_{C_{r-1}}x^{67-7r}$$

x^{18} occurs in this term

$\Rightarrow 18 = 67 - 7r \Rightarrow 7r = 49 \Rightarrow r = 7$.

48. (d) $\mathrm{T}_{\mathrm{r}+1} = {}^{29}\mathrm{C}_\mathrm{r} \cdot 3^{29-\mathrm{r}} \cdot (7\mathrm{x})^\mathrm{r} = ({}^{29}\mathrm{C}_\mathrm{r} \cdot 3^{29-\mathrm{r}} \cdot 7^\mathrm{r})\mathrm{x}^\mathrm{r}$

$\therefore a_r =$ coefficient of $(r+1)^{\text{th}}$ term $= {}^{29}C_r \cdot 3^{29-r} \cdot 7^r$

Now, $a_r = a_{r-1}$

$$\Rightarrow {}^{29}C_r \cdot 3^{29-r} \cdot 7^r = {}^{29}C_{r-1} \cdot 3^{30-r} \cdot 7^{r-1} \Rightarrow \frac{{}^{29}C_r}{{}^{29}C_{r-1}} = \frac{3}{7} \Rightarrow \frac{30-r}{r} = \frac{3}{7} \Rightarrow r = 21$$

49\. (c) The binomial expansion of $(x+a)^n$ gives $(t+1)^{\text{th}}$ term $= T_{t+1} = {}^nC_t x^{n-t} a^t$

We have expansion of $\left(x^4 + \frac{1}{x^3}\right)^{15}$.

On comparing with $(x+a)^n$, we get $x = x^4, a = \frac{1}{x^3}, n = 15$

$\therefore t^{\text{th}}$ term $= T_t = {}^{15}C_{t-1}(x^4)^{15-(t-1)} \cdot \left(\frac{1}{x^3}\right)^{t-1}$

$= {}^{15}C_{t-1}(x)^{60-4t+4} \cdot (x)^{-3t+3} = {}^{15}C_{t-1}(x)^{67-7t}$

Since, x^4 occurs in the t^{th} term.

$\therefore 67 - 7t = 4 \Rightarrow 7t = 63, t = 9$

50\. (d) Since the coefficient of $(r+1)^{\text{th}}$ term in the expansion of $(1+x)^n = {}^nC_r$

$\therefore$ In the expansion of $(1+x)^{18}$

coefficient of $(2r+4)^{\text{th}}$ term $= {}^{18}C_{2r+3}$,

Similarly, coefficient of $(r-2)^{\text{th}}$ term in the expansion of $(1+x)^{18} = {}^{18}C_{r-3}$

If ${}^nC_r = {}^nC_s$ then $r + s = n$ So, ${}^{18}C_{2r+3} = {}^{18}C_{r-3}$ gives

$2r + 3 + r - 3 = 18 \Rightarrow 3r = 18 \Rightarrow r = 6.$

51\. (a) $T_{17} = {}^{50}C_{16} \times 2^{34} \times a^{16}; T_{18} = {}^{50}C_{17} \times 2^{33} \times a^{17}$

Given $T_{17} = T_{18}$

$$\Rightarrow \frac{{}^{50}C_{16}}{{}^{50}C_{17}} \times 2 = \frac{a^{17}}{a^{16}}$$

$$\Rightarrow a = \frac{50!}{34!\,16!} \times \frac{33!\,17! \times 2}{50!} = \frac{17}{34} \times 2 = 1$$

52\. (a) In the expansion of $(1+\alpha x)^4$

Middle term $= {}^4C_2(\alpha x)^2 = 6\alpha^2 x^2$

In the expansion of $(1-\alpha x)^6$,

Middle term $= {}^6C_3(-\alpha x)^3 = -20\alpha^3 x^3$

It is given that

Coefficient of the middle term in $(1+\alpha x)^4$

= Coefficient of the middle term in $(1-\alpha x)^6$

$$\Rightarrow 6\alpha^2 = -20\alpha^3 \Rightarrow \alpha = 0, \alpha = -\frac{3}{10}$$

53\.

(d) $\frac{(3-2x)}{(1+3x)^3} = (3-2x)(1+3x)^{-3}$

$= (3-2x)\left[1 - 9x + \frac{(-3)(-4)}{2!} \cdot 9x^2\right.$

[Expanding $(1+3x)^{-3}$]

$$\left. + \frac{(-3)(-4)(-5)}{3!} \cdot 27x^3 + \cdots.\right]$$

$= (3-2x)(1 - 9x + 54x^2 - 270x^3 + \cdots \ldots \ldots)$

$\therefore$ Coefficient of $x^3 = -270 \times 3 - 2 \times 54$

$= -810 - 108 = -918$

54. (b) Since $T_{r+1} = {}^nC_r a^{n-r} x^r$ in expansion of $(a+x)^n$, Therefore, $T_8 = {}^nC_7 (2)^{n-7} \left(\frac{x}{3}\right)^7 = {}^nC_7 \frac{2^{n-7}}{3^7} x^7$

and $T_9 = {}^nC_8 (2)^{n-8} \left(\frac{x}{3}\right)^8 = {}^nC_8 \frac{2^{n-8}}{3^8} x^8$

Therefore, ${}^nC_7 \frac{2^{n-7}}{3^7} = {}^nC_8 \frac{2^{n-8}}{3^8}$

(since it is given that coefficient of x^7 = coefficient of x^8)

$$\Rightarrow \frac{n!}{7!(n-7)!} \times \frac{8!(n-8)!}{n!} = \frac{2^{n-8}}{3^8} \cdot \frac{3^7}{2^{n-7}}$$

$$\Rightarrow \frac{8}{n-7} = \frac{1}{6} \Rightarrow n = 55$$

55. (a) The $(r+1)$ th term in the expansion of $\left(\sqrt{\frac{x}{3}} + \frac{3}{2x^2}\right)^{10}$ is given by $T_{r+1} = {}^{10}C_r \left(\sqrt{\frac{x}{3}}\right)^{10-r}$

$$\left(\frac{3}{2x^2}\right)^r = {}^{10}C_r \frac{x^{5-(r/2)}}{3^{5-(r/2)}} \cdot \frac{3^r}{2^r x^{2r}}$$

$$= {}^{10}C_r \frac{3^{(3r/2)-5}}{2^r} x^{5-(5r/2)}$$

For T_{r+1} to be independent of x, we must have $5-(5r/2)=0$ or $r=2$.

Thus, the 3rd term is independent of x and is equal to

$$ {}^{10}C_2 \frac{3^{3-5}}{2^2} = \frac{10 \times 9}{2} \times \frac{3^{-2}}{4} = \frac{5}{4}$$

56. (b) On comparing with the expansion of $(x+a)^n$, we get

$$\mathrm{x} = \mathrm{x}, \mathrm{a} = \frac{2}{\mathrm{x}^2}, \mathrm{n} = 15$$

Now, r^{th} term of $\left(\mathrm{x} + \frac{2}{\mathrm{x}^2}\right)^{15}$ is given as

$$T_{r+1} = {}^nC_r x^{n-r} a^r = {}^{15}C_r (x)^{15-r} \left(\frac{2}{x^2}\right)^r$$

$$= {}^{15}\mathrm{C_r x^{15-r} 2^r \cdot x^{2r}} = {}^{15}\mathrm{C_r x^{15-3r} 2^r}$$

Now, in the expansion of $\left(x + \frac{2}{x^2}\right)^{15}$, the term is independent of x if $15-3r=0$ i.e., $r=5$

$\therefore$ Term independent of $x = {}^{15}\mathrm{C}_5, 2^5$

57. (b) In the binomial expansion of $(1+x)^n$,

$\mathrm{T_r} = {}^n\mathrm{C_{r-1}} \cdot (\mathrm{x})^{\mathrm{r}-1}$ For $\mathrm{r} = 5, \mathrm{T_5} = {}^n\mathrm{C_4 x^4}$

$\mathrm{r} = 6, \mathrm{T_6} = {}^n\mathrm{C_5 x^5}$; and $\mathrm{r} = 7, \mathrm{T_7} = {}^n\mathrm{C_6 x^6}$

Since, the coefficients of these terms are in A.P.

$$\Rightarrow \mathrm{T_5} + \mathrm{T_7} = 2\mathrm{T_6} \Rightarrow {}^n\mathrm{C_4} + {}^n\mathrm{C_6} = 2 \times {}^n\mathrm{C_5}$$

$$\Rightarrow \frac{n!}{(n-4)!\,4!} + \frac{n!}{(n-6)!\,6!} = \frac{2 \times n!}{(n-5)!\,5!} \Rightarrow \frac{n(n-1)(n-2)(n-3)}{4!}$$

$$+ \frac{n(n-1)(n-2)(n-3)(n-4)(n-5)}{6!}$$

$$= \frac{2n(n-1)(n-2)(n-3)(n-4)}{5!}$$

$\Rightarrow \frac{1}{4!} + \frac{(n-4)(n-5)}{6!} = \frac{2(n-4)}{5!}$
$\Rightarrow \frac{1}{1} + \frac{(n-4)(n-5)}{5 \times 6} = \frac{2(n-4)}{5}$
$\Rightarrow \frac{30 + n^2 - 9n + 20}{5 \times 6} = \frac{2n-8}{5}$
$\Rightarrow n^2 - 9n + 50 = 6(2n-8)$
$\Rightarrow n^2 - 9n + 50 - 12n + 48 = 0 \Rightarrow n^2 - 21n + 98 = 0$
$\Rightarrow (n-7)(n-14) = 0 \Rightarrow n = 7$ or $n = 14$.

58. (c) $T_r = {}^nC_{r-1}(2x)^{r-1}(3y^2)^{n-r+1}$
$T_4 = T_{3+1} = {}^5C_3(2x)^3(3y^2)^2$
$= \frac{5!}{3!\,2!} 2^3 \cdot x^3 \cdot 9y^4 = \frac{5.4}{2.1} \times 8 \times 9 \times x^3y^4 = 720x^3y^4$
$\therefore$ Coefficient of $x^3y^4 = 720$

59. (a) Given expansion is $\left(\sqrt{x} + \frac{k}{x^2}\right)^{10}$
$(r+1)_{th}$ term, $T_{r+1} = {}^{10}C_r(\sqrt{x})^{10-r}\left(\frac{k}{x^2}\right)^r$
$\Rightarrow T_{r+1} = {}^{10}C_r x^{5-r/2} \cdot (k)^r \cdot x^{-2r}$
$\therefore T_{r+1} = {}^{10}C_r x^{(10-5r)/2}(k)^r$
Since, T_{r+1} is independent of x
$\therefore \frac{10-5r}{2} = 0 \Rightarrow r = 2 \therefore 405 = {}^{10}C_2(k)^2$
$405 = 45 \times k^2 \Rightarrow k^2 = 9 \Rightarrow k = \pm 3$

60. (a) $T_{r+1} = {}^9C_r(\sqrt[6]{x})^{9-r}\left(-\frac{1}{\sqrt[3]{x}}\right)^r$
$= {}^9C_r(-1)^r \cdot x^{\frac{9-r}{6} - \frac{r}{3}} = {}^9C_r \cdot x^{\left(\frac{9-3r}{6}\right)}$
Now $\frac{9-3r}{6} = 0 \Rightarrow r = 3;$
Thus, term independent of $x = -{}^9C_3$

61. (a) Here, $T_{r+1} = {}^{4n-2}C_r(ix)^r$
The term is real negative if $r = 2, 6, 10, \ldots$ ....
but $0 \le r \le 4n-2$
and $4n - 2 = 2 + (p-1)4$ [pth term of A.P.] $\Rightarrow p = n$
Hence, required number of terms $= n$

62. (a) Expression $= (1-x)^5 \cdot (1+x)^4(1+x^2)^4$
$= (1-x)(1-x^2)^4(1+x^2)^4$
$= (1-x)(1-x^4)^4$
$\therefore$ Coefficient of $x^{13} = -{}^4C_3(-1)^3 = 4$

63. (a) Given expansion is $(1+x)^m$. Now,
General term $= T_{r+1} = {}^mC_r x^r$
Put r = 2, we have $T_3 = {}^mC_2 x^2$
According to the question $C(m, 2) = 6$
or $\frac{m(m-1)}{2!} = 6$
$\Rightarrow m^2 - m = 12$ or $m^2 - m - 12 = 0$

$\Rightarrow m^2-4m+3m-12=0$ or $(m-4)(m+3)=0$
$\therefore m=4$, since $m \neq -3$

64. (b) $T_{r+1}={}^5C_r(x^2)^{5-r}(k/x)^r={}^5C_r k^r x^{10-3r}$
For coefficient of $x, 10-3r=1 \Rightarrow r=3$
coefficient of $x={}^5C_3k^3=270$
$\Rightarrow k^3=\frac{270}{10}=27. \therefore k=3$

65. (d) $a_0+a_1+a_2+\cdots..=2^{2n}$ and $a_0+a_2+a_4+\cdots=2^{2n-1}a_n={}^{2n}C_n=$ the greatest coefficient, being the middle coefficient
$a_{n-3}={}^{2n}C_{n-3}={}^{2n}C_{2n-(n-3)}={}^{2n}C_{n+3}=a_{n+3}$

66. (c) $T_{3+1}=\frac{5}{2}\Rightarrow {}^nC_3(ax)^{n-3}\left(\frac{1}{x}\right)^3=\frac{5}{2}$
$\Rightarrow {}^nC_3a^{n-3}\cdot x^{n-6}=\frac{5}{2}$
$\Rightarrow n-6=0\Rightarrow n=6$
($\because$ RHS of above equality is independent of x)
Put $n=6$ in (i), we get ${}^6C_3a^3=\frac{5}{2}\Rightarrow a^3=\frac{1}{8}$
$\Rightarrow a=\frac{1}{2}$ and $n=6$. Hence, $a\times\frac{1}{n}=\frac{1}{2}\times 6=3$

67. (d) $T_{r+1}={}^{18}C_r(9x)^{18-r}\left(-\frac{1}{3\sqrt{x}}\right)^r$
$=(-1)^r\,{}^{18}C_r9^{18-\frac{3r}{2}}x^{18-\frac{3r}{2}}$
is independent of x provided $r=12$ and then $a=1$.

68. (b) $1+2x+3x^2+\cdots=(1+x)^{-2}$
$\Rightarrow(1+2x+3x^2+\cdots)^{-3/2}=\{(1+x)^{-2}\}^{-7/2}=(1+x)^7$
$\therefore$ The coefficient of x^5 in $(1+2x+3x^2+\cdots\ldots.)^{-7/2}$
$=$ Coefficient of x^5 in $(1+x)^7={}^7C_5=21$

69. (d) $T_{r+1}=\frac{n(n-1)(n-2)\ldots\ldots(n-r+1)}{r!}(x)^r$
For first negative term, $n-r+1<0\Rightarrow r>n+1$
$\Rightarrow r>\frac{32}{5}\therefore r=7\cdot\left(\because n=\frac{27}{5}\right)$
Therefore, first negative term is T_8.

70.
(c) $S_n=a_0C_0{}^2+a_1C_1{}^2+a_2C_2{}^2+\cdots\ldots.+a_nC_n{}^2$
$S_n=a_nC_n{}^2+a_{n-1}C_{n-1}{}^2+a_{n-2}{}^2+\cdots\ldots.+a_0C_0{}^2$
$2S_n=(a_0+a_n)C_0{}^2+(a_1+a_{n-1})C_1{}^2+\cdots\ldots.+(a_n+a_0)C_n{}^1$
$=(2n+2)(C_0{}^2+C_1{}^2+C_2{}^2+\cdots\ldots\ldots+C_n{}^2)$
$\therefore S_n=(n+1)^{2nC_n}$
$[\because a_0+a_n=a_1+a_{n-1}+..$

71. (d) ${}^{50}C_4+\sum_{r=1}^{6}{}^{56-r}C_3$

$$= {}^{50}C_4 + \left[{}^{55}C_3 + {}^{54}C_3 + {}^{53}C_3 + {}^{52}C_3 + {}^{51}C_3 + {}^{50}C_3\right]$$

We know $[{}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r]$

$= ({}^{50}C_4 + {}^{50}C_3) + {}^{51}C_3 + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$

$= ({}^{51}C_4 + {}^{51}C_3) + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$

Proceeding in the same way, we get

${}^{55}C_4 + {}^{55}C_3 = {}^{56}C_4.$

72. (b) Binomial expansion of

$(1+x)^{50} = C_0 + C_1x + C_2x^2 + C_3x^3 + \cdots + C_{50}x^{50}$

and in given expression

Putting $x = 1$, we get

$2^{50} = C_0 + C_1 + C_2 + C_3 \ldots + C_{50}$

and putting $x = -1$

$0 = C_0 - C_1 + C_2 - C_3 \ldots\ldots + C_{50}$

Subtracting (ii) from (i), we get

$2^{50} = 2(C_1 + C_3 + C_5 + \cdots C_{49})$

$\Rightarrow C_1 + C_3 + C_5 + \cdots C_{49} = \frac{2^{50}}{2} = 2^{49}$

Sum of the coefficient of odd powers of $x = 2^{49}$

73. (a)

$$\frac{r.{}^nC_r}{{}^nC_r - 1} = \frac{n \cdot {}^{n-1}C_{r-1}}{{}^nC_{r-1}}$$

$$= n \cdot \frac{(n-1)!}{(r-1)!\,(n-r)!} \times \frac{(r-1)!\,(n-r+1)!}{n!}$$

$$= n - r + 1$$

Sum $= n + (n-1) + \cdots. + (n-9) = 10n - 45$

EXERCISE - 2

1. (c) Since, $(x+a)^{100} + (x-a)^{100}$

$= ({}^{100}C_0x^{100} + {}_{100}C_1x^{99}a + \cdots + {}^{100}C_{100}a^{100})$

$+({}^{100}C_0x^{100} - {}^{100}C_1x^{99}a + \cdots + {}^{100}C_{100}a^{100})$

$= 2({}^{100}C_0x^{100} + {}^{100}C_2x^{98}a^2 + \cdots + {}^{100}C_{100}a^{100})$

So, total number of terms $= \frac{100}{2} + 1 = 51$

2. (a) $t_{r+2} = {}^{2n}C_{r+1}x^{r+1}; t_{3r} = {}^{2n}C_{3r-1}x^{3r-1}$

Given ${}^{2n}C_{r+1} = {}^{2n}C_{3r-1}; \Rightarrow {}^{2n}C_{2n-(r+1)} = {}^{2n}C_{3r-1}$

$\Rightarrow 2n - r - 1 = 3r - 1 \Rightarrow 2n = 4r \Rightarrow n = 2r$

3. (c) Suppose two successive terms in the exspansion of $(1+x)^{24}$ are $(r+1)^{\text{th}}$ and $(r+2)^{\text{th}}$ terms.

So, $T_{r+1} = {}^{24}C_rx^r$ and $T_{r+2} = {}^{24}C_{r+1}x^{r+1}$

$$\text{As, } \frac{{}^{24}C_r}{{}^{24}C_{r+1}} = \frac{1}{4} \Rightarrow \frac{\frac{(24)!}{r!(24-r)!}}{\frac{(24)!}{(r+1)!(24-r-1)!}} = \frac{1}{4}$$

$$\Rightarrow \frac{r+1}{24-r} = \frac{1}{4} \Rightarrow 5r = 20 \Rightarrow r = 4$$

Therefore, required terms are 5^{th} and 6^{th} terms.

4. (d) The required ratio $= \frac{{}^{2n}C_n}{{}^{2n-1}C_n} = \frac{\frac{(2n)!}{n!n!}}{\frac{(2n-1)!}{n!(n-1)!}} = \frac{(2n)!(n-1)!}{(2n-1)!n!} = \frac{2n}{n} = \frac{2}{1}$

5. (b) $2\,{}^{n}C_2 = {}^{n}C_1 + {}^{n}C_3 \Rightarrow n^2 - 9n + 14 = 0 \Rightarrow n = 2$ or 7

6. (b) We have $(1+x)^{2n} = {}^{2n}C_0 + {}^{2n}C_1 x + {}^{2n}C_2 x^2 + \cdots . {}^{2n}C_n x^n + \cdots . + {}^{2n}C_{2n} x^{2n}$ $(1+x)^{2n-1} = {}^{2n-1}C_0 + {}^{2n-1}C_1 x + {}^{2n-1}C_2 x^2 + \cdots . + {}^{2n-1}C_n x^n + \cdots . + {}^{2n-1}C_{2n-1} x^{2n-1}$

According to the given data and equations (i) and (ii), we can claim that

$A = {}^{2n}C_n$ and $B = {}^{2n-1}C_n$

$$\Rightarrow \frac{A}{B} = \frac{{}^{2n}C_n}{{}^{2n-1}C_n} = \frac{\frac{2n!}{n!n!}}{\frac{(2n-1)!}{n!(n-1)!}}$$

$$\Rightarrow \frac{A}{B} = \frac{2n(2n-1)!}{n(n-1)!} \times \frac{(n-1)!}{(2n-1)!} = 2 \Rightarrow A = 2B$$

7. (b) Total number of terms $= {}^{n+2}C_2 = 28$

$(n+2)(n+1) = 56; n = 6$

$\left(1 - \frac{2}{1} + \frac{4}{12} = 729\right)$

8. (a) We have $({}^{21}C_1 + {}^{21}C_2 \ldots\ldots + {}^{21}C_{10})$

$= \frac{1}{2}[({}^{10}C_1 + {}^{10}C_2 \ldots\ldots {}^{10}C_{10})$

$= \frac{1}{2}[2^{21} - 2] - (2^{10} - 1) = (2^{20} - 1) - (2^{10} - 1) = 2^{20} - 2^{10}C_{11} + \cdots . {}^{21}C_{20})\Big] - (2^{10} - 1)$

9. (c) Since we know that,

$$(x+a)^5 + (x-a)^5 = 2[{}^{5}C_0 x^5 + {}^{5}C_2 x^3 \cdot a^2 + {}^{5}C_4 x \cdot a^4]$$

$$\therefore \left(x + \sqrt{x^3 - 1}\right)^5 + \left(x - \sqrt{x^3 - 1}\right)^5$$

$$= 2[{}^{5}C_0 x^5 + {}^{5}C_2 x^3 (x^3 - 1) + {}^{5}C_4 x (x^3 - 1)^2]$$

$$= 2[x^5 + 10x^6 - 10x^3 + 5x^7 - 10x^4 + 5x]$$

$\therefore$ Sum of coefficients of odd degree terms $= 2$.

10. (b) Consider the expression

$$\left(\frac{1-t^6}{1-t}\right)^3 = (1-t^6)^3 (1-t)^{-3}$$

$$= (1 - 3t^6 + 3t^{12} - t^{18})\left(1 + 3t + \frac{3 \cdot 4}{2!} t^2 + \frac{3 \cdot 4 \cdot 5}{3!} t^3 + \frac{3 \cdot 4 \cdot 5 \cdot 6}{4!} t^4 + \cdots \infty\right)$$

Hence, the coefficient of $t = 1 \cdot \frac{3 \cdot 4 \cdot 5 \cdot 6}{4!}$

$= \frac{3 \times 4 \times 5 \times 6}{4 \times 3 \times 2 \times 1} = 15$

11. (b) General term of the given expansion

$T_{r+1} = {}^{16}C_r \left(\frac{x}{\sin\ \theta}\right)^{16-r} \left(\frac{1}{x\cos\ \theta}\right)^r$

For $r = 8$ term is free from ' x '

$T_9 = {}^{16}C_8 \frac{1}{\sin^8\ \theta \cos^8\ \theta}$ $T_9 = {}^{16}C_8 \frac{2^8}{(\sin\ 2\theta)^8}$

When $\theta \in \left[\frac{\pi}{8}, \frac{\pi}{4}\right]$, then least value of the term independent of x,

$l_1 = {}^{16}C_8 2^8$

[$\because$ min. value of l_1 at $\theta = \pi 4$]

When $\theta \in \left[\frac{\pi}{16}, \frac{\pi}{8}\right]$, then least value of the term independent of x,

$l_2 = {}^{16}C_8 = \frac{2^8}{\left(\frac{1}{\sqrt{2}}\right)^8} = {}^{16}\mathrm{C}_8 \cdot 2^8 \cdot 2^4$

[$\because$ min. value of l_2 at $\theta = \pi/8$]

Now, $\frac{l_2}{l_1} = \frac{{}^{16}C_8 \cdot 2^8 \cdot 2^4}{{}^{16}C_8 \cdot 2^8} = 16$

12. (4) $(2021)^{3762} = (2023 - 2)^{3762}$

$= m(17) + 2^{3762}$

$(\because 2023 = 17 \times 119)$

Where $m(17)$ denotes "multiple of 17 "

Required remainder = remainder on dividing 2^{3762} by 17.

Then $2^{3762} = 4.16^{940} = 4.(1 - 17)^{40} = m(17) + 4$

Here required remainder is 4 .

13. (a) Given expression is $\sum_{\substack{i,j=0 \\ i \neq j}}^{n} {}^nC_i {}^nC_j$

$$= \sum_{i=0}^{n} {}^nC_i \cdot \sum_{j=0}^{n} {}^nC_j - \sum_{i=j=0}^{n} ({}^nC_i)^2$$

$$= (2^n)(2^n) - {}^{2n}C_n = 2^{2n} - {}^{2n}C_n$$

14. (23)Given expression is $(1+x)^p(1-x)^q$ and coefficient of x and x^2 is -3 and -5 respectively.

$(1+x)^p(1-x)^q = (1 + {}^pC_1 x + {}^pC_2 x^2) \ldots)(1 - {}^qC_1 \mathrm{x} + {}^qC_2 x^2 \ldots)$

According to question,

${}^{-q}C_1 + {}^pC_1 = -3 \Rightarrow p - q = -3$

${}^9C_2 + {}^PC_2 - {}^PC_1 {}^9C_1 = -5$

$\frac{\mathrm{q(q-1)}}{2} - \mathrm{pq} + \frac{\mathrm{p(p-1)}}{2} = -5$

$q^2 - q - 2pq + p^2 - p = -10$

$(p-q)^2 - (p+q) = -10$

from(i),

$(-3)^2 - (p+q) = -10$

$9 + 10 = (p+q) = 19$

Add (i) & (ii), $\mathrm{p} = 8, \mathrm{q} = 11$.

Coefficient of $x^3 = (1+x)^8(1-x)^{11} = (1-x^2)^8(1-x)^3$

$= 1 \times (-1) + (-8) \times (-3) = -1 + 24 = 23$

15. (Bonus) T_{1011} from beginning $= T_{1010+1}$

$= {}^{2022}C_{1010}\left(\frac{4x}{5}\right)^{1012}\left(\frac{-5}{2x}\right)^{1010}$ $[\because T_{r+1} = {}^{n}C_r a^{n-r} b]$

T_{1011} from end $= {}^{2022}C_{1010}\left(\frac{-5}{2x}\right)^{1012}\left(\frac{4x}{5}\right)^{1010}$

ATQ $\Rightarrow {}^{2022}C_{1010}\left(\frac{-5}{2x}\right)^{1012}\left(\frac{4x}{5}\right)^{1010}$

$= 2^{10.2022}C_{1010}\left(\frac{-5}{2x}\right)^{1010}\left(\frac{4x}{5}\right)^{1012}$

$\Rightarrow \left(\frac{-5}{2x}\right)^2 = 2^{10}\left(\frac{4x}{5}\right)^2 \Rightarrow x^4 = \frac{5^4}{2^{16}} \Rightarrow |x| = \frac{5}{16}$

16.

(c) $\sum_{r=1}^{26} \frac{1}{(2r-1)!(51-(2r-1))!}$

$= \sum_{r=1}^{26} \frac{51!}{51!\,(2r-1)!\,(51-(2r-1))!} = \sum_{r=1}^{26} {}^{s1}C_{(2r-1)} \frac{1}{51!}$

$= \frac{1}{51!}\{{}^{51}C_1 + {}^{51}C_3 + \cdots \ldots + {}^{51}C_{51}\} = \frac{1}{51!}(2^{50})$

EXERCISE - 3

1. (c) Let $x = 2^{\frac{153}{2}} : \alpha = x^2 + \sqrt{2}x + 1$

Let $y = 2^{204}$ $\therefore \beta = y^2 - y + 1$

$\Rightarrow N = x^{16} - 1 \Rightarrow N = (x^4-1)(x^4+1)(x^8+1)$

$\Rightarrow N = (x^4-1)(x^2+\sqrt{2}x+1)(x^2-\sqrt{2}x+1)(x^8+1)$

$\Rightarrow N = y^6 - 1 = (y^3-1)(y^3+1) = (y^3-1)(y+1)(y^2-y+1)$

2. (b) We know that $\frac{C_k}{C_{k-1}} = \frac{{}^{n}C_k}{{}^{n}C_{k-1}} = \frac{n-k+1}{k}$

$\therefore \sum_{k=1}^{n} k^3 \left(\frac{C_k}{C_{k-1}}\right)^2 = \sum_{k=1}^{n} k^3 \left(\frac{n-k+1}{k}\right)^2 = \sum_{k=1}^{n} k(n-k+1)^2$

Put $n - k + l = p \Rightarrow k = n - p + l$

when $k = 1$, then $p = n$ and when $k = n, p = 1$.

$\therefore$ Series $= \sum_{p=n}^{1}(n-p+1)p^2 = \sum_{p=1}^{n}(np^2 - p^3 + p^2)$

$= \sum_{p=1}^{n}(n+1)p^2 - \sum_{p=1}^{n} p^3$

$= \frac{(n+1)n(n+1)(2n+1)}{6} - \frac{n^2(n+1)^2}{4}$

$= \frac{n(n+1)^2}{2}\left[\frac{2n+1}{3} - \frac{n}{2}\right] = \frac{n(n+1)^2(n+2)}{12}$

3. (b) Given $(2+\sqrt{3})^n = I + f$, where I is integer and $0 \le f < 1$. We note that $(2+\sqrt{3})(2-\sqrt{3}) = 1$. So let us assume that $F = (2-\sqrt{3})^n$. Clearly $0 < F < 1$. Now,

$I + f + F = (2+\sqrt{3})^n + (2-\sqrt{3})^n$

$= 2[{}^{n}C_0 2^n + {}^{n}C_2 2^{n-2} \cdot 3 + {}^{n}C_4 \cdot 2^{n-4} \cdot 3^2 +$

$= 2 \times \text{Integer} = \text{Integer}$
$\because I + f + F$ is integer $\Rightarrow f + F$ must be integer.
$\therefore 0 \leq f < 1$ and $0 < F < 1 \Rightarrow 0 < f + F < 2$
$\Rightarrow f + F = 1 \Rightarrow F = 1 - f$
$\therefore (I + f)(1 - f) = (I + f)F = (2 + \sqrt{3})^n \cdot (2 - \sqrt{3})^n = 1$

4.

(b) $x + \frac{1}{x} = 1 \Rightarrow x^2 - x + 1 = 0 \Rightarrow x = \frac{1 \pm \sqrt{3}i}{2}$
$\Rightarrow x = -\omega, -\omega^2$
Now, $p = \omega^{1000} + \frac{1}{\omega^{1000}} = (\omega^3)^{333} \cdot \omega + \frac{1}{(\omega^3)^{333} \cdot \omega}$
$= \omega + \frac{1}{\omega} = \omega + \omega^2 = -1$
Similarly, for $x = -\omega^2$, also $p = -1$
For $n > 1, 2^n = 4k, k \in N$
$\therefore 2^{2^n} = 2^{4k} = (16)^k =$ a number with last digit $= 6$
$\therefore q = ($ the digit at unit place in $2^{2^n}) + 1 = 6 + 1 = 7$
$\therefore p + q = 7 + (-1) = 6$

5. (d) ${}^nC_r + 4 \cdot {}^nC_{r+1} + 6 \cdot {}^nC_{r+2} + 4.{}^nC_{r+3} + {}^nC_{r+4}$
$= ({}^nC_r + {}^nC_{r+1}) + 3 \cdot ({}^nC_{r+1} + {}^nC_{r+2})$
$+3({}^nC_{r+2} + {}^nC_{r+3}) + ({}^nC_{r+3} + {}^nC_{r+4})$
$= {}^{n+1}C_{r+1} + 3.{}^{n+1}C_{r+2} + 3.{}^{n+1}C_{r+3} + {}^{n+1}C_{r+4}$
$= ({}^{n+1}C_{r+1} + {}^{n+1}C_{r+2}) + 2 \cdot ({}^{n+1}C_{r+2} + {}^{n+1}C_{r+3}) + ({}^{n+1}C_{r+3} + {}^{n+1}C_{r+4})$
$= {}^{n+2}C_{r+2} + 2.{}^{n+2}C_{r+3} + {}^{n+2}C_{r+4}$
$= ({}^{n+2}C_{r+2} + {}^{n+2}C_{r+3}) + ({}^{n+2}C_{r+3} + {}^{n+2}C_{r+4})$
$= {}^{n+3}C_{r+3} + {}^{n+3}C_{r+4} = {}^{n+4}C_{r+4} = \frac{n+4}{r+4}\,{}^{n+3}C_{r+3}$
Similarly, ${}^nC_r + 3.{}^nC_{r+1} 3 \cdot {}^nC_{r+2} + {}^nC_{r+3} = {}^{n+3}C_r + 3$
$\therefore \frac{n+4}{r+4} = \frac{n+\lambda}{r+\lambda} \Rightarrow \lambda = 4$

6. (c) Genral term $T_{r+1} = {}^{16}C_r \left(\frac{x}{\cos\theta}\right)^{16-r} \left(\frac{1}{x\sin\theta}\right)^r$
$= {}^{16}C_r \frac{1}{(\cos\theta)^{16-r}(\sin\theta)^r} x^{16-2r}$
If this term is independent of x, then $16 - 2r = 0$
$\therefore$ The term independent of $x = {}^{16}C_8 \frac{1}{\cos^8\theta \sin^8\theta}$
$= {}^{16}C_8 \frac{2^8}{\sin^8 2\theta}; l_1 = {}^{16}C_8 \frac{2^8}{\sin^8 \frac{\pi}{2}} = {}^{16}C_8 2^8$
$l_2 = {}^{16}C_8 \frac{2^8}{\sin^8 \frac{\pi}{4}} = {}^{16}C_8 \cdot \frac{2^8}{\left(\frac{1}{\sqrt{2}}\right)^8} = {}^{16}C_8 2^{12}$
$\therefore \frac{l_2}{l_1} = \frac{2^{12}}{2^8} = 2^4 = 16$

7. (c) Since $f(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n + \cdots$
and $f(x)(1-x)^{-1} = b_0 + b_1x + b_2x^2 + \cdots \ldots + b_nx^n + \cdots$

then $f(x)\{1+x+x^2+\cdots+x^n+\cdots\infty\}$

$= b_0+b_1x+b_2x^2+\cdots+b_nx^n$

Such that the coefficient of x^n on both sides is

$a_0+a_1+a_2+\cdots+a_{n-1}+a_n=b_n$

Also, coefficient of x^{n-1} on both sides is

$a_0+a_1+a_2+\cdots+a_{n-1}=b_{n-1} \Rightarrow b_n-b_{n-1}=a_n$

Also, coefficient of b_0 and b_1 on both sides respectively

are $b_0=a_0$ and $b_1=a_0+a_1$

$\Rightarrow b_0=1$ and $3=a_0+a_1 \Rightarrow a_1=2$

Since, given that $\frac{a_0}{a_1}=\frac{a_1}{a_2}=\cdots=$ constant

$\therefore b_{10}=a_0+a_1+a_2+\cdots+a_{10}$

$=1+2+2^2+\cdots+2^{10}=2^{11}-1=2047$

8. (d) $\sum_{r=3}^{52}\frac{r}{(r+1)!}=\sum_{r=3}^{52}\left[\frac{1}{r!}-\frac{r}{(r+1)!}\right]=\frac{1}{3!}-\frac{1}{53!}$

$\Rightarrow k=50$

9. (d) $(1+\omega)^n={}^nC_0+{}^nC_1\omega+{}^nC_2\omega^2+\cdots..+{}^nC_n\omega^n$

$(1+1)^n={}^nC_0+{}^nC_1+{}^nC_2+\cdots..+{}^nC_n$

$(1+\omega)^n+(1+1)^n=2C_0+C_1(1+\omega)+C_2(1+\omega^2)+C_3(1+\omega^3)$

$+C_4(1+\omega)+C_5(1+\omega^2)+C_6(1+\omega^3)+\cdots..C_n(1+\omega^n)$

$2(C_0+C_3+C_6+\cdots.)+(C_1+C_4+C_7+\cdots)(1+\omega)+(C_2+C_5+C_8+\cdots..)(1+\omega^2)=-\omega^{2n}+2^n$

$=(2^n-1)(\because n$ in a multiple of $3, \omega^n=1)$

10. (d) $a_5X^{P_5}={}^{15}C_{10}X^{10}; a_{11}X^{P_{11}}={}^{15}C_{13}\cdot X^{13}$

$a_9X^{P_9}={}^{15}C_{12}\cdot X^{12}; a_{14}X^{P_{14}}={}^{15}C_0\cdot X^0$

11. (c) Consider

$\sum_{k=1}^{n}\frac{k^2}{k!\,(n-k)!}=\frac{1}{n!}\sum_{k=1}^{n}\frac{n!\,k^2}{k!\,(n-k)!}$

$=\frac{1}{n!}\sum_{k=1}^{n}k^2\,{}^nC_k=\frac{1}{n!}\sum_{k=1}^{n}k\cdot k^nC_k$

$=\frac{1}{n!}\sum_{k=1}^{n}k\cdot n\cdot{}^{n-1}C_{k-1}=\frac{n}{n!}\sum_{k=1}^{n}(k-1+1)^{n-1}C_{k-1}$

$=\frac{1}{(n-1)!}\sum_{k=1}^{n}[(k-1)^{n-1}C_{k-1}+{}^{n-1}C_{k-1}]$

$=\frac{1}{(n-1)!}\sum_{k=1}^{n}[(n-1)^{n-2}C_{k-2}+{}^{n-1}C_{k-1}]$

$=\frac{1}{(n-2)}\sum_{k=1}^{n}{}^{n-2}C_{k-2}+\frac{1}{(n-1)!}\sum_{k=1}^{n}{}^{n-1}C_{k-1}$

$=\frac{2^{n-2}}{(n-2)!}+\frac{2^{n-1}}{(n-1)!}$

$\therefore a_n=(\log_e 3)^n\left[\frac{2^{n-2}}{(n-2)!}+\frac{2^{n-1}}{(n-1)!}\right]$

$\therefore a_1+a_2+a_3+\cdots\ldots\ldots$

$=(\log 3)^2\sum_{n=1}^{\infty}\frac{(2\log 3)^{n-2}}{(n-2)!}+(\log 3)\sum_{n=1}^{\infty}\frac{(2\log 3)^{n-1}}{(n-1)!}$

$=(\log 3)^2e^{2\log 3}+(\log 3)e^{2\log 3}$

$= (\log_e 3)^2(9) + (\log_e 3)(9) = 9\log_e 3(\log_e 3 + 1)$

12. (c) (a) $(\sqrt{2}+1)^6 = \mathrm{I} + \mathrm{F}$, where I is integer and $0 \le \mathrm{F} < 1$ and $(\sqrt{2}-1)^6 = \mathrm{G}$, where $0 < G < 1$

$\therefore \mathrm{I} + \mathrm{F} + \mathrm{G} = (\sqrt{2}+1)^6 + (\sqrt{2}-1)^6$

$= 2[{}^6C_0 2^3 + {}^6C_2 2^2 + {}^6C_4 2 + {}^6C_6] = 198$

Now $0 < F + G < 2$

But $\mathrm{F} + \mathrm{G} = 198 - \mathrm{I}$ is an integer

$\therefore \mathrm{F} + \mathrm{G} = 1 \Rightarrow \mathrm{I} = 198 - 1 = 197$

(b) $(\sqrt{3}+1)^{2m} = \left[(\sqrt{3}+1)^2\right]^m = (4 + 2\sqrt{3})^m = 2^m(2+\sqrt{3})^m$

Now, $(\sqrt{3}-1)^{2m} < 1$ as $\sqrt{3} - 1 = 0.732$

Also, $(\sqrt{3}+1)^{2m} + (\sqrt{3}-1)^{2m}$

$= 2[3^m + {}^{2m}C_2 3^{m-1} + \cdots.]$ is an integer

$\Rightarrow$ required integer.

$= (\sqrt{3}+1)^{2m} + (\sqrt{3}-1)^{2m}$

$= 2^m(2+\sqrt{3})^m + 2^m(2-\sqrt{3})^m$

$= 2^m[2(2^m + {}^mC_2 2^{m-2} \cdot 3 + {}^mC_4 \cdot 2^{m-4} 3^2 + \cdots)]$

$= 2^{m+1} \times$ integer

(c) Let I $+ \mathrm{f} = (7 + 4\sqrt{3})^m 0 \le \mathrm{f} < 1$

Consider, $F = (7 - 4\sqrt{3})^m \; 0 \le \mathrm{F} < 1$

$\therefore \mathrm{I} + \mathrm{f} + \mathrm{F} = 2\left(7^m + {}^mC_2 7^{m-2}(4\sqrt{3})^2 + \cdots..\right]$

$= 2\mathrm{k}$, where k is an integer

$\Rightarrow \mathrm{f} + \mathrm{F} = 2\mathrm{k} - \mathrm{I}$ is an integer

$\because 0 \le \mathrm{f} + \mathrm{F} < 2 \Rightarrow f + F = 1$

$\Rightarrow \mathrm{I} = 2\mathrm{k} - 1$, an odd integer.

Thus I cannot be a multiple of 2 .

(d) Let $F = (6\sqrt{6} - 14)^{2n+1}$

Then, $R - F = (6\sqrt{6} + 14)^{2n+1} - (6\sqrt{6} - 14)^{2n+1}$

$2\left[{}^{2n+1}C_1(6\sqrt{6})^{2n}(14) + \cdots.\right]$ = an even integer

$\Rightarrow [\mathrm{R}] + \mathrm{f} - \mathrm{F}$ = an even integer

Also $0 \le \mathrm{f} < 1$ and $0 < F < 1$

$\therefore -1 < f - F < 1$ and $\mathrm{f} - \mathrm{F}$ is integer

$\therefore \mathrm{f} - \mathrm{F} = \mathbf{0} \Rightarrow \mathrm{f} = \mathrm{F}$

So, $Rf = RF = (6\sqrt{6} + 14)^{2n+1}(6\sqrt{6} - 14)^{2n+1} = 20^{2n+1}$

13. (a) We have,

$$\sum_{r=1}^{m} {}^{m+1}C_r s_r = \sum_{r=1}^{m} {}^{m+1}C_r(1^r + 2^r + \cdots + n^r)$$
$$= \sum_{k=1}^{n} \left\{\sum_{r=1}^{m} {}^{m+1}C_r k^r\right\}$$
$$= \sum_{k=1}^{n} \left[\left\{\sum_{r=0}^{m+1} {}^{m+1}C_r k^r\right\} - {}^{m+1}C_0 - {}^{m+1}C_{m+1}k^{m+1}\right]$$
$$= \sum_{k=1}^{n} \{(1+k)^{m+1} - 1 - k^{m+1}\} = \sum_{k=1}^{n} \{(1+k)^{m+1} - k^{m+1}\} - \sum_{k=1}^{n} 1$$
$$= \sum_{k=1}^{n} \{(1+k)^{m+1} - k^{m+1}\} - n$$
$$= \{(2^{m+1} - 1^{m+1}) + (3^{m+1} - 2^{m+1}) + \cdots$$
$$+\{(n+1)^{m+1} - n^{m+1}\}\} - n$$
$$= \{(n+1)^{m+1} - 1\} - n = (n+1)^{m+1} - (n+1)$$

Now put $n = 10$ and $m = 11$ then

$(n+1)^{m+1} - (n+1) = 11^{12} - 11$

14. (c) We know that $({}^nC_0)^2 + ({}^nC_1)^2 + \cdots + ({}^nC_n)^2 = {}^{2n}C_n$ and $({}^nC_0)^2 - ({}^nC_1)^2 + \cdots + ({}^nC_n)^2$

$= \begin{cases} 0 & \text{if } n \text{ is odd} \\ {}^nC_{n/2}(-1)^{n/2} & \text{, if } n \text{ is even} \end{cases}$

From this $({}^{31}C_0)^2 - ({}^{31}C_0)^2 + ({}^{31}C_2)^2 - \cdots - ({}^{31}C_{31})^2 = 0$

$({}^{32}C_0)^2 - ({}^{32}C_1)^2 + ({}^{32}C_2)^2 - \cdots + ({}^{32}C_{32})^2 = -{}^{32}C_{16}$

$({}^{34}C_0)^2 - ({}^{34}C_1)^2 + ({}^{34}C_2)^2 - \cdots + ({}^{34}C_{32})^2 = -{}^{34}C_{17}$

$({}^{32}C_0)^2 + ({}^{32}C_1)^2 + ({}^{32}C_2)^2 - \cdots + ({}^{32}C_{32})^2 = {}^{64}C_{32}$

Obviously ${}^{64}C_{32}$ is greatest.

15. (b) If **n** is odd, then numerically greatest coefficient in the expansion of $(1-x)^n$ is $\frac{{}^nC_{n-1}}{2}$ or $\frac{{}^nC_{n+1}}{2}$.

Therefore in $(1-x)^{21}$, the numerically greatest coefficient is ${}^{21}C_{10}$ or ${}^{21}C_{11}$. So, the numerically greatest term

$= {}^{21}C_{11}x^{11}$ or ${}^{21}C_{10}x$ So,

$|{}^{21}C_{11}x^{11}| > |{}^{21}C_{12}x^{12}|$ and $|{}^{21}C_{10}x^{10}| > |{}^{21}C_9 \cdot x^9|$

$\Rightarrow \frac{21!}{10!11!} > \frac{21!}{9!12!}x$ and $\frac{21!}{11!10!}x > \frac{21!}{9!12!} (\because x > 0)$

$\Rightarrow x < \frac{6}{5}$ and $x > \frac{5}{6} \Rightarrow x \in \left(\frac{5}{6}, \frac{6}{5}\right)$

EXERCISE - 4

1. (5) $\sum_{r=0}^{n} \frac{r+2}{r+1} {}^nC_r = \frac{2^8-1}{6}$

$$\Rightarrow \sum_{r=0}^{n} \left[1 + \frac{1}{r+1}\right] {}^nC_r = \frac{2^8-1}{6}$$

$\Rightarrow 2^{n}+\sum_{r=0}^{n} \frac{1}{n+1} \cdot {}^{n+1}C_{r+1}=\frac{2^{8}-1}{6}$

$\Rightarrow 2^{n}+\frac{2^{n+1}-1}{n+1}=\frac{2^{8}-1}{6}$

$\Rightarrow \frac{2^{n}(n+1+2)-1}{n+1}=\frac{2^{5}(6+2)-1}{6}$

Comparing we get $n+1=6 \Rightarrow n=5$

2. (12.25) $(x+10)^{50}+(x-10)^{50}$

$=a_{0}+a_{1} x+a_{2} x^{2}+\equiv+a_{50} x^{50}$

$\therefore a_{0}+a_{1} x+a_{2} x^{2}+\equiv+a_{50} x^{50}$

$=2\left({}^{50}C_{0} x^{50}+{}^{50}C_{2} x^{48} \cdot 10^{2}+{}^{50}C_{4} x^{46} \cdot 10^{4}+\cdots\right)$

$\therefore \quad a_{0}=2 . {}^{50}C_{50} 10^{50}$

$a_{2}=2 . {}^{50}C_{2} \cdot 10^{48}$

$\therefore \quad \frac{a_{2}}{a_{0}}=\frac{{}^{50}C_{2} \times 10^{48}}{{}^{50}C_{50} 10^{50}}$

$\frac{50 \times 49}{2 \times 100}=\frac{49}{4}=12.25$

3. (24) $\left(x+\sqrt{x^{3}-1}\right)^{6}+\left(x-\sqrt{x^{3}-1}\right)^{6}$

$=2\left[{}^{6}C_{0} x^{6}+{}^{6}C_{2} x^{4}\left(x^{3}-1\right)+{}^{6}C_{4} x^{2}\left(x^{3}-1\right)^{2}+{}^{6}C_{6}\left(x^{3}-1\right)^{3}\right]$

$=2\left[x^{6}+15 x^{7}-15 x^{4}+15 x^{8}-30 x^{5}+15 x^{2}+x^{9}-3 x^{6}+3 x^{3}-1\right]$

Hence, the sum of coefficients of even powers of $x=2[1-15+15+15-3-1]=24$

4. (10) $\therefore$ Fourth term is equal to 200 .

$$T_{4}={}^{6}C_{3}\left(\sqrt{\left(\frac{1}{x^{1+\log_{10} x}\right)}}\right)^{3}\left(x^{\frac{1}{12}}\right)^{3}=200$$

$$\Rightarrow 20 x^{\frac{3}{2(1+\log_{10} x)}} x^{\frac{1}{4}}=200$$

$$\Rightarrow x^{\frac{1}{4}+\frac{3}{2(1+\log_{10} x)}}=10$$

Taking $\log_{10}$ on both sides and putting $\log_{10} x=t$

$\left(\frac{1}{4}+\frac{3}{2(1+t)}\right) t=1 \Rightarrow t^{2}+3 t-4=0$

$\Rightarrow t^{2}+4 t-t-4=0 \Rightarrow t(t+4)-1(t+4)=0$

$\Rightarrow t=1$ or $t=-4$

$\log_{10} x=1 \Rightarrow x=10$

or $\log_{10} x=-4 \Rightarrow x=10^{-4}$

According to the question $x>1, \therefore x=10$.

5. (7) Given expression

$\left[x+\left(x^{3}-1\right)^{1/2}\right]^{5}+\left[x-\left(x^{3}-1\right)^{1/2}\right]^{5}$

$\because \{\left[(x+y)^{n}+(x-y)^{n}\right]=2\left[{}^{n}C_{0} x^{n} y^{0}+{}^{n}C_{2} x^{n-2} y^{2}\right.$

$\left.\left.+{}^{n}C_{4} x^{n-4} y^{4}+\cdots\right]\right\}$

$=2\left[x^{5}+{}^{5}C_{2} x^{3}\left\{\left(x^{3}-1\right)^{1/2}\right\}^{2}+{}^{5}C_{4} x\left\{\left(x^{3}-1\right)^{1/2}\right\}^{4}\right]$

$=2\left[x^{5}+10 x^{3}\left(x^{3}-1\right)+5 x\left(x^{3}-1\right)^{2}\right]$

$= 2[5x^7 + 10x^6 + x^5 - 10x^4 - 10x^3 + 5x]$,
which is a polynomial of degree 7 .

6. (9) $27^{40} = 3^{120}$

$$3^{119} = (4-1)^{119} = {}^{119}C_0 4^{119} - {}^{119}C_1 4^{118} + {}^{119}C_2 4^{117} - {}^{119}C_3 4^{116} + \cdots + (-1)$$

$\therefore 3^{119} = 4k - 1$

$\therefore 3^{120} = 12k - 3 = 12(k-1) + 9$

$\therefore$ the required remainder is 9 .

7. (10) $T_3 = {}^nC_2 (x)^{n-2} \left(-\frac{1}{2x}\right)^2$

and $T_4 = {}^nC_3 (x)^{n-3} \left(-\frac{1}{2x}\right)^3$

But according to the condition,

$\frac{-n(n-1)\times 3\times 2\times 1\times 8}{n(n-1)(n-2)\times 2\times 1\times 4} = \frac{1}{2} \Rightarrow n = -10$.

8. (12) Since n is even therefore $\left(\frac{n}{2}+1\right)^{\text{th}}$ term is middle term, hence ${}^nC_{n/2} (x^2)^{n/2} \left(\frac{1}{x}\right)^{n/2}$

$= 924x^6 \Rightarrow x^{n/2} = x^6 \Rightarrow n = 12$

9. (80) $(1-y)^m (1+y)^n$

$= [1 - {}^mC_1 y + {}^mC_2 y^2 - \cdots..]$

$$= 1 + (n-m) + \left\{\frac{m(m-1)}{2} + \frac{n(n-1)}{2} - mn\right\} y^2 + \cdots..$$

$$\therefore a_1 = n - m = 10$$

$$\text{and } a_2 = \frac{m^2 + n^2 - m - n - 2mn}{2} = 10$$

So, $n - m = 10$ and $(m-n)^2 - (m+n) = 20$

$\Rightarrow m + n = 80$

10. (0.3) The middle term in the expansion of

$(1 + \alpha x)^4 = T_3 = {}^4C_2 (\alpha x)^2 = 6\alpha^2 x^2$

The middle term in the expansion of

$(1 - \alpha x)^6 = T_4 = {}^6C_3 (-\alpha x)^3 = -20\alpha^3 x^3$

According to the question

$$6\alpha^2 = -20\alpha^3 \Rightarrow \alpha = -\frac{3}{10}$$

11.

(33) $T_{r+1} = {}^{256}C_r (\sqrt{3})^{256-r} (\sqrt[8]{5})^r$

$= {}^{256}C_r (3)^{\frac{256-r}{2}} (5)^{r/8}$

Terms will be integral if $\frac{256-r}{2}$ & $\frac{r}{8}$ both are +ve integer, which is so if r is an integral multiple of 8 .

As $0 \leq r \leq 256$
$r = 0,8,16,24,32, \ldots .256.$
n^{th} term $= 256$
$\Rightarrow a + (n-1)d = 256$
$\Rightarrow 0 + (n-1)8 = 256$
$\Rightarrow n - 1 = 32$
$\Rightarrow n = 33$

There are 33 integral terms in the expansion.

12\. (10) T_7 in $\left[\sqrt[6]{3}\sqrt{2} + \frac{1}{\sqrt[3]{3}}\right]^n = {}^nC_6\left(2^{1/2}3^{1/6}\right)^{n-6}\left[\frac{1}{3^{1/3}}\right]^6$

7^{th} term from the end in $\left[\sqrt[6]{3}\sqrt{2} + \frac{1}{\sqrt[3]{3}}\right]^n$

$= T_{n-6}$ in $\left[\frac{1}{\sqrt[3]{3}} + \sqrt[6]{3}\sqrt{2}\right]^n = {}^nC_6\left(\frac{1}{3^{1/3}}\right)^{n-6}\left(3^{1/6}2^{1/2}\right)^6$

$$\therefore \frac{{}^nC_6\left[2^{1/2}3^{1/6}\right]^{n-6}\left[\frac{1}{3^{1/3}}\right]^6}{{}^nC_6\left[\frac{1}{3^{1/3}}\right]^{n-6}\left[2^{1/2}3^{1/6}\right]^6} = \frac{1}{6}$$

$$\Rightarrow \frac{\left[2^{\frac{n-6}{2}}3^{\frac{n-6}{2}}\right] \times [3^{-2}]}{3^{-\left(\frac{n-6}{3}\right)}2^3 \cdot 3^1} = \frac{1}{6} \Rightarrow \frac{2^{\frac{n-6}{2}}3^{\frac{n-6}{6}}}{3^{\frac{6-n}{3}}2^3 \cdot 3^1}\frac{1}{3^2} = \frac{1}{6}$$

$$\Rightarrow \frac{2^{\frac{n-6}{2}}}{3^{\frac{6-n}{3}}} \cdot \frac{1}{3^{-\left(\frac{n-6}{6}\right)}} \cdot \frac{1}{2^3 \cdot 3^3} = \frac{1}{6}$$

$$\Rightarrow \frac{2^{\frac{n-6}{2}}}{3^{\frac{6-n}{2}}} = \frac{2^3 \cdot 3^3}{6} \Rightarrow 2^{\frac{n-6}{2}} \cdot 3^{\frac{n-6}{2}} = 2^2 \times 3^2$$

Comparing the powers of 2 and 3 , we get

$\frac{n-6}{2} = 2 \Rightarrow n = 10.$

13\. (24) Given binomial expression is

$(3 + 6x)^n = {}^nC_0 3^n + {}^nC_1 3^{n-1}(6x)^1 + \cdots$

General term is shown below.

$T_{r+1}\,{}^nC_r 3^{n-r} \cdot (6x)r = {}^nC_r 3^{n-r} \cdot 6^r \cdot x^r$

$= {}^nC_r 3^{n-r} \cdot 3^r \cdot 2^r \cdot \left(\frac{3}{2}\right)^r = {}^nC_r 3^n \cdot 3^r \left[\text{ for } x = \frac{3}{2}\right]$

T_9 is greatest of $x = \frac{3}{2}$

So, $T_9 > T_{10}$ and $T_9 > T_8$

Here,

$\frac{T_9}{T_{10}} > 1$ and $\frac{T_9}{T_8} > 1 \Rightarrow \frac{{}^nC_8 3^n \cdot 3^8}{{}^nC_9 3^n \cdot 3^9} > 1$ and $\frac{{}^nC_8 3^n \cdot 3^8}{{}^nC_7 3^n \cdot 3^7} > 1$

So, $\frac{{}^nC_8}{{}^nC_7} > \frac{1}{3}$ and $\frac{n-7}{8} > \frac{1}{3} \Rightarrow \frac{29}{3} < n < 11 \Rightarrow n = 10 = n_0$

So, in $(3 + 6x)^n$ for $n = n_0 = 10$

Now, Take $(3 + 6x)^{10}$, here $T_{r+1} = {}^{10}C_r\,{}^{10-r}6^r x^r$

$T_7 = {}^{10}C_6 3^4 \cdot 6^6 \cdot x^6 = 210.3^{10}.2^6x^6$

$T_4 = {}^{10}C_3 3^7 \cdot 6^3 \cdot x^3 = 120.3^{10} \cdot 2^3x^3$

Ratio of coefficient of x^6 and coefficient of $x^3 = k \therefore \mathrm{k} = \frac{210.3.^{10}2^6}{120.3^{10}.2^3} = \frac{7}{4} \times 2^3 = 14$

Therefore, $\mathrm{k} + \mathrm{n}_0 = 14 + 10 = 24$.

14. (5) $T_{r+1} = (-1)^r \cdot {}^{15}C_r \cdot 2^{15-r} x^{\frac{15-2r}{5}} \Rightarrow m = {}^{15}C_{10} 2^5$ for Coefficient of x^{-1}

$$\frac{15-r}{5} - \frac{r}{5} = -1 \Rightarrow r = 10 \Rightarrow n = -1$$

Given, $mn^2 = {}^{15}C_5 2^5$

15. (5) Given binomial expansion is $\left(\frac{\sqrt{x}}{5^{\frac{1}{4}}} + \frac{\sqrt{5}}{x^{\frac{1}{3}}}\right)^{60}$.

Take general term of binomial expansion

$$T_{r+1} = {}^{60}C_r \left(\frac{x^{\frac{1}{2}}}{5^{\frac{1}{4}}}\right)^{60-r} \left(\frac{5^{\frac{1}{2}}}{x^{\frac{1}{3}}}\right)^r$$

$$= {}^{60}C_r 5 \frac{3r-60}{4} \cdot x \frac{180-5r}{6}$$

The power of x should be equal to 10 .

$$\frac{180-5r}{6} = 10 \Rightarrow r = 24$$

Coeff. of $x^{10} = {}^{60}C_{24} 5^3 = \frac{60}{\underline{2436}} 5^3$

Powers of 5 in $= {}^{60}C_{24} \cdot 5^3 = \frac{5^{14}}{5^4 \times 5^8} \times 5^3 = 5^5$

16. (83) Given binomial expansion is $\left(2x^3 + \frac{3}{x}\right)^{10}$.

$$T_{r+1} = {}^{10}C_r (2x^3)^{10-r} \left(\frac{3}{x}\right)^r = {}^{10}C_r 2^{10-r} 3^r x^{30-4r}$$

Put $r = 0,1,2, \ldots 7$

$$= {}^{10}C_0 2^{10} 3^0 + {}^{10}C_1 2^9 3 + {}^{10}C_2 2^8 3^2 + \cdots$$
$$+ {}^{10}C_{10} 2^0 3^{10} - ({}^{10}C_8 2^2 3^8 + {}^{10}C_9 \cdot 2 \cdot 3^9 + {}^{10}C_0 3^{10})$$

use $(a+b)^n$ expansion,

$$= (2+3)^{10} - (3 \times 5 \times 4 \times 3^9 + 2 \times 5 \times 2 \times 3^9 + 3 \cdot 3^9)$$
$$= (5)^{10} - 3^9(60 + 20 + 3) = 5^{10} - 8^3 9^3$$

Compare with given equation. Then, $B = 83$.

17.

(84) $\frac{T_5}{T_{n-3}} = \frac{{}^nC_4 (2^{1/4})^{n-4} (3^{-1/4})^4}{{}^nC_{n-4} (2^{1/4})^4 (3^{-1/4})^{n-4}} = \frac{\sqrt[4]{6}}{1}$

$$\Rightarrow (6)^{\frac{n-8}{4}} = 6^{1/4} \Rightarrow 6^{n-8} = 6$$
$$\Rightarrow \mathrm{n} - 8 = 1 \Rightarrow \mathrm{n} = 9$$
$$\mathrm{T}_6 = {}^9\mathrm{C}_5 (2^{1/4})^4 (3^{-1/4})^5 = \frac{84}{\sqrt[4]{3}}$$
$$\therefore \alpha = 84$$

18. (57) Coefficient of middle term

${}^4C_2 \times \frac{\beta^2}{6}, -6\beta, -{}^6C_3 \times \frac{\beta^3}{8}$ are in A.P

$$2(-6\beta) = {}^4C_2 \frac{\beta^2}{6} - 6C_3 \frac{\beta^3}{8}$$

$$\beta^2-\frac{5}{2}\beta^3=-12\beta$$

$\beta=\frac{12}{5}$ or $\beta=-2 \therefore \beta=\frac{12}{5}$

common difference

$$d=\frac{72}{5}-\frac{144}{25}=-\frac{504}{25} \therefore 50-\frac{2d}{\beta^2}=57$$

19. (6006) $\left(t^2x^{\frac{1}{5}}+\frac{(1-x)^{\frac{1}{10}}}{t}\right)^{15}$

$$T_{r+1}={}^{15}C_r\left(t^2x^{\frac{1}{5}}\right)^{15-r}\frac{(1-x)^{\frac{r}{10}}}{t^{r}}$$

For independent of t,

$r+2r-30=0$ So $r=10$

So, maximum value of ${}^{15}C_{10}x(1-x)$ will be at $x=\frac{1}{2}$

i.e. 6006

20. (2) Given binomial expansion is

$$\left(2x^3+\frac{3}{x^k}\right)^{12}$$

Take general term $t_{r+1}={}^{12}C_r(2x^3)^r\left(\frac{3}{x^k}\right)^{12-r}$

$x^3r-(12-r)^k \rightarrow$ constant

$$\therefore 3r-12k+rk=0, \Rightarrow k=\frac{3r}{12-r}$$

Here, possible values of r are $3,6,8,9,10$ and corresponding values of k are $1,3,6,9,15$

Take, ${}^{12}C_r=220,924,495,220,66$

Therefore, possible value of k for which we will get 2^8 are 3,6

21. (221) Given expression is $\sum_{K=1}^{10}K^2({}^{10}C_K)^2$

$$\Rightarrow \sum_{K=1}^{10}(K\cdot{}^{10}C_K)^2=\sum_{K=1}^{10}(10.{}^{9}C_{K-1})^2$$

$$=100\sum_{K=1}^{10}{}^{9}C_{K-1}\cdot{}^{9}C_{10-K}=100({}^{18}C_9)=100\left(\frac{18!}{9!\,9!}\right)$$

$$\Rightarrow 4862000=22000L$$

Therefore, $L=221$

22. (99) From given expression $1+(1+2^{49})(2^{49}-1)=2^{98}$

$$\Rightarrow m=1, n=98 \Rightarrow m+n=99$$

23. (57) Given binomial expansion is $\left(x^n+\frac{2}{x^5}\right)^7$

Take general term as,

$$=\sum_{r=0}^{7}C_r(x^n)^{7-r}\left(\frac{2}{x^5}\right)^r=\sum_{r=0}^{7}{}^{7}C_r2^rx^{7n-nr-5r}$$

For positive powers of x,

$$7n-nr-5r\geq 0, n\geq\frac{5r}{7-r}$$

From(i),

$$={}^{7}C_02^{\circ}x^{7n}+{}^{7}C_12x^{6x-5}+{}^{7}C_22^2x^{5x-10}+{}^{7}C_32^3x^{4x-15}$$
$$+{}^{7}C_42^4\cdot x^{3x-20}+{}^{7}C_52^5x^{2x+25}+\cdots\ldots+{}^{7}C_72^7\cdot x^{-35}$$

According to question,

$= {}^{7}C_0 + {}^{7}C_1 \cdot 2 + {}^{7}C_2 \cdot 2^2 + {}^{7}C_3 \cdot 2^3 + {}^{7}C_4 \cdot 2^4$

$= 1 + 7.2 + \frac{7.6}{2} \times 4 + \frac{7 \times 6 \times 5}{6} \times 8 + \frac{7 \times 6 \times 5}{6} \times 2^4 = 939$

Then, r should be equal to 4

From(ii),

$n \geq \frac{5\times4}{3}, n \geq \frac{20}{3}$ Here, $n > 6$

Then, possibilities of n would be 7, 8,9,10,11,12

Required sum $= 7 + 8 + 9 + 10 + 11 + 12 = 57$

24. (102) ${}^{40}C_0 + {}^{41}C_1 + {}^{42}C_2 + \cdots .. + {}^{59}C_{19} + {}^{60}C_{20}$

$= {}^{40}C_{40} + {}^{41}C_{40} + {}^{42}C_{40} + \cdots .. + {}^{60}C_{40}$.

25. (286) Given expansion is

$(1 + x)^{10} = C_0 + C_1x + C_2x^2 + \cdots . + C_{10}x^{10}$

Differentiating

$10(1 + x)^9 = C_1 + 2C_2x + 3C_3x^2 + \cdots . + 10C_{10}x^9$

Replace x by x^2

$10(1 + x^2)^9 = C_1 + 2C_2x^2 + 3C_3x^4 + \cdots . + 10C_{10}x^{18}$.

$10x(1 + x^2)^9 = C_1x + 2C_2x^3 + 3C_3x^5 + \cdots .. + 10C_{10}x^{19}$

Differentiate w.r.t. x.

$10((1 + x^2)^9 . 1 + x . 9(1 + x^2)^8 2x)$

$= C_1x + 2C_2 \cdot 3x^3 + 3 \cdot 5 \cdot C_3x^4 + \cdots . + 10.19C_{10}x^{18}$

Put $x = 1$,

$10(2^9 + 18.2^8)$

$= C_1 + 3.2 . C_2 + 5.3 . C_3 + \cdots . + 19.10C_{10}$

$C_1 + 3.2 . C_2 + \cdots . + 19.10C_{10} = 10.2^9 . 10 = 100.2^9$

Take, $C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \cdots + \frac{C_9}{11} + \frac{C_{10}}{11} = \frac{2^{11}-1}{11}$

$C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \cdots . + \frac{C_9}{11} = \frac{2^{11} - 2}{11}$

Now, $100.2^9 = \frac{\alpha.2^{11}}{2^\beta-1}\left(\frac{2^{11}-2}{11}\right)$;

$25.2^{11} = \frac{\alpha \cdot 2^{11}}{2^\beta - 1} \cdot \left(\frac{2^{11} - 2}{11}\right)$

compare the above equation,

$\alpha = 25 \times 11 = 275 \& \beta = 11 \Rightarrow \alpha + \beta = 275 + 11 = 286$