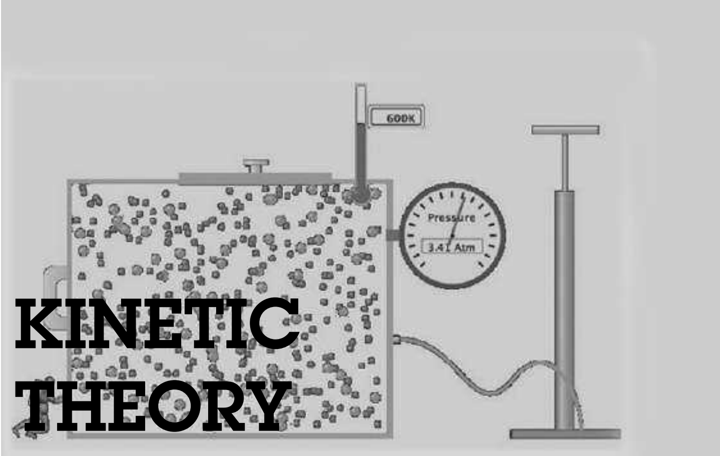
CHAPTER

13 KINETIC THEORY



Chapter Objective

This chapter will help you understand :

- *Equation of state of a perfect gas, Work done in compressing a gas. Kinetic theory of gases–assumptions, Concept of pressure, Kinetic interpretation of temperature, r.m.s. speed of gas molecules.*
- *Degrees of freedom, Law of equi-partition of energy (statement only) and application of specific heat capacities of gases, Concept of mean free path.*

TOPIC-1
Equation of State and Kinetic Theory of Gases

Quick Review

- **Ideal Gases :**

(a) Ideal gas or perfect gas is that gas which strictly obeys gas laws, like Boyle's law, Charles' law etc.

(b) For ideal gas, the size of the gas molecules is almost zero and the volume of the gas molecule is also almost zero.

(c) There is no force of attraction or repulsion amongst the molecules of an ideal gas.

(d) There is no intermolecular potential energy for the molecules of an ideal gas.

(e) The molecules of an ideal gas consists only kinetic energy.

(f) The ideal gas cannot be liquefied or solidify, which supports the absence of intermolecular forces of ideal gas at very low pressure and high temperature.

(g) The internal energy of an ideal gas does not depend on volume.

(h) The internal energy of an ideal gas depends upon the temperature alone.

(i) The specific heat of an ideal gas is independent of temperature.

(j) No gas available in the universe is strictly an ideal gas.

(k) The gases such as H_2, O_2, N_2 etc. and monoatomic inert gases behave very similar to ideal gases at very low pressure and high temperature

(l) The real gases at low pressure and high temperature behave as ideal gases due to negligible intermolecular force of attraction and volume of gas molecules.

- **Real Gases :**

(a) Real gases which are actually found in nature are known as real gases.

(b) The size of the molecules of a real gas is finite and hence the volume of the molecules of a real gas is finite.

(c) There is a force of attraction or repulsion between the molecules of a real gas. The intermolecular force between molecules is attractive for large intermolecular separation and repulsive for small intermolecular separation.

(d) The molecules of a real gas have potential energy as well as kinetic energy.

(e) The internal energy of a real gas depends on pressure, volume and temperature of the gas.

(f) The real gas can be liquefied and solidified.

(g) The real gas do not obey gas equation but obey Van der Waal's gas equation :

$$\left(P + \frac{n^2 a}{V^2}\right)(V - nb) = nRT$$

where *a* and *b* are Van der Waal's gas constants of a real gas.

(h) The real gases like CO_2, NH_3, SO_2 etc. obey Van der Waal's equation at high pressure and low temperature.

(i) In Van der Waal's equation, the value of *a* depends upon the intermolecular force and the nature of the gas.

(j) The value of *b* depends upon the size of the gas molecules and represents the volume occupied by the molecules of a gas.

(k) In Van der Waal's equation (V – *b*) shows volume available to the molecules of the real gas, which is the effective volume of the gas.

(l) Real gases do not obey the gas laws at all temperatures.

- **Boyle's law :** It states that the volume V of the given mass of a gas is inversely proportional to its pressure P, when temperature is kept constant, *i.e.*,

$$V \propto 1/P \text{ or } V = K/P \text{ ...(Here, T is constant)}$$

or

$$PV = K = \text{Constant.}$$

- **Charles's law :** It states that the pressure remaining constant, the volume of the given mass of a gas is directly proportional to its kelvin temperature, *i.e.*, $V \propto T$, if P is constant.

or,

$$V = KT \text{ ...(Here, P is constant)}$$

or,

$$\frac{V}{T} = K = \text{Constant.}$$

- **Assumptions of Kinetic Theory of Gases :**

(a) A gas consists of a very large number of molecules which are perfectly elastic spheres and are identical in all respects for a given gas and are different for different gases.

(b) The molecules of a gas are in a state of continuous, rapid and random motion.

(c) The volume occupied by the molecules is negligible in comparison to the volume of the gas.

(d) The molecules do not exert any force of attraction or repulsion on each other, except during collision.

(e) The collisions of the molecules with themselves and with the walls of the vessel are perfectly elastic.

(f) Molecular density is uniform throughout the gas.

(g) A molecule moves along a straight line between two successive collisions.

(h) The collisions are almost instantaneous.

Know the Terms

- **Gram mole and kilogram mole :**

(i) The molecular weight expressed in gram is known as gram mole (g mol). The molecular weight expressed in kilograms is known as kilogram mole (kg mol).

(ii) The mass of 1 mole of a gas equal to its molecular weight in gram. And 1 kg mol = 1,000 g mole.

- **Most probable speed** of the molecules of a gas as that speed which is possessed by maximum fraction of total number of molecules of the gas.
- **Mean speed or average speed** is the average speed with which molecules of a gas move.
- **Root mean square speed** is defined as the square root of the mean of the squares of random velocities of individual molecules of a gas.

Know the Formulae

- **Boyle's Law :** $PV = \text{constant}$
- **Charles's Law :** $\frac{V}{T} = \text{constant}$
- **Standard gas equation :** $PV = nRT$

where *n* is the number of moles contained in the given ideal gas of volume V, pressure P and temperature T.

- **Gas constant :**

(i) R is a universal gas constant and *r* is a gas constant for 1 gram of a gas.

(ii) The universal gas constant is defined as the work done by (or on) a gas per mole per Kelvin *i.e.*

$$R = \frac{PV}{nT}$$

$$= \frac{\text{Pressure} \times \text{Volume}}{\text{No. of moles} \times \text{Temperature}}$$

$$= \frac{\text{Work done}}{\text{No. of moles} \times \text{Temperature}}$$

(iii) The value of R for every gas at S.T.P. = 8·31 J $mole^{-1}$ K^{-1} = 1.98 cal. mol^{-1} K^{-1}.

(iv) Dimensional formula for R = $[ML^2T^{-2}K^{-1}]\ mol^{-1}$.

- **Most probable speed :** $c_{mp} = \sqrt{\frac{2k_BT}{m}}$
- **Average speed** $c_{av} = \sqrt{\frac{8k_BT}{\pi m}}$
- **Root mean square speed** $c_{rms} = \sqrt{\frac{3k_BT}{m}}$

  k_B = Boltzman constant, T = Temperature, m = mass
- **Ratio among speeds,** $c_{mp} : c_{av} : c_{rms} = \sqrt{2} : \sqrt{\frac{8}{\pi}} : \sqrt{3}$.
- **Van der Waal's equation for one mole of a gas,** $\left(P + \frac{a}{V^2}\right)(V - b) = RT$

MCQ/Fillups/True or False (1 mark each)

(A) Multiple Choice Questions

Q. 1. A cubic vessel (with faces horizontal + vertical) contains an ideal gas at NTP. The vessel is being carried by a rocket which is moving at a speed of 500 ms^{-1} in vertical direction. The pressure of the gas inside the vessel as observed by us on the ground

(a) remains the same because 500 ms^{-1} is very much smaller than v_{rms} of the gas.

(b) remains the same because motion of the vessel as a whole does not affect the relative motion of the gas molecules and the walls.

(c) will increase by a factor equal to $(v^2_{rms} + (500)^2)/v^2_{rms}$ where v_{rms} was the original mean square velocity of the gas.

(d) will be different on the top wall and bottom wall of the vessel.

[NCERT Exemp. Q. 13.1, Page 90]

Ans. Correct option: (b)

Explanation:

$$P = \frac{nRT}{V} = P$$

P remains unaffected as n, R, T and V are constant.

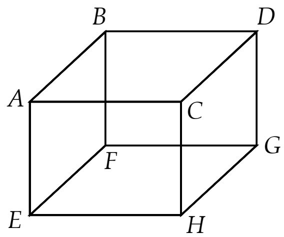
Q. 2. 1 mole of an ideal gas is contained in a cubical volume V. ABCDEFGH at 300 K as shown in figure. One face of the cube (EFGH) is made up of a material which totally absorbs any gas molecule incident on it. At any given time,

(a) the pressure on EFGH would be zero.

(b) the pressure on all the faces will the equal.

(c) the pressure of EFGH would be double the pressure on ABCD.

(d) The pressure on EFGH would be half that of ABCD.



[NCERT Exemp. Q. 13.2, Page 90]

Ans. Correct option: (d)

Explanation: The momentum transferred to the face ABCD = 2 mv and gas molecule is absorbed by the face EFGH. Hence it does not rebound. So, momentum transferred to the EFGH = mv.

Q. 3. Boyle's law is applicable for an

(a) adiabatic process.

(b) isothermal process.

(c) isobaric process.

(d) isochoric process.

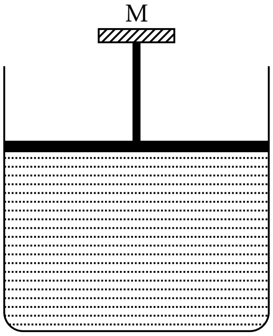
[NCERT Exemp. Q. 13.3, Page 91]

Ans. Correct option: (b)

Explanation:

Since $V \propto \frac{1}{\rho}$ at constant temperature.

Q. 4. A cylinder containing an ideal gas is in vertical position and has a piston of mass M that is able to move up or down without friction as shown in fig. If the temperature is increased.



(a) both P and V of the gas will change.

(b) only P will increase according to Charles's law.

(c) V will change but not P.

(d) P will change but not V.

[NCERT Exemp. Q. 13.4, Page 91]

Ans. Correct option: (c)

Explanation:

$$p = \frac{F}{A} = \frac{mg}{A} = \text{constant}$$

$\therefore$ $V \propto T$ (at constant) pressure

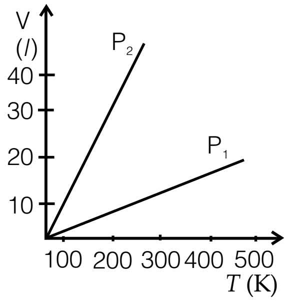
Q. 5. Volume versus temperature graphs for a given mass of an ideal gas are shown in figure at two different values of constant pressure. What can be inferred about relation between P_1 & P_2 ?

(a) $P_1 > P_2$

(b) $P_1 = P_2$

(c) $P_1 < P_2$

(d) data is insufficient



[NCERT Exemp. Q. 13.5, Page 91]

Ans. Correct option: (a)

Explanation:

$V \propto T$

$$\frac{V}{T} = \text{constant} = \frac{1}{P}$$

In graph, slope at P_2 is more than slope at P_1,

$\therefore$ $P_1 > P_2$

Q. 6. 1 mole of H_2 gas is contained in a box of volume V = 1.00 m^3 at T = 300 K. The gas is heated to a temperature of T = 3000 K and the gas gets converted to a gas of hydrogen atoms. The final pressure would be (considering all gases to be ideal)

(a) Same as the pressure initially.

(b) 2 times the pressure initially.

(c) 10 times the pressure initially.

(d) 20 times the pressure initially.

[NCERT Exemp. Q. 13.6, Page 92]

Ans. Correct option: (d)

Explanation: At constant volume $P \propto nT$

$$\therefore \quad \frac{P_2}{P_1} = \frac{n_2T_2}{n_1T_1} = \frac{2n}{n} \times \frac{3000}{300} = 20$$

$\Rightarrow$ $P_2 = 20\,P_1$

Q. 7. A vessel of volume V contains a mixture of 1 mole of Hydrogen and 1 mole of Oxygen (both considered as ideal). Let $f_1(v)dv$, denote the fraction of molecules with speed between v and $(v + dv)$ with $f_2(v)dv$, similarly for oxygen. Then :

(a) $f_1(v) + f_2(v) = f(v)$ obeys the Maxwell's distribution law

(b) $f_1(v)$, $f_2(v)$ will obey the Maxwell's distribution law separately

(c) Neither $f_1(v)$, nor $f_2(v)$ will obey the Maxwell's distribution law

(d) f_2 (v) and f_1(v) will be the same.

[NCERT Exemp. Q. 13.7, Page 92]

Ans. Correct option: (b)

Explanation: The masses of hydrogen and oxygen molecules $dx = f(v)$, which are having speeds between v and $v + dv$. The Maxwell Boltzmann speed distribution function NV = $\frac{dn}{dv}$ depends an the mass of the gas molecules.

For each function $f_1(v)$ and $f_2(v)$ n will be different, hence each function $f_1(v)$ and $f_2(v)$ will obey the Maxwell's distribution law separately.

Q. 8. An inflated rubber balloon contains one mole of an ideal gas, has a pressure p, volume V and temperature T. If the temperature rises to 1.1. T, and the volume is increases to 1.05 V, the final pressure will be

(a) 1.1 p **(b) p**

(c) less than p **(d) between p and 1.1 p**

[NCERT Exemp. Q. 13.8, Page 92]

Ans. Correct option: (d)

Explanation: Ideal gas equation PV = nRT. Here n is the numbers of moles,

$$\text{So,} \quad n = \frac{P_1V_1}{RT_1} = \frac{P_2V_2}{RT_2} \quad \text{or} \quad \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\therefore \quad P_2 = P_1V_1\frac{T_2}{V_2T_1}$$

here $P_1 = p$, $V_2 = 1.05$ V, $V_1 = V$, $T_2 = 1.1$ T, $T_1 = T$

$$\therefore \quad P_2 = \frac{P \times V \times 1.1T}{1.05V \times T} = \frac{1.1}{1.05}P = 1.05\,p$$

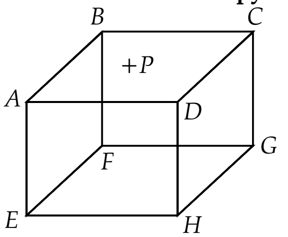
Q. 9. ABCDEFGH is a hollow cube made of an insulator as shown in fig). Face ABCD has positive charge on it. Inside the cube, we have ionized hydrogen.
The usual kinetic theory expression for pressure

(a) will be valid

(b) will not be valid since the ions would experience forces other than due to collisions with the walls

(c) will not be valid since collisions with walls would not be elastic

(d) will not be valid because isotropy is lost



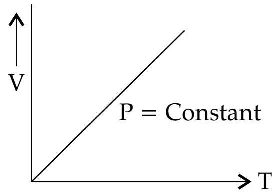
[NCERT Exemp. Q. 13.9, Page 92]

Ans. Correct option: (b) and (d)

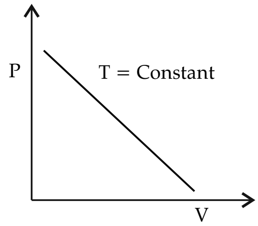
Explanation: According to the problem ionized hydrogen is present inside the cube, they are having charge, Now, due to the preference of Positive charge on the surface ABCD hydrogen ions would experience forces other then the forces due to collision with the walls of container. So, these forces must be of electrostatic nature Hence, Isotropy of system is last at only are face ABCD because of the pressure of external positive charge. The usual expression for pressure are the basis of Kinetic they will be valid.

Q. 10. Which of the following diagrams (figure) depicts ideal as behaviour ?

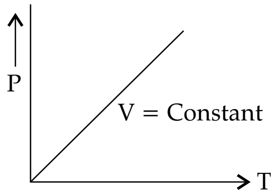
(a)



(b)



(c)



(d)

[NCERT Exemp. Q. 13.12, Page 93]

Ans. Correct option: (a) and (c)

Explanation:

(a) At constant P, V∝T it is charles' law.

(c) At constant V, P∝T it is pressure law

Very Short Answer Type Questions **(1 mark each)**

Q. 1. The volume of a given mass of a gas at 27°C, 1 atm is 100 cc. What will be its volume at 327°C ?

[NCERT Exemp. Q. 13.15, Page 94]

Ans. Given that

$$T_1 = 27 + 273 = 300 \text{ K}$$
$$T_2 = 327 + 273 = 600 \text{ K}$$
$$V_1 = 100 \text{ cc}, V_2 = ?$$

If P is constant then V∝T or $\frac{V_1}{T_1} = \frac{V_2}{T_2}$

$$\Rightarrow \quad V_2 = \frac{V_1 T_2}{T_1} = \frac{100 \times 600}{300}$$

$$= 200 \text{ c.c.}$$

Q. 2. The molecules of a given mass of a gas have root mean square speeds of 100 ms^{-1} at 27°C and 1.00 atmospheric pressure. What will be the root mean square speeds of the molecules of the gas at 127°C and 2.0 atmospheric pressure?

[NCERT Exemp. Q. 13.16, Page 94]

Ans.

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\frac{V_1}{V_2} = \frac{P_2T_1}{P_1T_2} = \frac{2\times 300}{400} = \frac{3}{2}$$

$$P_1 = \frac{1}{3}\frac{M}{V_1}c_1^{-2};\ P_2 = \frac{1}{3}\frac{M}{V_2}c_2^{-2}$$

$$\therefore c_2^{\ 2} = c_1^{\ 2} \times \frac{V_2}{V_1} \times \frac{P_2}{P_1}$$

$$= (100)^2 \times \frac{2}{3} \times 2$$

$$c_2 = \frac{200}{\sqrt{3}} \text{ms}^{-1}$$

Q. 3. Two molecules of a gas have speed of 9 × 10^{16} ms^{-1} and 1 × 10^{16} ms^{-1} respectively. What is the root mean source speed of these molecules ?

[NCERT Exemp. Q. 13.17, Page 94]

Ans. Root mean square speed

$$V_{rms} = \sqrt{\frac{V_1^2 + V_2^2 + V_3^2 + ...V_n^2}{n}}$$

For two molecules

$$V_{rms} = \sqrt{\frac{V_1^2 + V_2^2}{2}} = \sqrt{\frac{(9\times10^6)^2 + (1\times10^6)^2}{2}}$$

$$= \sqrt{\frac{(81\times1)\times10^{12}}{2}} = \sqrt{41}\times10^6 \text{ ms}^{-1}$$

Short Answer Type Questions

(2 or 3 marks each)

Q. 1. The container shown in Fig. 13.6 has two chambers, separated by a partition, of volumes V_1= 2.0 litre and V_2 = 3.0 litre. The chambers contain μ_1 = 4.0 and μ_2 = 5.0 moles of a gas at pressures P_1 = 1.00 atm and P_2 = 2.00 atm. Calculate the pressure after the partition is removed and the mixture attains equilibrium.

V_1	V_2
μ_1, P_1	μ_2, P_2

[NCERT Exemp. Q. 13.20, Page 94]

Ans. V_1 = 2.0 litre, V_2 = 3.0 litre

μ_1= 4.0 moles, μ_2 = 5.0 moles

P_1 = 1.00 atm, P_2 = 2.00 atm

$P_1V_1 = \mu_1RT_1$, $P_2V_2 = \mu_2RT_2$

$\mu = \mu_1 + \mu_2 \qquad V = V_1 + V_2$

For 1 mole, $PV = \frac{2}{3}E$

For μ_1 moles, $P_1V_1 = \frac{2}{3}\mu_1E_1$

For μ_2 moles, $P_2V_2 = \frac{2}{3}\mu_2E_2$

Total energy is $(\mu_1E_1 + \mu_2E_2) = \frac{3}{2}(P_1V_1 + P_2V_2)$

$$PV = \frac{2}{3}E_{total} = \frac{2}{3}\mu E_{per\ mole}$$

$$P(V_1+V_2) = \frac{2}{3}\times\frac{3}{2}(P_1V_1 + P_2V_2)$$

$$P = \frac{P_1V_1 + P_2V_2}{V_1+V_2}$$

$$= \left(\frac{1.00\times2.0 + 2.00\times3.0}{2.0+3.0}\right)\text{atm}$$

$$= \frac{8.0}{5.0} = 1.60 \text{ atm.}$$

Comment: This form of Ideal gas law represented by Equation marked* becomes very useful for adiabatic changes.

Q. 2. When air is pumped into a cycle tyre the volume and pressure of the air in the tyre both are increased. What about Boyle's law in this case?

[NCERT Exemp. Q. 13.23, Page 95]

Ans. When air is pumped, more molecules are pumped in. Boyle's law is stated for situation where number of molecules remain constant.

Q. 3. A gas mixture consists of molecules of types A, B and C with masses $m_A > m_B > m_C$. Rank the three of molecules is decreasing under of :

(a) Average KE (b) rms speeds.

[NCERT Exemp. Q. 13.21, Page 94]

Ans. (a) $\quad$ K.E. $= \frac{3}{2}$kT

or $\quad$ K.E. $\propto \sqrt{T}$

Which remains same for all three types of molecules

(b) $\quad v_{rms} = \sqrt{\frac{3RT}{m}} = \sqrt{\frac{3kT}{m}}$

or $\quad v_{rms} \propto \frac{1}{\sqrt{m}}$

here $\quad m_A > m_B > m_C$

so $\quad (v_{rms})_C > (v_{rms})_B > (v_{rms})_A$

Long Answer Type Questions

(5 marks each)

Q. 1. Explain why

(a) there is no atmosphere on moon.

(b) there is fall in temperature with altitude.

[NCERT Exemp. Q. 13.27, Page 95]

Ans. (a) The moon has small gravitational force and hence the escape velocity is small. As the moon is in the proximity of the Earth as seen from the Sun, the moon has the same amount of heat per unit area as that of the Earth. The air molecules have large range of speeds. Even though the rms speed of the air molecules is smaller than the escape velocity on the moon, a significant number of molecules have speed greater than escape velocity and they escape. Now rest of the molecules arrange the speed distribution for the equilibrium temperature. Again a significant number of molecules escape as their speeds exceed escape speed. Hence, over a long time the moon has lost most of its atmosphere.

At 300 K,

$$V_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3\times1.38\times10^{-23}\times300}{7.3\times10^{-26}}} = 1.7\ km/s$$

V_{esc} for moon = 4.6 km/s

(b) As the molecules move higher their potential energy increases and hence kinetic energy decreases and hence temperature reduces.

At greater height more volume is available and gas expands and hence some cooling takes place.

Q. 2. A box of 1.00m^3 is filled with nitrogen at 1.50 atm at 300K. The box has a hole of an area 0.010 mm^2. How much time is required for the pressure to reduce by 0.10 atm, if the pressure outside is 1 atm.

[NCERT Exemp. Q. 13.30, Page 96]

Ans. V_{ix} = speed of molecule inside the box along x direction

n_1 = number of molecules per unit volume

In time Δt, particles moving along the wall will collide if they are within $(V_{ix}\Delta t)$ distance. Let a = area of the wall. No. of particles colliding in time $\Delta t = \frac{1}{2}n_i(V_{ix}\Delta t)a$ (factor of 1/2 due to motion towards wall).

In general, gas Is In equilibrium as the wall is very large as compared to hole.

$$\therefore V_{1x}^2 + V_{1y}^2 + V_{1z}^2 = V^2_{rms}$$

$$\therefore V_{1x}^2 = \frac{V^2_{rms}}{3}$$

$$\frac{1}{2}mV^2_{rms} = \frac{3}{2}kT \Rightarrow V^2_{rms} = \frac{3kT}{m}$$

$$\therefore V^2_{1x} = \frac{kT}{m}$$

$\therefore$ No. of particles colliding in time $\Delta t = \frac{1}{2}n_1\sqrt{\frac{kT}{m}}\Delta t\, a.$

If particles collide along hole, they move out. Similarly outer particles colliding along hole will move in.

$\therefore$ Net particle flow in time $\Delta t = \frac{1}{2}(n_1 - n_2)\sqrt{\frac{kT}{m}}\Delta t\, a$ as temperature is same in and out.

$$pV = \mu RT \Rightarrow \mu = \frac{PV}{RT}$$

$$n = \frac{\mu N_A}{V} = \frac{PN_A}{RT}$$

After some time τ pressure changes to p_1' inside

$$\therefore n_1' = \frac{P_1'N_A}{RT}$$

$n_1V - n_1'V$ = no.of particle gone out

$$= \frac{1}{2}(n_1 - n_2)\sqrt{\frac{kT}{m}}\tau a$$

$$\therefore \frac{P_1N_A}{RT}V - \frac{P_1'N_A}{RT}V = \frac{1}{2}(P_1 - P_2)\frac{N_A}{RT}\sqrt{\frac{kT}{m}}a$$

$$\therefore \tau = 2\left(\frac{P_1 - P_1'}{P_1 - P_2}\right)\frac{V}{a}\sqrt{\frac{m}{kT}}$$

$$= 2\left(\frac{1.5-1.4}{1.5-1.0}\right)\frac{5\times1.00}{0.01\times10^{-6}}\sqrt{\frac{46.7\times10^{-27}}{1.38\times10^{-23}\times300}}$$

$$= 1.38\times10^5 s$$

Q. 3. Consider a rectangular block of wood moving with a velocity v_0 in a gas at temperature T and mass density ρ. Assume the velocity is along x-axis and the area of cross-section of the block perpendicular to v_0 is A. Show that the drag force on the block is $4\rho Av_0\sqrt{\frac{kT}{m}}$, where m is the mass of the gas molecule.

[NCERT Exemp. Q. 13.31, Page 96]

Ans. n = no. of molecules per unit volume

v_{rms} = rms speed of gas molecules

When block is moving with speed v_o, relative speed of molecules w.r.t. front face = $v + v_o$

Coming head on, momentum transferred to block per collision

= $2m(v + v_o)$, where m = mass of molecule.

No. of collision in time $\Delta t = \Delta t\frac{1}{2}(v+v_o)n\Delta tA$,

where A = area of cross section of block and factor of 1/2 appears due to particles moving towards block.

$\therefore$ Momentum transferred in time $\Delta t = m(v+v_o)^2 nA\Delta t$ from front surface.

Similarly momentum transferred in time $\Delta t = m(v-v_o)^2 nA\Delta t$ from back surface.

$\therefore$ Net force (drag force) $= mnA\left[(v+v_o)-(v-v_o)^2\right]$

from front $= mnA(vv_o) = (4mnAv)v_o$

$= (4\rho Av)v_o$

We also have $\frac{1}{2}mv^2 = \frac{1}{2}kT$ ($\because$ v is the velocity along x-axis)

Therefore, $v = \sqrt{\frac{kT}{m}}$.

Thus, drag $= 4\rho A\sqrt{\frac{kT}{m}}v_o$.

Q. 4. Consider an ideal gas with following distribution of speeds.

Speed (m/s)	% of molecules
200	10
400	20
600	40
800	20
1000	10

(i) Calculate V_{rms} and hence h. (m = 3.0 × 10^{-26} kg)

(ii) If all the molecules with speed 1000 m/s escape from the system, calculate new V_{rms} and hence T.

[NCERT Exemp. Q. 13.28, Page 95]

Ans.

(i) $$v_{rms}^2 = \frac{\sum_i n_i v_i^2}{\sum_i n_i}$$

$$= \frac{10\times(200)^2+20\times(400)^2+40\times(600)^2 +20\times(800)^2+10\times(1000)^2}{10+20+40+20+10}$$

$= 10 \times 400 + 20 \times 1600 + 40 \times 3600 + 20 \times 6400 + 10 \times 10000$

$= 4 \times 1000 + 32 \times 1000 + 144 \times 1000 + 128 \times 1000 + 100 \times 1000$

$= 408 \times 1000$

$\therefore\ V_{rms} = \sqrt{408\times1000} = 639$ m/sec

now, $\frac{1}{2}mv_{rms}^2 = \frac{3}{2}kT$

$\therefore\ T = \frac{mv_{rms}^2}{3K} = \frac{3\times10^{-26}\times4.08\times10^5}{3\times1.38\times10^{-23}}$

$= 2.96 \times 10^2 = 296$ K.

(ii) remaining particles root mean speed

$$v_{rms}^2 = \frac{10\times(200)^2+20\times(400)^2+40\times(600)^2+20\times(800)^2}{90}$$

$= 342000$

$\therefore\ v_{rms} = \sqrt{342000} = 584$ m/sec.

$\therefore\ T = \frac{mv_{rms}^2}{3k} = \frac{3\times10^{-26}\times342000}{3\times1.38\times10^{-23}}$

$= 248$ K.

TOPIC-2

Law of Equi-partition energy and Brownian Motion

Quick Review

- **Gay Lussac's Law or Regnault's Law :** When volume of a certain mass of a gas is kept constant, the pressure P exerted by gas is directly proportional to temperature T of gas *i.e.* P & T.
- **Avogadro's Law :** It states that equal volumes of all gases. Under identical conditions of temperature and pressure contain the same no. of molecules. $n_1 = n_2$
- **Graham's Law of Diffusion—**It states that rates of diffusion of two gases are inversely proportional to the square roots of their densities $r \propto \frac{1}{\sqrt{\rho}}$ or $\frac{r_1}{r_2} = \sqrt{\frac{\rho_2}{\rho_1}}$
- **Dalton's Law of Partial Pressure :** It states that total pressure exerted by a mixture of non-reactive ideal gases is equal to sum of partial pressures which each would exert, if it alone occupied the same volume at the given temperature.

$$P_1 + P_2 + P_3 \ldots\ldots\ldots\ldots = P$$

- **Law of Gaseous Volumes :** It states that when gases react together, they do so in volumes which will be a simple ratio to one another and also to the volumes of product.
- **Law of Equipartition of energy :** It states that the energy for each degree of freedom in thermal equilibrium is $\frac{1}{2}k_BT$.
- **Brownian Motion :** It is defined as continuous zig-zag motion of particles of macroscopic size ($\approx10^{-5}$ m) suspended in water or air or some other fluid. Brownian motion increases :
  (a) When size of suspended object is smaller.
  (b) When density of fluid is smaller.
  (c) When temperature of medium is higher.
  (d) When viscosity of medium is smaller.

Know the Terms

- **Pressure** exerted by gas is due to continuous bombardment of gas molecules against the walls of container.
- **Degrees of freedom** of a dynamic system is defined as the total no. of co-ordinates or independent quantities required to describe completely the position & configuration of the system.
- **Mean free path** is the average distance covered between two successive collisions by the gas molecule moving along the straight line.

- **Absolute zero of temperature** may be defined as that temperature at which the root mean square velocity of gas molecules reduces to zero.

Know the Formulae

- **Pressure exerted by Ideal Gas.** $P = \frac{1}{3} mnc^2$

or $P = \frac{1}{3}\frac{M}{V}c^2 = \frac{1}{3}\rho c^2$

- **Relation between Pressure & K.E. of gas**

$$P = \frac{2}{3}E$$

- **Average K.E. of translation of 1 mole**

$$= \frac{1}{2}Mc^2 = \frac{3}{2}RT.$$

- **Average K.E. of translation per molecule of gas**

$$= \frac{1}{2}mc^2 = \frac{3}{2}k_BT.$$

- **Boyle's Law** PV = Constant at constant T.
- **Charles's Law** $V \propto c^2 \therefore V \propto T$ at constant pressure
- **Avogadro's Law** $n_1 = n_2$ at equal temperature , pressure and volume.
- **Graham's Law of diffusion** $r \propto \frac{1}{\sqrt{\rho}}$ or $\frac{r_1}{r_2} = \sqrt{\frac{\rho_2}{\rho_1}}$
- **Dalton's Law of partial pressure**

$$P_1 + P_2 + P_3 + ... = P$$

- **Degrees of freedom**
For
(a) Mono-atomic gas = 3
(b) Di-atomic gas = 5
(c) Tri-atomic gas = 7
(d) Non-linear triatomic gas = 6

- **Law of equipartition of energy** $E_t = \frac{1}{2}k_BT$

- **Specific Heat Capacity of :**

(a) Monoatomic gas $\gamma = \frac{C_P}{C_V}$

$$\gamma = \frac{5}{3} = 1.67$$

(b) Diatomic gas $\gamma = \frac{C_P}{C_V} = \frac{7}{5} = 1.4$

(c) Triatomic gas

Linear gas molecules $\gamma = \frac{9}{7} = 1.28$

Non-linear gas molecules $\gamma = \frac{4}{3} = 1.33.$

(d) Polyatomic gas $\gamma = \left(1+\frac{2}{n}\right)$, where n is the degree of freedom

(e) Solids $c = 3R = 24.93 J\ mole^{-1} K^{-1}$
(f) Water $c = 9R = 74.7\ J\ mole^{-1} K^{-1}$

- **Mean free path**

$$\lambda = \frac{\lambda_1 + \lambda_2 + \lambda_3 + ... + \lambda_n}{n} = \frac{ct}{n}$$

$$\lambda = \frac{1}{\sqrt{2}n\pi d^2} = \frac{R_B T}{\sqrt{2}n\pi d^2 P}$$

Here, d is diameter, P is the pressure, T is temperature & k_B is Boltzmann constant.

MCQ/Fillups/True or False **(1 mark each)**

(A) Multiple Choice Questions

Q. 1. Diatomic molecules like hydrogen have energies due to both translational as well as rotational motion. From the equation in kinetic theory pV = $\frac{2}{3}$ E, E is

(a) the total energy pen unit volume

(b) Only the translational part o energy because rotational energy is very small compared to the translational energy

(c) Only translational part of the energy because during collisions with the well pressure relates to change in linear momentum

(d) The translational part of the energy because rotational energies of molecules can be with sign and its average our all the molecules is zero.

[NCERT Exemp. Q. 13.10, Page 92]

Ans. Correct option: (c)

Explanation: According to kinetic theory equation PV = $\frac{2}{3}$ E. Here E is representing only translational part of energy. Internal energy contains all types of energies like translational, rotational vibrational etc. But the molecules of an θ ideal gas is treated as point masses in kinetic theory, so its kinetic energy is only due to translational or vibrational motion. Here we assumed that the walls only exert perpendicular forces a molecules. They do not exert any parallel force hence there there will not be any type of rotation present. The wall produces only change in translational motion.

Q. 2. In a diatomic molecule, the rotational energy at a given temperature.

(a) obeys Maxwell's distribution

(b) have the same value for all molecules

(c) equals the translational kinetic energy for each molecule

(d) is $\left(\frac{2}{3}\right)^{rd}$ the translational kinetic energy for each molecule. **[NCERT Exemp. Q. 13.11, Page 93]**

Ans. Correct option: (a) and (d)

Explanation:

(a) Translational kinetic energy and rotational kinetic energy both obey Maxwell's distribution independent of each other.

(d) Here 2 rotational and 3 translational energies are associated with each molecule. Translational kinetic energy of each molecule

$$K_T = \frac{3}{2} kT$$

and rotational kinetic energy $K_R = 2\left(\frac{1}{2}kT\right)$

now $$\frac{K_R}{K_T} = \frac{kT}{\frac{3}{2}kT} = \frac{2}{3}$$

$$\therefore\ K_R = \frac{2}{3} kT$$

Q. 3. When an ideal gas is compressed adiabatically. Its temperature rises the molecules on the average have more kinetic energy than before. The kinetic energy increases.

(a) because of collisions with mouing parts of the wall only

(b) because of collisions with the entire wall

(c) because the molecules gets accelerated in their motion inside the volume

(d) because of redistribution of energy amongst the molecules

[NCERT Exemp. Q. 13.13, Page 93]

Ans. Correct option: (a)

Explanation: Since the gas is ideal and the collisions of the molecules are elastic. When the molecules collides with the moving parts of the wall. its kinetic energy increases. But the total kinetic energy of the system will remain conserved. When the gas is compressed adiabatically, the total work done on the gas increases, its internal energy which in turn increases the KE of gas molecules and hence, the collision between molecules also increases.

Very Short Answer Type Questions (1 mark each)

Q. 1. Calculate the number of atoms in 39.4 g gold. Molar mass of gold is 197g mole^{-1}.

[NCERT Exemp. Q. 13.14, Page 94]

Ans. $\therefore$ Molar mass of gold is 197 g mole^{-1}, the number of atoms = 6.0 x 10^{23}

$\therefore$ No. of atoms in 39.4g $= \frac{6.0\times10^{23}\times39.4}{197} = 1.2\times10^{23}$

Q. 2. A gas mixture consists of 2.0 moles of oxygen and 4.0 moles of neon at temperature *T*. Neglecting all vibrational modes, calculate the total internal energy of the system. (Oxygen has two rotational modes.)

[NCERT Exemp. Q. 13.18, Page 94]

Ans. O_2 has 5 degrees of freedom. Therefore, energy per mole = $\frac{5}{2}RT$

$\therefore$ For 2 moles of O_2, energy = $5RT$

Neon has 3 degrees of freedom

$\therefore$ Energy per mole = $\frac{3}{2}RT$

$\therefore$ For 4 mole of neon, energy = $4\times\frac{3}{2}RT = 6RT$

$\therefore$ Total energy = $11RT$.

Q. 3. Calculate the ratio of the mean free paths of the molecules of two gases having molecular diameters 1 Å and 2 Å. The gases may be considered under identical conditions of temperature, pressure and volume.

[NCERT Exemp. Q. 13.19, Page 94]

Ans. $l\,\alpha\frac{1}{d^2}$

$d_1 = 1Å$, $\alpha_2 = 2Å$

$l_1 : l_2 = 4 : 1$

Short Answer Type Questions (2 or 3 marks each)

Q. 1. A balloon has 5.0 g mole of helium at 7°C. Calculate (a) the number of atoms of helium in the balloon, (b) the total internal energy of the system.

[NCERT Exemp. Q. 13.24, Page 95]

Ans. **(a)** $\mu = 5.0T$

$T = 280K$

No. of atoms = $\mu N_A = 5.0 \times 6.02 \times 10^{23}$

$= 30 \times 10^{23}$

(b) Average kinetic energy per molecule = $\frac{3}{2}kT$

$\therefore$ Total internal energy = $\frac{3}{2}kT\times N$

$= \frac{3}{2}\times30\times10^{23}\times1.38\times10^{-23}\times280$

$= 1.74 \times 10^4$ J

Q. 2. Calculate the number of degrees of freedom of molecules of hydrogen in 1 cc of hydrogen gas at NTP. **[NCERT Exemp. Q. 13.25, Page 95]**

Ans. Volume occupied by 1 gram mole of gas at NTP = 22400cc

$\therefore$ Number of molecules in 1cc of hydrogen

$= \frac{6.023\times10^{23}}{22400} = 2.688\times10^{19}$

As each diatomic molecule has 5 degrees of freedom, hydrogen being diatomic also has 5 degrees of freedom

$\therefore$ Total no. of degrees of freedom = $5 \times 2.688 \times 10^{19}$

$= 1.344 \times 10^{20}$

Q. 3. An insulated container containing monoatomic gas of molar mass m is moving with a velocity vo. If the container is suddenly stopped, find the change in temperature.

[NCERT Exemp. Q. 13.25, Page 95]

Ans. Loss in kinetic energy

$\Delta K = \frac{1}{2}(mn)v_0^2$

It temperature changes ΔT.

then $n \times \frac{3}{2} R \times \Delta T = \frac{1}{2}mnv_0^2$

$\therefore \quad \Delta T = \frac{mv_0^2}{3R}$

Q. 4. We have 0.5 g of hydrogen gas in a cubic chamber of size 3 cm kept at NTP. The gas in the chamber is compressed keeping the temperature constant till a final pressure of 100 atm. Is one justified in assuming the ideal gas law in the final state ? (Hydrogen molecules can be consider as sphere of radius)Å. **[NCERT Exemp. Q. 13.22, Page 95]**

Ans. Volume of 1 molecule = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times 3.14 \times (10^{-10}) = 4.2 \times 10^{-30}$ m^3

Number of mole in 0.5 g H_2 gas

$= \frac{0.5}{2} = 0.25$ moles

$\therefore$ Volume of H_2 molecules in 0.25 moles

$= 0.25 \times 6.023 \times 10^{23} \times 4.2 \times 10^{-30}$

$V_1 = 1.05 \times 6.023 \times 10^{-7} = 6.3 \times 10^{-7}\ m^3$

Now, $P_1V_1 = P_2V_2$

$$V_2 = \frac{P_1V_1}{P_2} = \frac{1}{100}\ (3 \times 10^{-2})^3$$

$$= \frac{27 \times 10^{-6}}{100} = 2.7 \times 10^{-7}\ m^3$$

TIPS... & TRICKS...

- *Understand the concept of Ideal gas and real gas.*
- *Study about Boyle's law, charle's law and Pressure law.*
- *Study and understand about most probable speed. Average speed and root mean square speed.*
- *Study and understand about Avogodro law and Avogadro number.*
- *Learn about Degree of freedom.*
- *Study Law of Equi-Partition of energy.*