CHAPTER 3

MOTION IN A PLANE

Scalars and Vectors

Scalar: A physical quantity which has only magnitude and no direction is called a scalar quantity or scalar.

Vector: A physical quantity which constitutes of magnitude as well as direction is called a vector quantity or vector. If it follows law of vector addition too.

- **Unit vector:**

(i) A unit vector of $\vec{A}$ is written as $\hat{A}$ and is given by $\hat{A} = \vec{A}/|\vec{A}|$

(ii) A unit vector is dimensionless quantity of magnitude equal to unity. Its magnitude is 1 and it can have any direction.

(iii) In cartesian co-ordinates, $\hat{i}, \hat{j}, \hat{k}$ are the unit vectors along x, y and z-axes, respectively.

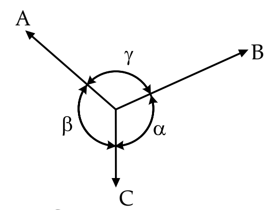
- **Polar vectors:** These are those vectors which have a linear directional effect. For example, force, linear momentum, linear velocity etc.
- **Axial vectors or rotational vectors:** These vectors represent rotational effect. They are always directed along the axis of rotation in accordance with right hand screw rule. Angular velocity, torque, angular momentum etc. are few examples of axial vectors.
- **Some vector laws:**

(1) General law for addition of vector: It states that the vectors to be added are arranged in such a way so that the head of first vector coincides with the tail of second vector, whose head coincides with tail of third vector and so on then resultant vector is represented in magnitude and direction by the line starting from tail of first vector to head of last vector.

(2) Triangle Law: It states that if two vectors acting on a particle at the same time are represented in magnitude and direction by the two sides of a triangle taken in one order, their resultant vector is represented in magnitude and direction by the third side of triangle taken in opposite order.

(3) Parallelogram Law: It states that if two vectors acting on a particle at the same time be represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point, their resultant vector is represented in magnitude and direction by the diagonal of the parallelogram drawn from the same point.

- **Lami's Theorem:** It states that if three forces acting at a point are in equilibrium, then each force is proportional to the sine of the angle between the other two forces, *i.e.*,



$$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{C}{\sin\gamma}$$

Projectile Motion

- **Projectile:**

(a) Projectile is defined as a body thrown with some initial velocity except vertically upward or downward and then allowed to move under the action of gravity alone, without being propelled by an engine or fuel or any source. The path followed by a projectile is known as its **trajectory**.

(b) The motion of a projectile may be thought of as the result of two separate, simultaneously occurring components of motions. One component is along a horizontal direction without any acceleration and the other along the vertical direction with constant acceleration due to the force of gravity.

(c) Due to the vertical component of velocity, the body rises vertically upward and due to the horizontal component of velocity the body shifts horizontally simultaneously.

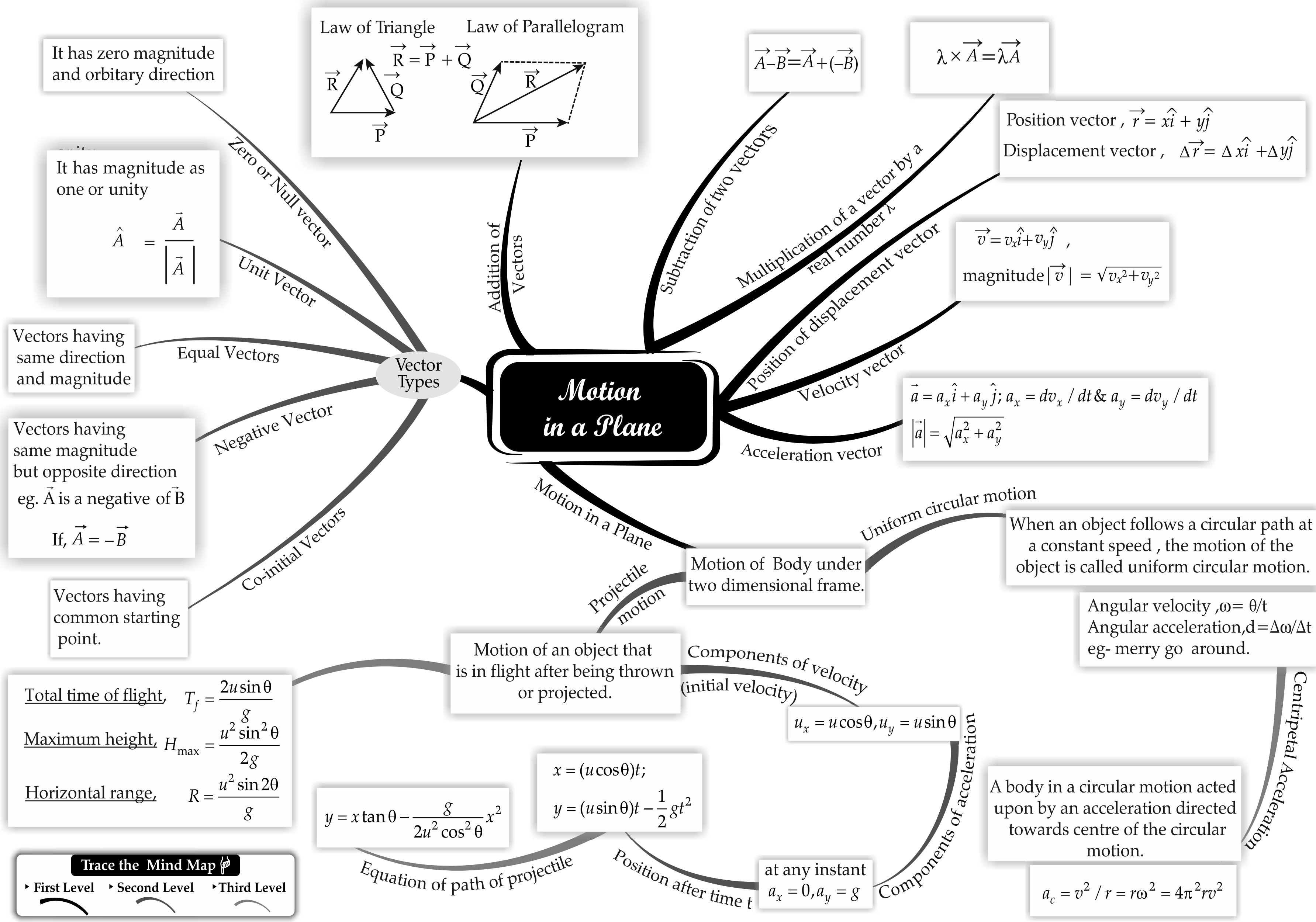
- The motion of a projectile may be thought of as the result of two separate, simultaneously occurring components of motions. One component is along a horizontal direction without any acceleration and the other along the vertical direction with constant acceleration due to the force of gravity.
- Due to the vertical component of velocity, the body rises vertically upward and due to the horizontal component of velocity the body shifts horizontally simultaneously.
- **Centripetal force:**

(a) It is the force required to move the body in circular path with a constant angular velocity.

(b) The centripetal force acts on the particle along the radius which is directed towards the centre of circular path.

(c) The centripetal force does not increase the kinetic energy and angular momentum of the particle moving in a circular path, therefore the work done by the centripetal force is zero.

(d) The centripetal force is provided in different ways, in different types of circular motions.


Motion in a Plane
Vector Types
Zero or Null vector
It has zero magnitude and orbitary direction
Unit Vector
It has magnitude as one or unity
$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$
Equal Vectors
Vectors having same direction and magnitude
Negative Vector
Vectors having same magnitude but opposite direction eg. $\vec{A}$ is a negative of $\vec{B}$
If, $\vec{A} = -\vec{B}$
Co-initial Vectors
Vectors having common starting point.
Addition of Vectors
Law of Triangle
Law of Parallelogram
$\vec{R} = \vec{P} + \vec{Q}$
$\vec{R}$
$\vec{Q}$
$\vec{P}$
Subtraction of two vectors
$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$
Multiplication of a vector by a real number λ
$\lambda \times \vec{A} = \lambda\vec{A}$
Position of displacement vector
Position vector , $\vec{r} = x\hat{i} + y\hat{j}$
Displacement vector , $\Delta\vec{r} = \Delta x\hat{i} + \Delta y\hat{j}$
Velocity vector
$\vec{v} = v_x\hat{i} + v_y\hat{j}$,
magnitude $|\vec{v}| = \sqrt{v_x^2 + v_y^2}$
Acceleration vector
$\vec{a} = a_x\hat{i} + a_y\hat{j}$; $a_x = dv_x / dt$ & $a_y = dv_y / dt$
$|\vec{a}| = \sqrt{a_x^2 + a_y^2}$
Motion in a Plane
Motion of Body under two dimensional frame.
Uniform circular motion
When an object follows a circular path at a constant speed , the motion of the object is called uniform circular motion.
Angular velocity ,ω= θ/t
Angular acceleration,d=Δω/Δt
eg- merry go around.
Centripetal Acceleration
A body in a circular motion acted upon by an acceleration directed towards centre of the circular motion.
$a_c = v^2 / r = r\omega^2 = 4\pi^2 r v^2$
Projectile motion
Motion of an object that is in flight after being thrown or projected.
Components of velocity (initial velocity)
$u_x = u\cos\theta, u_y = u\sin\theta$
Components of acceleration
at any instant
$a_x = 0, a_y = g$
Position after time t
$x = (u\cos\theta)t$;
$y = (u\sin\theta)t - \frac{1}{2}gt^2$
Equation of path of projectile
$y = x\tan\theta - \frac{g}{2u^2\cos^2\theta}x^2$
Total time of flight, $T_f = \frac{2u\sin\theta}{g}$
Maximum height, $H_{max} = \frac{u^2\sin^2\theta}{2g}$
Horizontal range, $R = \frac{u^2\sin 2\theta}{g}$
Trace the Mind Map
First Level
Second Level
Third Level

Key Words

- **Modulus of vector** is the magnitude of vector.
- **Equal vectors** are those vectors which have equal magnitude and same direction.
- **Negative vector** of a given vector is a vector of same magnitude but acting in a direction opposite to that of given vector.
- **Co-initial vectors** are those vectors which have common initial point.
- **Collinear vectors** are those vectors which are having equal or unequal magnitudes and are acting along parallel straight lines.
- **Coplanar vectors** are those vectors which are acting in the same plane.
- **Localized vector** is that vector whose initial point is fixed.
- **Non-Localized vector** is that vector whose initial point is not fixed.
- **Zero or Null vector** is that vector which has zero magnitude and an arbitrary direction and represented by 0.
- **Displacement vector** is that vector which tells how much and in which direction an object has changed its position in a given interval of time.
- **Resultant vector** is defined as that single vector which produces the same effect as is produced by two or more individual vectors together.
- **Equilibrate vector** is a single vector which balances two or more vectors acting on a body at the same time.
- **Angular displacement** of the object moving around a circular path is defined as the angle traced out by radius vector at the centre of circular path in given time. It is vector quantity.
- **Angular velocity** of an object in circular motion is defined as the time rate of change of its angular displacement.
- **Angular acceleration** of an object in circular motion is defined as the time rate of change of its angular velocity.
- **Uniform circular motion** is the motion when a point object is moving on a circular path with a constant speed.

Key Formulae

- $\vec{R} = \vec{A} + \vec{B}$

  $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

  $\tan\beta = \dfrac{B\sin\theta}{A + B\cos\theta}$; β = angle of $\vec{R}$ with $\vec{A}$.

- $\vec{R} = \vec{A} - \vec{B} = \vec{A} + (-\vec{B})$

  $R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$

  $\tan\beta = \dfrac{B\sin(180°)}{A + B\cos(180°)} = \dfrac{B\sin\theta}{A - B\cos\theta}$

- $\vec{A} = A_x\hat{i} + A_y\hat{j}$ and $A_x = A\cos\theta$, $A_y = A\sin\theta$ (in 2*D*)

- $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$, $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ (in 3*D*)

  $|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$, $|\vec{B}| = \sqrt{B_x^2 + B_y^2 + B_z^2}$

- $\vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$

- Unit Vector or $\hat{A}$ is

  $$\hat{A} = \frac{\vec{A}}{|A|} = \frac{A_x\hat{i} + A_y\hat{j} + A_z\hat{k}}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

- $\vec{A}\cdot\vec{B} = AB\cos\theta$

  $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$

- If two vectors are parallel to each other i.e., $\vec{\theta} = 0°$

  $\vec{A}\cdot\vec{B} = AB\cos 0° = AB$

  $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

- If two vectors are perpendicular to each other i.e., θ = 90°.

  $\vec{A}\cdot\vec{B} = AB\cos 90° = 0$

  $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{j} = 0$

- If two vectors are parallel to each other i.e., θ = 0°

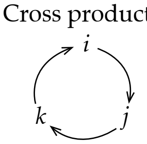
- $\vec{A}\times\vec{B} = AB\sin 0°\,\hat{n} = \vec{0}$.

  $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$

- If two vectors are perpendicular to each other *i.e.*, θ = 90°

  $\therefore \vec{A}\times\vec{B} = AB\sin 90° = AB$

- Trick to remember Cross product



  $\hat{i}\times\hat{j} = \hat{k},\ \hat{j}\times\hat{k} = \hat{i},\ \hat{k}\times\hat{i} = \hat{j}$

  and $\hat{i}\times\hat{k} = -\hat{j},\ \hat{k}\times\hat{j} = -\hat{i},\ \hat{j}\times\hat{i} = -\hat{k}$

- Area of triangle = $\dfrac{1}{2}|\vec{A}\times\vec{B}|$

- Area of parallelogram = $|\vec{A}\times\vec{B}|$

- Unit vector perpendicular to $\vec{A}$ & $\vec{B}$

  $$\hat{n} = \frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}$$

where, $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

- If $\vec{A} + \vec{B} + \vec{C} = \vec{0}$

then $\vec{A} \times \vec{B} = \vec{B} \times \vec{C} = \vec{C} \times \vec{A}$

- $\sin\theta = \dfrac{|\vec{A} \times \vec{B}|}{|\vec{A}|\,|\vec{B}|}$
- $\cos\theta = \dfrac{\vec{A} \cdot \vec{B}}{|\vec{A}|\,|\vec{B}|}$
- $\tan\theta = \dfrac{\vec{A} \times \vec{B}}{\vec{A} \cdot \vec{B}}$

- **For motion along *X*-axis,**

$v_x = u_x + a_x t$ and $x = x_0 + u_x t + \frac{1}{2} a_x t^2$

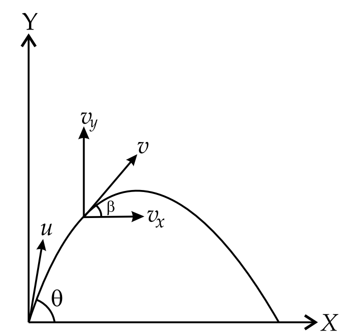
- **For motion along *Y*-axis,**

$v_y = u_y + a_y t$ and $y = y_0 + u_y t + \frac{1}{2} a_y t^2$

- **Velocity of projective at an instant of its flight is**

$$v = \sqrt{v_x^{\,2} + v_y^{\,2}}$$

and $\tan\gamma = \dfrac{v_y}{v_x}$



u = initial speed
θ = angle of projection

- **Angular projection of projectile :**

(i) Time of flight, $T = \dfrac{2u\sin\theta}{g}$

(ii) Maximum height, $h = \dfrac{u^2\sin^2\theta}{2g}$

(iii) Horizontal range, $R = \dfrac{u^2\sin 2\theta}{g}$

(iv) Maximum horizontal range $R_{max} = \dfrac{u^2}{g}$ for $\theta = 45°$

(v) Range is same for for angles θ and (90° – θ) if *u* & *g* remains unchanged

- **Circular Motion**
  - $\omega = \theta/t$
  - $\omega = 2\pi\nu = \dfrac{2\pi}{t}$
  - $a_c = \omega^2 r = \omega v = v^2/r$
  - $a_T = r\alpha$

where, a_c = centripetal acceleration
a_T = tangential acceleration
ω = angular velocity
ν = frequency
T = Time period

Mnemonics

Concept: Cross and dot product of two vectors.
Mnemonics: A and **B c**rossed **S**ikkim and **d**rove to **C**alcutta.
Interpretation:

A - $\vec{A}$

B - $\vec{B}$

c - Cross product
s - sin θ
d - dot product
c - cos θ

$\vec{A} \times \vec{B}$ = AB sin θ

$\vec{A} . \vec{B}$ = AB cos θ

NCERT CORNER

Q. 1. State, for each of the following physical quantities, if it is a scalar or a vector:
Volume, mass, speed, acceleration, density, number of moles, velocity, angular frequency, displacement, angular velocity. **[NCERT Ex., Q. 3.1, Page 46]**

Ans.

Scalars	Vectors
volume mass speed density number of moles	acceleration velocity displacement angular velocity angular frequency

Q. 2. Pick out the two scalar quantities in the following list:

Force, angular momentum, work, current, linear momentum, electric field, average velocity, magnetic moment, relative velocity. [NCERT Ex., Q. 3.2, Page 46]

Ans. Work, Current.

Q. 3. Pick out the only vector quantity in the following list:

Temperature, pressure, impulse, time, power, total path length, energy, gravitational potential, coefficient of friction, charge.

Ans. Impulse, as Impulse = Force × time

Q. 4. State with reasons, whether the following algebraic operations with scalar and vector physical quantities are meaningful:

(a) adding any two scalars, (b) adding a scalar to a vector of the same dimensions, (c) multiplying any vector by any scalar, (d) multiplying any two scalars, (e) adding any two vectors, (f) adding a component of a vector to the same vector. [NCERT Ex., Q. 3.4, Page 46]

Ans. (a) No, adding any two scalars is not meaningful, but the scalars of same dimensions, (i.e., of same nature) can be added.

(b) No, adding a scalar to a vector of the same dimension is not meaningful as a scalar cannot be added to a vector.

(c) Yes, multiplying any vector by any scalar is meaningful in algebraic operation. It is because when any vector is multiplied by any scalar, then we get a vector having magnitude equal to scalar number of times the magnitude of the given vector. For example, when acceleration a is multiplied by mass m, we get force $F = ma$ which is a meaningful operation.

(d) Yes, it is product of two scalar gives a meaningful result. For example, when power P is multiplied by time t, then we get work done (W), i.e., $W = Pt$, which is a useful algebraic operation.

(e) No, as the two vectors of same dimensions (i.e., of the same nature) can only be added. So addition of any two vectors of same dimension is a meaningful algebraic operation.

(f) Addition of a component of a vector to the same vector can be done by the law of vector addition. So algebraic operation is a meaningful operation.

Q. 5. Read each statement below carefully and state with reasons, if it is true or false:

(a) The magnitude of a vector is always a scalar.

(b) Each component of a vector is always a scalar.

(c) The total path length is always equal to be magnitude of the displacement vector of a particle.

(d) The average speed of a particle (defined as total path length divided by the time taken to cover the path) is either greater or equal to the magnitude of average velocity of the particle over the same interval of time.

(e) Three vectors not lying in a plane can never add up to give a null vector. [NCERT Ex., Q. 3.5, Page 46]

Ans. (a) True, magnitude is pure number.

(b) False, each component is vector.

(c) True, only if particle moves along a straight line and in same direction otherwise false.

(d) True, because-

Total path length ≥ Magnitude of displacement vector

(e) True, because these three vectors cannot represent three sides of triangle which taken in same order.

Q. 6. Establish the following vector inequalities geometrically or otherwise:

(a) $|\vec{a}+\vec{b}| \le |\vec{a}|+|\vec{b}|$

(b) $|\vec{a}+\vec{b}| \ge ||\vec{a}|+|\vec{b}||$

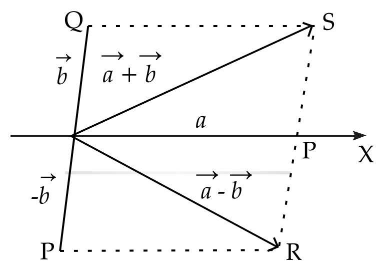
(c) $|\vec{a}-\vec{b}| \le |\vec{a}|+|\vec{b}|$

(d) $|\vec{a}-\vec{b}| \ge ||\vec{a}|-|\vec{b}||$

When does the equality sign above apply?

[NCERT Ex., Q. 3.6, Page 46]

Ans. (a) As-



Length of one side of triangle is always less than sum of lengths of other two sides.

So, $|\vec{a}+\vec{b}| < |\vec{a}|+|\vec{b}|$ **(I)**

If the two vectors are acting along same straight line and in same direction, then

$|\vec{a}+\vec{b}| = |\vec{a}|+|\vec{b}|$ (II)

(b) From Δ OPS,

$OS+PS>OP$

$\Rightarrow OS > |OP-PS|,\ \ OS > |OP-OQ|$ **(III)**

Modulus of ($OP-PS$) has been taken, as LHS is positive but RHS may be negative if $OP < PS$.

From eq.(III)

$|\vec{a}+\vec{b}| > ||\vec{a}|-|\vec{b}||$ (IV)

If $\vec{a}$ and $\vec{b}$ are acting along straight line but in opposite direction,

$|\vec{a}+\vec{b}| = ||\vec{a}|-|\vec{b}||$ (V)

from eq (IV) &(V)

$|\vec{a}+\vec{b}| \ge ||\vec{a}|-|\vec{b}||$

(c) From ΔORP, $OR<OP+PR$

So, $|\vec{a}-\vec{b}| < ||\vec{a}|+|\vec{b}||$ (VI)

Now, two vectors are acting along straight line in opposite direction-

$\left|\vec{a}-\vec{b}\right| = \left|\left|\vec{a}\right|+\left|\vec{b}\right|\right|$ (VII)

from eq. (VI) & (VII)

$\left|\vec{a}-\vec{b}\right| \le \left|\vec{a}\right|+\left|\vec{b}\right|$

(d) From ΔOPR, $OR > |OP - PR|$

$\left|\overline{OP}-\overline{PR}\right|$ has been taken, because LHS = positive, RHS may be negative

if $OP<PR$,

$\therefore \quad \left|\vec{a}-\vec{b}\right| > \left|\left|\vec{a}\right|-\left|\vec{b}\right|\right|$

Q. 7. Given $\vec{a}+\vec{b}+\vec{c}-\vec{d} = 0$, which of the following statements are correct:

(a) $(\vec{a},\vec{b},\vec{c})$ and $\vec{d}$ must each be a null vector,

(b) The magnitude of $(\vec{a}+\vec{c})$ equals the magnitude of $(\vec{b}+\vec{d})$,

(c) The magnitude of $\vec{a}$ can never be greater than the sum of the magnitudes of $\vec{b}$, $\vec{c}$ and $\vec{d}$,

(d) $\vec{b}+\vec{c}$ must lie in the plane of $\vec{a}$ and $\vec{d}$ if $\vec{a}$ and $\vec{d}$ are not collinear, and in the line of $\vec{a}$ and $\vec{d}$, if they are collinear? [NCERT Ex., Q. 3.7, Page 47]

Ans. (a) Incorrect

In order to make, $\vec{a}+\vec{b}+\vec{c}+\vec{d}=\vec{0}$ it is not necessary to have all the four given vectors to be null vectors. There are many other combinations which can give the sum zero.

(b) Correct

$\vec{a}+\vec{b}+\vec{c}+\vec{d}=\vec{0}$

$\vec{a}+\vec{c}=-\left(\vec{b}+\vec{d}\right)$

Taking modulus on both the sides, we get:

$\left|\vec{a}+\vec{c}\right| = \left|-(\vec{b}+\vec{d})\right| = \left|\vec{b}+\vec{d}\right|$

Hence, the magnitude of $\left(\vec{a}+\vec{c}\right)$ is the same as the magnitude of $\left(\vec{b}+\vec{d}\right)$

(c) Correct

$\vec{a}+\vec{b}+\vec{c}+\vec{d}=\vec{0}$

$\vec{a}=-\left(\vec{b}+\vec{c}+\vec{d}\right)$

Taking modulus both sides, we get the magnitude of $\vec{a}$ to be equal to the magnitude of $\left(\vec{b}+\vec{c}+\vec{d}\right)$:

$\left|\vec{a}\right| = \left|\left(\vec{b}+\vec{c}+\vec{d}\right)\right|$

$\left|\vec{a}\right| \le \left|\vec{b}\right|+\left|\vec{c}\right|+\left|\vec{d}\right|$ ...(i)

Now, $\left(\vec{b}+\vec{c}+\vec{d}\right)$ is the sum of vectors $\vec{b}$, $\vec{c}$ and $\vec{d}$. Therefore, the magnitude of $\left(\vec{b}+\vec{c}+\vec{d}\right)$ is less than, or equal to the sum of the magnitudes of $\vec{b}$, $\vec{c}$ and $\vec{d}$. Hence, the magnitude of $\vec{a}$ can never be greater than the sum of the magnitudes of $\vec{b}$, $\vec{c}$ and $\vec{d}$. Equation (i) shows that the magnitude of $\vec{a}$ is equal to or less than the sum of the magnitudes of $\vec{b}$, $\vec{c}$ and $\vec{d}$.

(d) Correct

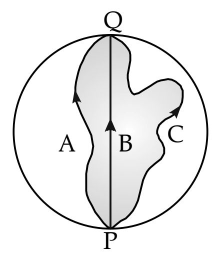
For, $\vec{a}+\vec{b}+\vec{c}+\vec{d}=0$

$\vec{a}+\left(\vec{b}+\vec{c}\right)+\vec{d}=0$

The resultant sum of the three vectors $\vec{a}$, $\left(\vec{b}+\vec{c}\right)$, and $\vec{d}$ can be zero only if $\left(\vec{b}+\vec{c}\right)$ lie in the same plane as $\vec{a}$ and $\vec{d}$.

If $\vec{a}$ and $\vec{d}$ are collinear, then it implies that the vector $\left(\vec{b}+\vec{c}\right)$ is in the line of $\vec{a}$ and $\vec{d}$. This implication holds true in this scenario and the vector sum of all the vectors will be zero.

Q. 8. Three girls skating on a circular ice ground of radius 200 m start from a point P on the edge of the ground and reach a point Q diametrically opposite to P following different paths as shown in Figure. (i) What is the magnitude of the displacement vector for each? (ii) For which girl is this equal to the actual length of path skate ?



[NCERT Ex., Q. 3.8, Page 47]

Ans. (i) PQ = diameter

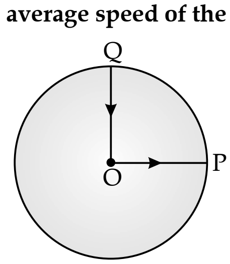
= displacement for each girl

= $2r = 2 \times 200 = 400$ m

Since, displacement vector does not depend upon the actual path length and it is the shortest distance between initial and final position, so in the case of each girl the displacement is 400 m.

(ii) For girl B, the displacement is equal to the actual length of path skate.

Q. 9. A cyclist starts from the centre O of a circular park of radius 1 km, reaches the edge P of the park, then cycles along the circumference and returns to the centre along QO as shown. If the round trip takes 10 min, what is the (a) net displacement, (b) average velocity and (c) average speed of the cyclist ?



[NCERT Ex., Q. 3.9, Page 47]

Ans. Given, radius of circular park = 1 km

(a) As cyclist returns to its initial state, therefore the net displacement of the cyclist is zero.

(b) Average velocity $= \dfrac{\text{Total displacement}}{\text{Total time taken}}$

$$= \frac{0}{\text{Total time taken}}$$

$$= 0$$

(c) Total distance travelled by the cyclist

$$= OP + \text{arc } PQ + QO$$

$$= r + \left(\frac{1}{4} \times 2\pi r\right) + r$$

$$= 1 + \left(\frac{1}{2} \times \frac{22}{7} \times 1\right) + 1$$

$$= 2 + \frac{11}{7}$$

$$= \frac{25}{7} \text{ km}$$

Total time taken = 10 min

$$= \frac{10}{60} \text{ h} = \frac{1}{6} \text{ h}$$

$\therefore$ Average speed of the cyclist

$$= \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

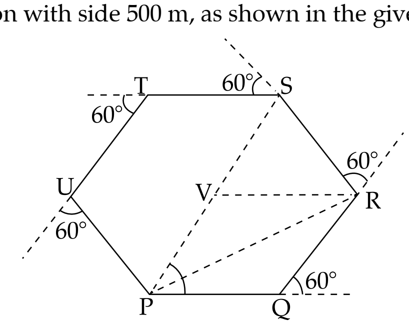
$$= \frac{25/7}{1/6} = \frac{150}{7}$$

$$= 21.43 \text{ km/h}$$

Q. 10. On an open ground, a motorist follows a track that turns to his left by an angle of 60° after every 500 m. Starting from a given turn, specify the displacement of the motorist at the third, sixth and eighth turn. Compare the magnitude of the displacement with the total path length covered by the motorist in each case.

[NCERT Ex., Q. 3.10, Page 47]

Ans. The path followed by the motorist is a regular hexagon with side 500 m, as shown in the given figure



Let the motorist start from point P.

The motorist takes the third turn at S.

$\therefore$ Magnitude of displacement $= PS = PV + VS = 500 + 500 = 1000$ m

Total path length $= PQ + QR + RS = 500 + 500 + 500 = 1500$ m

The motorist takes the sixth turn at point P, which is the starting point.

$\therefore$ Magnitude of displacement = 0

Total path length $= PQ + QR + RS + ST + TU + UP$

$= 500 + 500 + 500 + 500 + 500 + 500 = 3000$ m

The motorist takes the eight turn at point R

$\therefore$ Magnitude of displacement $= PR$

$$= \sqrt{PQ^2 + QR^2 + 2(PQ).(QR)\cos 60^\circ}$$

$$= \sqrt{500^2 + 500^2 + (2 \times 500 \times 500 \times \cos 60^\circ)}$$

$$= \sqrt{250000 + 250000 + \left(500000 \times \frac{1}{2}\right)}$$

$$= 866.03 \text{ m}$$

Total path length

= Circumference of the hexagon $+ PQ + QR$

$= 6 \times 500 + 500 + 500 = 4000$ m

The magnitude of displacement and the total path length corresponding to the required turns is shown in the given table

Turn	Magnitude of displacement (m)	Total path length (m)
Third	1000	1500
Sixth	0	3000
Eighth	866.03	4000

Q. 11. A passenger arriving in a new town wishes to go from the station to a hotel located 10 km away on a straight road from the station. A dishonest cabman takes him along a circuitous path 23 km long and reaches the hotel in 28 min. What is

(a) the average speed of the taxi?

(b) the magnitude of average velocity? Are the two equal? [NCERT Ex., Q. 3.11, Page 47]

Ans. **(a)** Total distance travelled = 23 km

Total time taken = 28 min $= \dfrac{28}{60}$ h

$\therefore$ Average speed of the taxi $= \dfrac{\text{Total distance travelled}}{\text{Total time taken}}$

$$\Rightarrow \frac{23}{\left(\frac{28}{60}\right)} = 49.29 \text{ km/h}$$

(b) Distance between the hotel and the station = 10 km = Displacement of the car

$$\therefore \text{Average velocity} = \frac{10}{\left(\frac{28}{60}\right)} = 21.43 \text{ km/h}$$

Therefore, the two physical quantities (average speed and average velocity) are not equal.

Q. 12. The ceiling of a long hall is 25 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 ms^{-1} can go without hitting the ceiling of the hall ? [NCERT Ex., Q. 3.12, Page 47]

Ans. Step 1: Using $H = \frac{u^2 \sin^2 \theta}{2g}$

when $H = 25\ m, u = 40$ m/s

and $g = 9.8$ m/s^2

$$25 = \frac{40^2 \sin^2 \theta}{2 \times 9.8}$$

i.e., $$\sin^2 \theta = \frac{490}{40^2}$$

$$\sin \theta = \frac{\sqrt{490}}{40} = 0.5534$$

i.e., $$\theta = 33.6°$$

Step 2: $$R = \frac{u^2 \sin 2\theta}{g}$$

$$R = \frac{40^2 \sin 2(33.6)}{9.8} \text{ m}$$

$$= \frac{40^2 \sin 67.2}{9.8} \text{ m}$$

$$= \frac{40^2 \times 0.9219}{9.8} \text{ m}$$

$$= 150.514 \text{ m}$$

Q. 13. A cricketer can throw a ball to a maximum horizontal distance of 100 m. How much high above the ground can the cricketer throw the same ball ?

[NCERT Ex., Q. 3.13, Page 47]

Ans. From the formula the horizontal range is given by

$$R = \frac{u^2 \sin 2\theta}{g} \quad ..(i)$$

For $R = R_{max}$, $\theta = 45°$, i.e., $\sin 2\theta = \sin 90° = 1$

Putting the given value is eq. (i), we

$$\therefore \quad R_{max} = \frac{u^2}{g}$$

$$\Rightarrow \quad 100 = \frac{u^2}{g} \quad (\because R_{max} = 100 \text{ given}) \ ...(ii)$$

Suppose H = height upto which the ball goes when the cricketer throws it with velocity u. Since, the final velocity of the ball, $v = 0$.

$\therefore$ Applying the relation, $v^2 - u^2 = 2as$,

($\because$ here, $v = 0, a = -g, s = H$)

or $$H = \frac{u^2}{2g}$$

$$H = \frac{1}{2}\left(\frac{u^2}{g}\right)$$

$$= \frac{1}{2} \times (100) \quad \text{[by using (ii)]}$$

$$H = 50 \text{ m.}$$

Q. 14. A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, what is the magnitude and direction of acceleration of the stone ?

[NCERT Ex., Q. 3.14, Page 48]

Ans. Given, Radius of the horizontal circle, $r = 80$ cm

$= 0.80$ m, frequency $= f = \frac{14}{25}$ rps

Angular speed of revolution of the stone is given by

$$\omega = 2\pi f$$

$$\Rightarrow \quad \omega = 2 \times \frac{22}{7} \times \left(\frac{14}{25}\right)$$

$$\Rightarrow \quad \omega = \frac{88}{25} \text{ rad/sec.}$$

$\therefore$ Magnitude of acceleration produced in the stone will be equal to the magnitude of centripetal acceleration.

$$= r\omega^2$$

$$= 0.80 \times \left(\frac{88}{25}\right)^2$$

$$= 0.80 \times \frac{88}{25} \times \frac{88}{25}$$

$$= 9.90 \text{ ms}^{-2}$$

We know that, the direction of the acceleration is towards the centre of the circle along its radius.

Q. 15. An aircraft executes a horizontal loop of radius 1.00 km with a steady speed of 900 km/h. Compare its centripetal acceleration with the acceleration due to gravity. **[NCERT Ex., Q. 3.15, Page 48]**

Ans. Given: $r = 1$ km $= 1000$ m;

$$v = 900 \text{ kmh}^{-1}$$

$$= 900 \times \frac{1000}{3600} \text{ ms}^{-1}$$

$$= 250 \text{ ms}^{-1}$$

The centripetal acceleration of the aircraft is

$$a = \frac{v^2}{r} = \frac{(250)^2}{1000}$$

$$= \frac{62500}{1000} = 62.5 \text{ ms}^{-2}$$

Acceleration due to gravity,

$$g = 9.8 \text{ ms}^{-2}$$

$$\therefore \quad \frac{\text{Centripetal acceleration}}{\text{Acceleration due to gravity}} = \frac{a}{g}$$

$$= \frac{62.5}{9.8}$$

or $$a = 6.38\ g$$

Q. 16. Read each statement below carefully and state, with reasons, if it is true or false:

(a) The net acceleration of a particle in circular motion is always along the radius of the circle towards the centre.

(b) The velocity vector of a particle at a point is always along the tangent to the path of the particle at that point.

(c) The acceleration vector of a particle in uniform circular motion over one cycle is a null vector.

[NCERT Ex., Q. 3.16, Page 48]

Ans. (a) False—The net acceleration of a particle in circular motion is towards the centre only if its speed is constant.

(b) True—A particle released at any point of its path will always move along the tangent to the path at the point.

(c) True—For any two diametrically opposite points on the circumference, the acceleration vectors are equal and opposite. Hence, the acceleration vector average over one completely cycle is null vector.

Q. 17. The position vector of a particle is given by

$$\vec{r} = 3.0t\,\hat{i} - 2.0t^2\,\hat{j} + 4.0\hat{k}\text{ m}$$

where t is in seconds and the coefficients have the proper units for $\vec{r}$ to be in metres.

(a) Find the $\vec{v}$ and $\vec{a}$ of the particle ?

(b) What is the magnitude and direction of velocity of the particle at t = 2.0 s ? [NCERT Ex., Q. 3.17, Page 48]

Ans. The position vector (r) of the particle is

$$\vec{r} = 3.0t\,\hat{i} - 2.0t^2\,\hat{j} + 4.0\hat{k}\text{ m} \quad ...(i)$$

(a) velocity $\vec{v}$ (t) of the particle is given by

$$\vec{v}\,(t) = \frac{d\vec{r}}{dt} = \frac{d}{dt}\,(\vec{r}\,)$$

$$= \frac{d}{dt}\,(3.0t\,\hat{i} - 2.0t^2\,\hat{j} + 4.0\hat{k}\,)$$

$$\vec{v}\,(t) = 3i - 4t\,\hat{j} + 0 \quad ....(ii)$$

Also, acceleration $\vec{a}$ (t) of the participle is given by

$$\vec{a}\,(t) = \frac{d\,\vec{v}\,(t)}{dt} = \frac{d}{dt}\vec{v}(t)$$

$$= \frac{d}{dt}\left(3\hat{i} - 4t\,\hat{j}\right) \quad \text{[by using (ii)]}$$

$$= 0 - 4\,\hat{j}$$

$$\vec{a}\,(t) = -4\hat{j}\ \text{ms}^2 \quad ....(iii)$$

(b) At time t, the velocity of the particle is given by using to equation (ii).

$$\vec{v}\,(t) = 3.0\hat{i} - 4t\,\hat{j} \quad \text{[by using (ii)]}$$

$\therefore$ At t = 2s,

$$v = 3.0\hat{i} - 4 \times 2\,\hat{j}$$

$$= 3.0\hat{i} - 8.0\,\hat{j}$$

$\therefore$ Its magnitude is

$$v = \sqrt{3^2 + (-8)^2}$$

$$= \sqrt{9 + 64}$$

$$= \sqrt{73} = 8.544\text{ ms}^{-1}$$

and, direction of v is given by

$$\therefore \quad \theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

$$= \tan^{-1}\left(\frac{-8}{3}\right)$$

$$= -70° \text{ with } x\text{-axis}$$

Q. 18. A particle starts from the origin at t = 0 s with a velocity of 10.0 $\hat{j}$ and moves in the x-y plane with a constant acceleration of (8.0 $\hat{i}$ +2.0 $\hat{j}$) ms^{-2}

(a) At what time is the x-coordinate of the particle 16 m? What is the y-coordinate of the particle at that time?

(b) What is the speed of the particle at the time?

[NCERT Ex., Q. 3.18, Page 48]

Ans. (a) Velocity of the particle, $\vec{u} = 10.0\,\hat{j}$ *m/s*

Acceleration of the particle $\vec{a} = (8.0\,\hat{i} + 2.0\,\hat{j})$

Also,

But, $\vec{a} = \dfrac{d\vec{v}}{dt} = 8.0\,\hat{i} + 2.0\,\hat{j}$

$d\vec{v} = (8.0\,\hat{i} + 2.0\,\hat{j}\,)dt$

Integrating both sides:

$\vec{v} = 8.0\,\hat{i}\ t + 2.0\,\hat{j}t + \vec{u}$

Where,

$\vec{u}$ = Velocity vector of the particle at t = 0

$\vec{v}$ = Velocity vector of the particle at time t

Again, $\vec{v} = \dfrac{d\vec{r}}{dt}$

$d\vec{r} = \vec{v}\,dt = (8.0\,\hat{i}\ t + 2.0\,\hat{j}\ t + \vec{u}\,)dt$

Integrating

$$\vec{r} = \vec{u}t + \frac{1}{2}8.0\,t^2\,\hat{i} + \frac{1}{2}2.0\,t^2\hat{j}$$

$$= \vec{u}t + 4.0t^2\,\hat{i} + t^2\,\hat{j}$$

$$= (10.0\,\hat{j}\,)t + 4.0\,t^2\,\hat{i} + t^2\,\hat{j}$$

$$x\,\hat{i} + y\,\hat{j} = 4.0\,t^2\hat{i} + (10t + t^2)\,\hat{j}$$

Since, the motion of the particle is confined to the x-y plane, on equating the coefficients of $\hat{i}$ and $\hat{j}$, we get:

$x = 4t^2$

or, $t = \left(\dfrac{x}{4}\right)^{1/2}$

And $y = 10t + t^2$

When x = 16m:

$$t = \left(\frac{16}{4}\right)^{\frac{1}{2}} = 2s$$

$\therefore y = 10 \times 2 + (2)^2 = 24\text{m}$

(b) Velocity of the particle is given by:

$\vec{v} = 8.0t\,\hat{i} + 2.0t\,\hat{j} + \vec{u}$

at t = 2**s**

$\vec{v} = 8.0 \times 2\,\hat{i} + 2.0 \times 2\,\hat{j} + 10\,\hat{j}$

$= 16\,\hat{i} + 14\,\hat{j}$

$\therefore$ Speed of the particle:

$$\left|\vec{v}\right| = \sqrt{(16)^2 + (14)^2}$$

$$= \sqrt{256 + 196} = \sqrt{452}$$

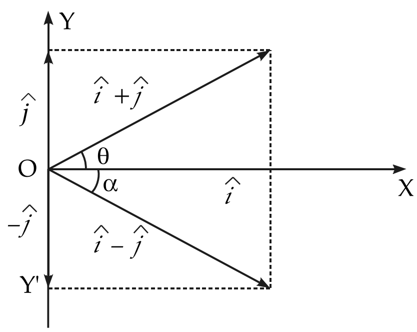
$= 21.26\ m/s$

Q. 19. (i) $\hat{i}$ and $\hat{j}$ are unit vector along *X*-and *Y*-axis respectively. What is the magnitude and direction of the vectors $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$?

(ii) What are the components of a vector $A = 2\hat{i} + 3\hat{j}$ along the directions of $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$? You may use graphical method. [NCERT Ex., Q. 3.19, Page 48]

Ans. (i) Magnitudes of $(\hat{i} + \hat{j})$ and $(\hat{i} - \hat{j})$ vectors: $\hat{i}$ and $\hat{j}$ are the unit vectors along X-and Y-axis respectively.

$$|\hat{i}| = |\hat{j}| = 1$$



The magnitude of vector $\hat{i} + \hat{j}$ is given by

$$|\hat{i} + \hat{j}| = \sqrt{1^2 + 1^2 + 2 \times 1 \times 1 \times \cos 90°}$$

$$= \sqrt{2 + 0} = \sqrt{2}$$

$= 1.414$ units.

The magnitude of vector $\hat{i} - \hat{j}$ is given by

$$|\hat{i} - \hat{j}| = \sqrt{1^2 + 1^2 - 2 \times 1 \times 1 \times \cos 90°}$$

$$= \sqrt{2 - 0} = \sqrt{2}$$

$= 1.414$ units.

Directions of $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$:

Let θ be the angle made by $\hat{i} + \hat{j}$ with $\hat{i}$, i.e., with X-axis.

By definition of scalar product of two vectors

$$(\hat{i} + \hat{j}).\hat{i} = |\hat{i} + \hat{j}|.|\hat{i}|\cos\theta$$

or $\hat{i}.\hat{i} + \hat{i}.\hat{j} = \sqrt{2} \times 1 \times \cos\theta$

or $1 + 0 = \sqrt{2}\cos\theta$

$(\because \hat{i}.\hat{i} = 1 \text{ and } \hat{i}.\hat{j} = 0)$

or $\cos\theta = \dfrac{1}{\sqrt{2}} = \cos 45°$

$\theta = 45°$

Also let α be the angle made by $\hat{i} - \hat{j}$ with $\hat{i}$, *i.e.,* with X-axis

Then $(\hat{i} - \hat{j}).\hat{i} = |\hat{i} - \hat{j}|.|\hat{i}|\cos\alpha$

or $\hat{i}.\hat{i} - \hat{i}.\hat{j} = \sqrt{2} \times 1 \times \cos\alpha$

or $1 - 0 = \sqrt{2}\cos\alpha$

or $\cos\alpha = \dfrac{1}{\sqrt{2}} = \cos 45°$

$\alpha = 45°$

As $-\hat{j}$, is – ve, so $\hat{i} - \hat{j}$ makes – 45° with $-\hat{j}$.

Thus, $(\hat{i} + \hat{j})$ and $(\hat{i} - \hat{j})$ act at 45° and –45° with X-axis respectively.

(ii) (a) Let us now determine the component of $\vec{A} = 2\hat{i} + 3\hat{j}$ in the direction of $(\hat{i} + \hat{j})$

Let $\vec{B} = \hat{i} + \hat{j}$

$\therefore$ $\vec{A}.\vec{B} = AB\cos\beta$

$= (A\cos\beta)B$

or $A\cos\gamma = \dfrac{\vec{A}\cdot\vec{B}}{B}$

Magnitude of the component of $\vec{A}$ in the direction of $\vec{B}$, i.e., $(\hat{i} + \hat{j})$ is A cos γ

$$= \frac{\vec{A}\cdot\vec{B}}{B} = \frac{(2\hat{i} + 3\hat{j}).(\hat{i} + \hat{j})}{|\hat{i} + \hat{j}|}$$

$$= \frac{(2\hat{i}\cdot\hat{i} + 3\hat{j}\cdot\hat{j})}{\sqrt{2}}$$

$(\because \hat{j}\cdot\hat{i} = \hat{i}\cdot\hat{j} = 0)$

$$= \frac{2+3}{\sqrt{2}} = \frac{5}{\sqrt{2}}$$

If $\hat{n}_1$ = unit vector along $(\hat{i} + \hat{j})$, then

$$\hat{n}_1 = \frac{(\hat{i} + \hat{j})}{|\hat{i} + \hat{j}|} = \frac{(\hat{i} + \hat{j})}{\sqrt{2}}$$

$\therefore$ Component of A along $(\hat{i} + \hat{j})$ = Magnitude of the component of A along $\hat{i} + \hat{j}$ i.e., $\hat{n}_1$

$$= \frac{5}{\sqrt{2}}\cdot\frac{(\hat{i} + \hat{j})}{\sqrt{2}} = \frac{5}{2}(\hat{i} + \hat{j})$$

(b) Let us now determine the component of $\vec{A}$ along $\hat{i} - \hat{j}$.

Let $\hat{n}_2$ = unit vector acting along $\hat{i} - \hat{j}$

$$\hat{i}-\hat{j} = |\hat{i}-\hat{j}|\,\hat{n}_2 = \sqrt{2}\hat{n}_2$$

$$\therefore \quad \hat{n}_2 = \frac{(\hat{i}+\hat{j})}{\sqrt{2}}$$

$\therefore$ Magnitude of the component of $\vec{A}$ along $(\hat{i}-\hat{j})$

$$= \vec{A}\cdot\hat{n}_2 = (2\hat{i}+3\hat{j})\cdot\frac{\hat{i}-\hat{j}}{\sqrt{2}}$$

$$= \frac{2-3}{\sqrt{2}} = -\frac{1}{\sqrt{2}}$$

$\therefore$ Component of $\vec{A}$ along $(\hat{i}-\hat{j})$

$$= \left(\vec{A}\cdot\hat{n}_2\right)\hat{n}_2$$

$$= \left(-\frac{1}{\sqrt{2}}\right)\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right)$$

$$= \frac{-1}{2}(\hat{i}-\hat{j})$$

Q. 20. For any arbitrary motion in space, which of the following relations are true:

(a) $\vec{v}_{average} = \left(\frac{1}{2}\right)(\vec{v}(t_1)+\vec{v}(t_2))$

(b) $\vec{v}_{average} = \frac{[\vec{r}(t_2)-\vec{r}(t_1)]}{(t_2-t_1)}$

(c) $\vec{v}(t) = \vec{v}(0)+\vec{a}t$

(d) $\vec{r}(t) = \vec{r}(0)+\vec{v}(0)t+\left(\frac{1}{2}\right)\vec{a}t^2$

(e) $\vec{a}_{average} = \frac{[(t_2)-\vec{v}(t_1)]}{(t_2-t_1)}$

(The 'average' stands for average of the quantity over the time interval t_1 to t_2)

[NCERT Ex., Q. 3.20, Page 48]

Ans. (a) False.

It is given that the motion of the particle is arbitrary. Therefore, the average velocity of the particle cannot be given by this equation.

(b) True.

The arbitrary motion of the particle can be represented by this equation.

(c) False.

The motion of the particle is arbitrary. The acceleration of the particle may also be non-uniform. Hence, this equation cannot represent the motion of the particle in space.

(d) False.

The motion of the particle is arbitrary; acceleration of the particle may also be non-uniform. Hence, this equation cannot represent the motion of particle in space.

(e) True.

The arbitrary motion of the particle can be represented by this equation.

Q. 21. Read each statement below carefully and state, with reasons and examples, if it is true or false:

A scalar quantity is one that

(a) is conserved in a process.

(b) can never take negative values.

(c) must be dimensionless.

(d) does not vary from one point to another in space.

(e) has the same value for observes with different orientations of the axes.

[NCERT Ex., Q. 3.21, Page 48]

Ans. (a) False, because energy is not conserved during inelastic collision.

(b) False, because temperature and electrical potential energy can be negative.

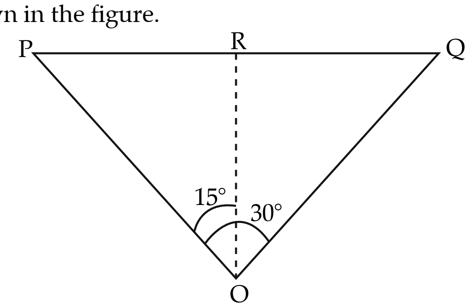
(c) False, scalar quantity like mass density are not dimensionless.

(d) False, because gravitational potential energy vary from point to point.

(e) True, as the R values of scalar does not change with orientation of axes.

Q. 22. An aircraft is flying at a height of 3400 m above the ground. If the angle subtended at a ground observation point by the aircraft positions 10.0 s apart is 30°, what is the speed of the aircraft? [NCERT Ex., Q. 3.22, Page 48]

Ans. The positions of the observer and the aircraft are shown in the figure.



Height of the aircraft from ground, OR = 3400 m

Angle subtended between the positions, $\angle POQ$ = 30°

Time = 10 s

In ΔPRO:

$$\tan 15° = \frac{PR}{OR}$$

$PR = OR \tan 15° = 3400 \times \tan 15°$

ΔPRO is similar to ΔRQO.

$\therefore PR = RQ$

$PQ = PR + RQ$

$= 2PR = 2 \times 3400 \tan 15°$

$= 6800 \times 0.268 = 1822.4$ m

$\therefore$ Speed of the aircraft $= \frac{1822.4}{10} = 182.24\ m/s$