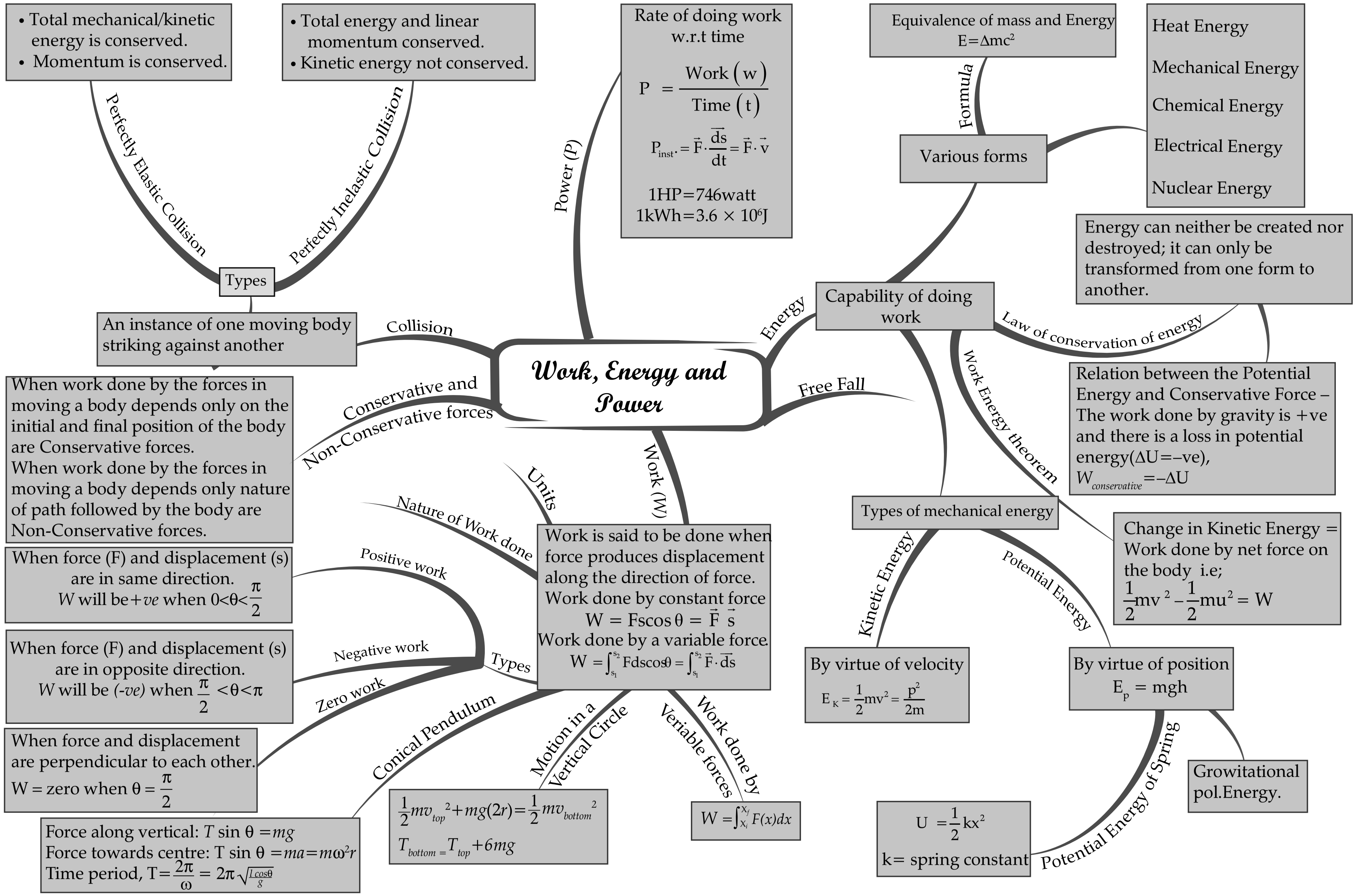

Work, Energy and Power
Power (P)
Rate of doing work w.r.t time
$P = \frac{Work(w)}{Time(t)}$
$P_{inst.} = \vec{F}\cdot\frac{\vec{ds}}{dt} = \vec{F}\cdot\vec{v}$
1HP=746watt
1kWh=3.6 × 10^6J
Collision
An instance of one moving body striking against another
Types
Perfectly Elastic Collision
• Total mechanical/kinetic energy is conserved.
• Momentum is conserved.
Perfectly Inelastic Collision
• Total energy and linear momentum conserved.
• Kinetic energy not conserved.
Conservative and Non-Conservative forces
When work done by the forces in moving a body depends only on the initial and final position of the body are Conservative forces.
When work done by the forces in moving a body depends only nature of path followed by the body are Non-Conservative forces.
Work (W)
Work is said to be done when force produces displacement along the direction of force.
Work done by constant force
$W = F s\cos\theta = \vec{F}\cdot\vec{s}$
Work done by a variable force
$W = \int_{s_1}^{s_2} F ds\cos\theta = \int_{s_1}^{s_2} \vec{F}\cdot\vec{ds}$
Units
Nature of Work done
Types
Positive work
When force (F) and displacement (s) are in same direction. W will be +ve when $0<\theta<\frac{\pi}{2}$
Negative work
When force (F) and displacement (s) are in opposite direction. W will be (-ve) when $\frac{\pi}{2}<\theta<\pi$
Zero work
When force and displacement are perpendicular to each other. W = zero when $\theta = \frac{\pi}{2}$
Conical Pendulum
Force along vertical: $T\sin\theta = mg$
Force towards centre: $T\sin\theta = ma = m\omega^2 r$
Time period, $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L\cos\theta}{g}}$
Motion in a Vertical Circle
$\frac{1}{2}mv_{top}^2 + mg(2r) = \frac{1}{2}mv_{bottom}^2$
$T_{bottom} = T_{top} + 6mg$
Work done by Veriable forces
$W = \int_{X_i}^{X_f} F(x)dx$
Energy
Capability of doing work
Free Fall
Various forms
Formula
Equivalence of mass and Energy $E=\Delta mc^2$
Heat Energy
Mechanical Energy
Chemical Energy
Electrical Energy
Nuclear Energy
Law of conservation of energy
Energy can neither be created nor destroyed; it can only be transformed from one form to another.
Relation between the Potential Energy and Conservative Force – The work done by gravity is +ve and there is a loss in potential energy(ΔU=–ve), $W_{conservative} = -\Delta U$
Work Energy theorem
Change in Kinetic Energy = Work done by net force on the body i.e; $\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = W$
Types of mechanical energy
Kinetic Energy
By virtue of velocity
$E_K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$
Potential Energy
By virtue of position
$E_p = mgh$
Growitational pol.Energy.
Potential Energy of Spring
$U = \frac{1}{2}kx^2$
k= spring constant

CHAPTER

6 WORK, ENERGY AND POWER

Chapter Objectives

Work done by a constant force and a variable force; kinetic energy, work energy theorem, power.
Potential energy, potential energy of a spring, conservative and non-conservative forces: mechanical energy and its conservation (kinetic and potential energies); different forms of energy: mass energy equivalence; dynamics of circular motion; Collision: elastic and inelastic collisions in one and two dimensions.

STUDY MATERIAL

Concepts Clarified :

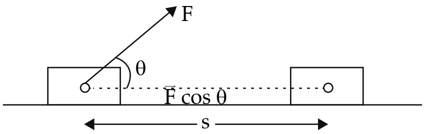
➢ **Work done by force :**

Work is done by the application of force when the body displaces from one place to another in the direction of applied force. In simple terms, Work stands for 'Mechanical Work'.

If the force is applied and there is no displacement then Work = 0, work is directly dependent on the displacement in the direction of force.

Let there be a body under the action of force $\vec{F}$ produces a displacement $\vec{s}$ in the x direction as shown in figure :

Force is applied making an angle θ with the horizontal *i.e.,* in the direction of displacement, then the component of $\vec{F}$ in the direction of displacement is $\vec{F}\cos\theta$. Hence, we can write work as :



$$W = (\vec{F}\cos\theta).\ s$$

If the force is applied in the direction of the displacement, *i.e.,* θ = 0° then

$$W = (\vec{F}\cos 0°).s = \vec{F}\,.\,\vec{s}$$

Mathematically, work can be defined as the dot product of force and displacement. In terms of Rectangular component, it can be written as :

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k} \ \textit{and}\ \vec{s} = x\hat{i} + y\hat{j} + z\hat{k}$$

Hence in rectangular components work can be defined as :

$$W = xF_x + yF_y + zF_z$$

Work is a scalar quantity has magnitude only just because of the dot (scalar) product of two vectors. It can be negative positive or zero.

• **Dimension and Unit of Work :**

Work = Force × Displacement hence W = $[M^1 L^1 T^{-2}] \times L = [ML^2T^{-2}]$

The units of work are of two types :

(a) Absolute Unit :

(i) Joule : It is absolute unit of work in S.I. "1 Joule can be defined as the work done when 1 Newton of force is applied and the body actually moves to 1 m in the direction of applied force" .

1 Joule = 1 Newton × 1 metre=1 N-m

(ii) Erg : It is absolute unit of work in CGS system. "When 1 dyne of force is applied and the 1 cm displacement is achieved in the direction of applied force, then 1 erg or work is done".

1 erg = 1 dyne × 1 cm = 1 dyne-cm

(b) Gravitational Units :

(i) Kilogram - metre - Gravitational Unit of work in S.I.
As per force units - 1 kg-f= 9.8 N, hence,1 kgf.m = 9.8 J

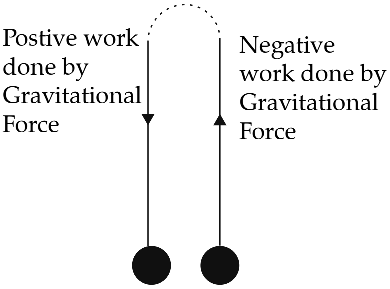
(ii) Gram - centimetre - Gravitation unit if work in CGS system.
As per force, 1 g-f = 980 dynes, hence 1 gf. cm = 980 ergs

- **Nature of Work done :**

Since, work is a scalar quantity but still it can be positive, negative or zero. Let's understand this with an example :

(a) Positive Work : As W = F. $s = \vec{F}\cdot\vec{s}$ = (Fcos θ) s, when θ is acute then cos θ is positive hence, work done is positive.
Example : Body falling under the action of gravity, θ = 0°, cos 0° = 1. Hence work done will be positive.

(b) Negative Work : As W = $\vec{F}\cdot\vec{s}$ = (Fcos θ) s, when θ is obtuse then cos θ is negative hence, work done is negative.
Example : When a ball is thrown against the gravity, the angle between the gravitational force $\vec{F}$ and displacement $\vec{s}$ will be 180° then cos 180° = –1, hence the work done by gravity on the body moving upward will be negative.



(c) Zero Work : When force applied or displacement or both are zero then it is said to be zero work. Angle between $\vec{F}$ & $\vec{s}$ will be 90° as cos 90° = 0.

Example : When we push a wall, we exert the force on wall but $\vec{s}$ = 0. So, $W = 0$

➢ **Work Done by Variable Force :**

(a) Graphical Method :

Usually we encounter variable force as constant force exist rarely. Hence there arises a need to find a technique to calculate work done by force acting in similar direction, but the magnitude keeps varying.

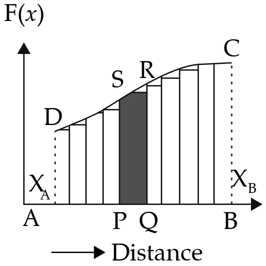
Let us consider there exist a body that moved from A to B as shown in figure. The net displacement from A to B is a cumulative effect of various small displacements.

From the fig. we consider PQ = dx is very small displacement force applied is PS in the same direction. So, the small amount of work done in moving body from P to Q will be given by

dW = F × dx = (PS)(PQ) = *area of the strip* PQRS

Net work done in moving body can be found using the summation *i.e.*,

$$W = \sum dW = \sum_{i=i}^{f} F \times dx_i$$



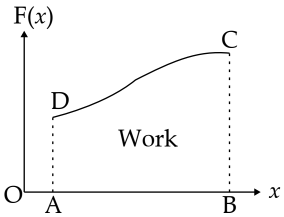
(a)

If the displacement can reach zero, then the no. of terms in the sum increases without limit. Sum will reach a definite value which is equal to the area in the curve CD.

Hence, $W = \lim_{dx \to x} \sum F(dx)$

➢ In Integral Terms, $W = \int_{x_A}^{x_B} F(dx)$ where $x_A = OA$ and $x_B = OB$

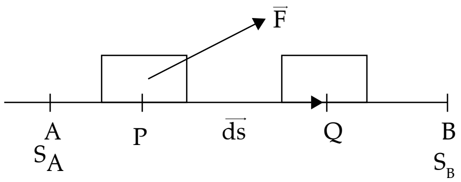
$$W = \int_{x_A}^{x_B} \text{Area of the strip PQRS} = \text{Area ABCDA.}$$



(b)

Note : Hence work done by a variable force can be calculated numerically by evaluating the area under the curve and displacement axis.

Mathematical Calculations (Work done of varying force) : Let there be a body moving from A (S_A) to B(S_B) with application of varying force, we must calculate the work done produced. S_A & S_B are the distance of the points A and B with respect to some reference point.



Let the body is at P, where force on the body is $\vec{F}$. Under the action of force, the body moves by an infinitesimally displacement is $\overrightarrow{PQ} = \overrightarrow{ds}$, if we consider that force remain constant for a displacement P to Q, then the small amount of work done in moving body P to Q is dW= $\vec{F}\cdot\overrightarrow{ds}$.

when $\overrightarrow{ds} \to 0$, total work done from moving body from A to B can be found by integrating the expression between S_A and S_B.

$$W = \int_{S_A}^{S_B} \vec{F}.\overrightarrow{ds}$$

Conservative & Non-Conservative Forces :

Conservative Force : A force is said to be conservative, if the work done by the force in moving a body depends only on the final points A and B *i.e.*, the initial and final positions of the body not on the nature of the path between them.

In simple words, Work done by a conservative force in moving a body between fixed initial and fixed final point will be same.

Example : Gravitational Force is a Conservative Force.

Properties :

(a) Work done by a Conservative Forces depends only on initial and final position of the body.

(b) Work done by a conservative force doesn't depend on the nature of the path in moving from initial to final point.

(c) Work done by or against the conservative force in closed loop or through round trip (*i.e.*, initial and final points coincide) is always zero.

Non-Conservative Force : A force is said to be non-conservative if the work done by the force in moving a body depends on the nature of path between the initial and final points.

Example : Frictional Force is non-conservative in nature.

Energy : Energy of a body can be defined as the ability to do work.

Kinetic Energy :

Kinetic Energy of a body can be defined as the energy possessed by the body by the virtue of its motion.

For Example :

(a) Kinetic Energy is the main cause of energy possessed by the Bullet fired from a bullet.

(b) Windmill works on the kinetic energy of air.

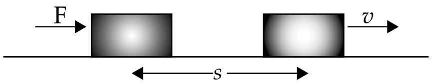
(c) A nail pierces the wooden block due to the kinetic energy of the hammer striking it.

Kinetic Energy in Mathematical terms : It can be derived using any of the below mentioned approach :

(1) The amount of work done in taking a moving body to rest.

(2) The amount of work done required to give the present state velocity to a body from rest.

Let m = mass of the body, F = Force applied, a = acceleration, v = velocity acquired by the body in moving through distance s (as per fig).



From $v^2 - u^2 = 2as \Rightarrow v^2 - 0 = 2as \Rightarrow a = \dfrac{v^2}{2s}$

As F = $ma \Rightarrow$ using F = $ma = m\left(\dfrac{v^2}{2s}\right)$

Work done by body, W = Force × Distance

$$W = F \times s = m\left(\frac{v^2}{2s}\right) \times s = \frac{1}{2}mv^2$$

Kinetic Energy of the body = Work done by the body

Relation between Kinetic Energy and Linear Momentum :

Let there be a body of mass = m, velocity of body = v,

Momentum, $p = mv$

KE of the body = $\dfrac{1}{2}mv^2 = \dfrac{1}{2m}\left(m^2v^2\right) = \dfrac{p^2}{2m}$

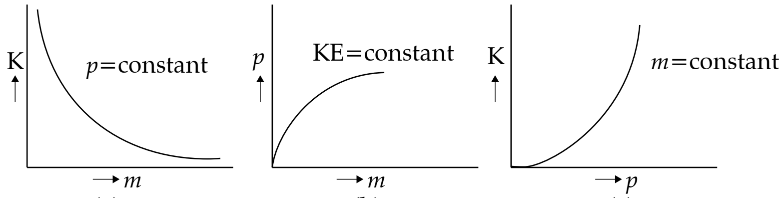
Note : Kinetic Energy is the mandatory factor for the existence of momentum and, is applicable reversibly as well.

Extending it further, if p = constant $\Rightarrow$ KE $\propto \dfrac{1}{m}$ **(a)**

Similarly, if KE = constant $\Rightarrow p \propto \sqrt{m}$ **(b)**

Thirdly, if m = constant $\Rightarrow p \propto \sqrt{KE}$ **(c)**

Graphical representation of all is as follows :



(a) **(b)** **(c)**

- **Work Energy Theorem**

As per this theorem, work done by force in displacing body is equivalent to the net change in kinetic energy of the body. Hence, as per work energy theorem, work and kinetic Energy are equivalent quantities.

Mathematical Proof - Assumption - Motion in one direction to prove the Work Energy theorem.

Let m = mass of the object, u= initial velocity of the object, F = force applied in the direction, a = acceleration, v = final velocity of the object after t seconds.

Then, small amount of work done by applied force on body, $dW = F(ds)$, where ds is the very small displacement achieved by applied force.

Now $F = ma = m\left(\frac{dv}{dt}\right)$

$$dW = F\,(ds) = m\left(\frac{dv}{dt}\right)ds = m\left(\frac{ds}{dt}\right)dv = mv\,dv \quad \left(\text{As } \frac{ds}{dt} = v\right)$$

Total work done by the applied force in increasing its velocity from u to v :

$$W = \int_u^v mvdv = m\int_u^v vdv = m\left[\frac{v^2}{2}\right]_u^v = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = KE_f - KE_i$$

$$W = KE_f - KE_i$$

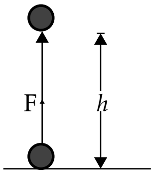
- **Potential Energy** :

It can be defined as the energy possessed by the body by virtue of its position. This can be defined as the force possessed by the system or configuration that one object exert on one another. If the configuration changes the potential energy changes. Two important types of potential energy are :

A. Gravitational Potential Energy : It can be defined as the potential energy possess by any object by virtue of its position above the surface of the Earth.

Let there be a body of mass 'm', 'g' be the acceleration of gravity and 'h' is the height up to which the object is raised.

If we consider that height is not too large then g will remain constant, then force applied will be :



$$F = mg$$

The distance moved by the application of force F is h

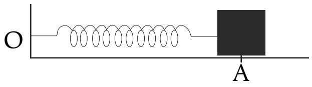
Hence, Work = Force × displacement

$$W = F \times s = mg \times h = mgh$$

Note : Work done taken against the gravity is taken as $+mgh$ and in the direction of gravity is $-mgh$. This work gets stored as potential energy.

B. Potential Energy of a Spring : It can be defined as the energy associated with state of compression or expansion of an elastic spring.

In order to calculate the mathematical expression, let there be an elastic spring OA of mass $\rightarrow 0$. O side of the spring is fixed to a rigid support and a body of a mass 'm' is attached at A.



When the spring is compressed or elongated, it tends to reach its original position by the virtue of elasticity. This force is known as restoring force or elastic force. For small stretch or change, the spring obeys hook's law.

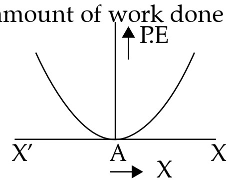
Restoring Force ∝ Elongation or compression

$$-F \propto x \text{ or } -F = kx$$

where, k is the proportionality constant or the spring constant. It is established for the spring as $k \propto \frac{1}{l}$

Smaller is the length of the spring, higher or larger is the spring constant.

The negative sign indicates that the restoring force is directed towards the equilibrium position. The body is displaced through an infinitesimally small distance dx, against the restoring force. Small amount of work done in increasing the length of spring by dx is



$$dW = -Fdx = kxdx$$

Total work done in giving displacement x to the body can be obtained by integrating from $x = 0$ to $x = x$ *i.e.,*

$$W = \int_{x=0}^{x=x} kxdx = k\left(\frac{x^2}{2}\right)_{x=0}^{x=x} = \frac{1}{2}kx^2$$

Mechanical Energy and Its Conservation :

Mechanical Energy is defined as the total energy possessed by the body *i.e.*, the summation of Kinetic Energy, and Potential Energy.

$$ME = KE + PE$$

It is a scalar quantity and measured in Joule. The total energy of the body or system remains conserved if the force doing work on the system is conservative. This is called Principle of Conservation of Mechanical Energy.

Let's try to find the mathematical expression, in order to keep this simple assume motion in one dimension only. Suppose a body undergoes a small displacement Δx under the action of a conservative force F. As per Work Energy theorem :

$$\Delta K = F(x)\,\Delta x$$

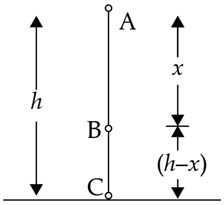
As the force is conservative, the potential energy function $V(x)$ can be defined as

$$-\Delta V = F(x)\,\Delta x \text{ or } \Delta V = F(x)\,\Delta x$$

Adding we get, $\Delta K + \Delta V = 0$ or $\Delta(K+V) = 0 \rightarrow (K + V) = E = \text{Constant}$

Illustration of law of conservation of Mechanical Energy :

To illustrate it further, let us calculate the KE, PE and ME of a body free falling under the action of gravity. Let there be mass held at A, at the height h above the ground.



At Point A,

$KE = 0$, $PE = mgh$, $TE_A = KE+PE = mgh$

In free fall if the body crosses any point B with velocity v_1 where, $AB = x$

Using

$$v^2 - u^2 = 2as$$
$$v_1^2 - 0 = 2gx$$
$$v_1^2 = 2gx$$

At Point B,

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}m(2gx) = mgx,$$

Height above the ground of body at B = CB = $(h - x)$

PE of the body at B = $mg(h - x) = mgh - mgx$

$TE_B = KE+PE = mgx + mgh - mgx = mgh$

When the body is at C it strikes the ground with velocity v, under the free fall.

Using, $v^2 - u^2 = 2as$

$v^2 - 0 = 2gh$

$v^2 = 2gh$

At Point C,

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}m(2gh) = mgh, PE = mgh(h = 0) \Rightarrow mg(0) = 0,$$

$$TE_C = KE + PE = mgh$$
$$\Rightarrow TE_C = TE_B = TE_A = mgh$$

Different Forms of Energy : Several forms of energy other than Kinetic energy and Potential Energy also exist in nature. Such as

(a) **Heat Energy :** This energy is possessed by the body because of the change in temperature due to which the random motion in particle arises.
Heat is also associated with Friction, If a block of mass m slides over a rough surface up to a distance x then the work done by the force will be given by $-f(x)$. We can say that the $KE = -f(x) = -\frac{1}{2}mv^2$. Here we are saying that KE converts to Frictional Energy completely but if we check the surface carefully. There is a slight conversion to heat energy as well.

(b) **Internal Energy :** It can be defined as the net energy possessed by the body by the virtue of configuration and random motion of particles. Thus, internal energy is the sum of potential and kinetic energy of the molecules.

(c) **Electrical Energy :** A definite amount of work is needed to move the electric charge carriers to move in a direction. Such energy is known as Electrical Energy.

(d) **Chemical Energy :** The energy arises due to interaction of molecules participating in chemical reactions. Each chemical possesses different binding energy.
Example : Coal when burnt releases 3×10^4 kJ of energy.

(e) **Nuclear Energy :** Energy possessed from an atomic nucleus. Two different types of nuclear energy are –

(i) **Nuclear Fission** : Splitting of heavy nucleus into two or more lighter nuclei.
(ii) **Nuclear Fusion** : Fusing of two light nuclei to form a heavy nucleus.

- **Mass Energy Equivalence :**

As per an incredible discovery by Einstein, Energy and mass can be transformed into one another.
The relation can be defined as

$$E = \Delta mc^2$$

where, E = Energy that appears, c= velocity of light, Δm = mass that disappears.

Note : Mass and Energy are not conserved individually but are conserved as a common entity mass - energy.

Principle of Conservation of Energy :

As per this principle, Total Energy of an isolated system remain constant.

Relation between Conservative Force and Potential energy :

We know that potential energy depends only on the position and there exists a corresponding potential energy function for every conservative force.

For every conservative force F_x that depends on the position x, there is a potential energy function U(x). Work done by conservative force will be given by –

$$F(x)\,\Delta x = -\Delta U$$

$$\Rightarrow \quad F(x) = -\frac{\Delta U}{\Delta x}$$

$\Rightarrow$ In limits it closes at : $$F(x) = \frac{dU}{dx}$$

Integrating under the limit $x = a$ to b, we have :

$$U_b - U_a = -\int_a^b F(x)dx$$

- **Power :**

Power is a defined as the rate at which the work is done .

$$\text{Power} = \text{Rate of doing work} = \frac{\text{Work done}}{\text{Time taken}}$$

In simple words, Power of a body can be defined as how fast work can be done.

$$P = \frac{dW}{dt}, \text{we know } dW = \vec{F}.\vec{ds} \rightarrow P = \frac{\vec{F}.\vec{ds}}{dt}$$

But $\frac{\vec{ds}}{dt} = \vec{v}$ *i.e.*, the instantaneous velocity.

Hence P= $\vec{F}.\vec{v}$

Dimension of power can be defined as : $P = \frac{W}{t} = \frac{M^1L^2T^{-2}}{T^1} = [ML^2T^{-3}]$

Unit of Power : The absolute unit of power in SI unit is Watt.

P = $\frac{W}{t}$, hence we can say that 1 Watt = 1 Joule/ 1 s $\Rightarrow$ 1 W = 1 Js^{-1}

One watt can be defined as the power to do one watt of work in one second.

$$1 \text{ HP} = 746 \text{ Watt}$$

Force on a particle : In uniform circular motion, acceleration of magnitude $\frac{v^2}{r}$ and it is directed towards center. Hence force of magnitude $\frac{mv^2}{r}$ acting towards the center is required to keep a particle in circular motion. This force is known as Centripetal force.

- **Example 1 :** In case of satellite revolving around Earth, the gravitational pull becomes the centripetal force.
- **Example 2 :** In case of electron revolving around the nucleus, the electrostatic attraction becomes the centripetal force.
- **Example 3 :** In case of Conical pendulum, T sin θ component of tension become the centripetal force.

Steps to analyse force in Uniform Circular Motion :

Take one axis along the radius of circle and other to the perpendicular to the radius. Resolve all forces into components.

Net force along perpendicular axis = 0

Net force along radial axis = $\frac{mv^2}{r} = m\omega^2 r$

Steps to analyse force in Non-Uniform Circular Motion :

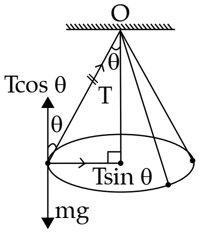
Resolving all the forces along the tangential and radial axes :

Net Tangential Force, $F_t = ma_t$

Net Radial Force, $F_r = ma_r = \frac{mv^2}{r}$

Example : Let there be a particle that revolve on a vertical circle. The forces are : Tension (T) – towards the centre, weight (mg). In case the particle moves outside the surface of the circular track (or sphere) the forces are : Normal Reaction (N) : away from the centre and weight (mg).

Conical Pendulum :



Conical Pendulum is an arrangement where a small block of mass m, rotating in a small circle with the help of a string of length l connected to m. Other end of the string is fixed at a point O vertically above the centre of the circle so that the string is always inclined with vertical at an angle θ. (See figure)

Let's configure all the acting forces,

Along the vertical – $T\cos\theta = mg$

Net force towards centre : $T\sin\theta = ma = m\omega^2 r$

We have,

$$\omega^2 = \frac{g\tan\theta}{r} = \frac{g\tan\theta}{l\sin\theta} = \frac{g}{l\cos\theta}$$

$\Rightarrow$ $$\text{Time period} = T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l\cos\theta}{g}}$$

Points to consider :

- If h is the height of the point O above the circle, then time period = $2\pi\sqrt{\frac{h}{g}}$
- For Conical pendulum, $\omega^2 l\cos\theta = g$
- $\omega > \sqrt{\frac{g}{l}}$ (because $\cos\theta < 1$)

Motion in a vertical Circle :

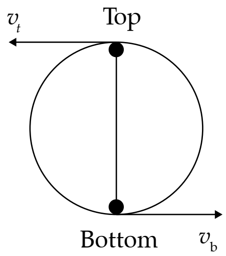
Let us consider a mass m, tied to a string of length l and is rotated in a vertical circle with centre at the other end of the string. Now we will try to find out the below mentioned values

- Minimum mass at the top so that it completes the circle.
- Minimum velocity at the bottom of the circle.

There will be two forces acting all along the mass: Own weight and tension in string.

Let the radius of the circle be 'r'

A. At the Top :



Let v_t = velocity at the top

Net force towards centre = $\frac{mv_t^2}{r}$

$$T + mg = \frac{mv_t^2}{r} \Rightarrow T = \frac{mv_t^2}{r} - mg$$

For the movement in circle the string should remain tight i.e. tension at all points should be positive.

As the tension is minimum at top, $T_{top} \geq 0$

$\Rightarrow$ $$\frac{mv_t^2}{r} - mg \geq 0 \Rightarrow v_t \geq \sqrt{rg}$$

$\Rightarrow$ Critical or Minimum Velocity at the top = $\sqrt{rg}$

B. At the bottom :

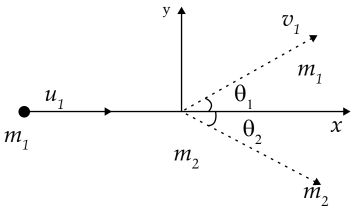
Let v_b be the velocity at the bottom, as particle goes up KE ↓ and GPE↑

- Loss in KE = Gain in GPE

- $\frac{1}{2}mv_b{}^2 - \frac{1}{2}mv_t{}^2 = mg(2r)$
- $v_b{}^2 = v_t{}^2 + 4gr$
- $(v_b)_{\min} = \sqrt{\left(v_t{}^2\right)_{\min} + 4gr} = \sqrt{5gr}$

Note : When a particle is in a vertical motion, its speed decreases when it goes up and increases when it comes down. Hence it is an example of Non-Uniform Circular Motion.

➢ Collision :



Collision means that for a very short period two objects encounter each other. In other words, collision is a reciprocal interaction between two masses for a very short interval in which the momentum and energy of the colliding masses change. You may have observed the impact of a striker on coins when they collide when playing.

Collision involves two masses m_1 and m_2. The v_1 is the speed of particle m_1, where the subscript '1' implies initial. The particle with m_2 is at rest. In this case, the object with mass m_1 collides with the stationary object of mass m_2.

Both masses move in different directions depending on the weight difference.

Types of Collision :

The law of momentum conservation remains true when two bodies collide, but there may be some accidents where Kinetic Energy is not preserved. Conservation can be classified into two, depending on energy conservation :

A. Elastic Collision : In the elastic collision, total momentum and the total kinetic energy are conserved. However, the total mechanical energy is not converted into any other energy form as the forces involved in the short interaction are conserved in nature.

The law of conservation of momentum will give :

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

The conservation of Kinetic Energy says : $\frac{1}{2}m_1u_1{}^2 + \frac{1}{2}m_2u_2{}^2 = \frac{1}{2}m_1v_1{}^2 + \frac{1}{2}m_2v_2{}^2$

B. Inelastic Collision : In the inelastic collision, the objects stick to each other or move in the same direction. The total kinetic energy in this form of collision is not conserved but the total momentum and energy are conserved. During this kind of collision, the energy is transformed into other energy forms like heat and light. Since during the phenomenon the two masses follow the law of conservation of momentum and move in the same direction with same the same v we have :

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

$$v = \frac{m_1u_1 + m_2u_2}{(m_1 + m_2)}$$

- The kinetic energy of the masses before the collision is,

$$\text{KE}_1 = \frac{1}{2}m_1u_1{}^2 + \frac{1}{2}m_2u_2{}^2$$

- While kinetic energy after the collision is,

$$\text{KE}_2 = \frac{1}{2}(m_1 + m_2)v^2$$

- But according to the law of conservation of energy,

$$\frac{1}{2}m_1u_1{}^2 + \frac{1}{2}m_2u_2{}^2 = \frac{1}{2}(m_1 + m_2)v^2 + Q$$

Where 'Q' is the change in energy that results in the production of heat or sound.

Coefficient of Restitution

The restitution coefficient is the proportion between the relative velocity of the bodies colliding before contact with the relative velocity of the electron followed by the crash. Represented by 'e,' the restitution coefficient relies on the content of the bodies that collide.

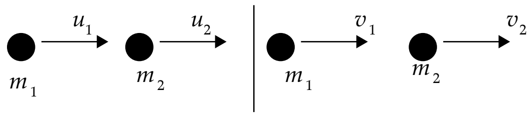
For elastic collisions, $e = 1$ while for inelastic collisions, $e = 0$. The value of $0 < e < 1$ in all other kinds of forceful interactions.

One Dimensional Collision

One dimensional sudden interaction of masses is that collision in which both the initial and final velocities of the masses lie in one line. All the variables of motion are contained in a single dimension.

A. Perfectly Elastic One-Dimensional Collision

The internal kinetic energy is conserved like the momentum, as already discussed in the elastic collisions. Only objects such as microscopic particles such as electrons, protons or neutrons can achieve elastic collisions.



Before collision **After collision**

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

We have,

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$m_1u_1^2 + m_2u_2^2 = m_1v_1^2 + m_2v_2^2$$

$$m_1u_1^2 - m_2u_2^2 = m_1v_1^2 - m_2v_2^2$$

Which finally reduces to $$m_1(u_1 - v_1) = m_2(v_2 - u_2)$$

We get, $$(v_2 - v_1) = -(u_2 - u_1)$$

Using the value in conservation of momentum, we get

$$v_2 = \frac{[2m_1u_1 + u_2(m_2 - m_1)]}{(m_1 + m_2)}$$

Reducing the same we finally get,

$$v_1 = \frac{[2m_2u_2 + u_1(m_1 - m_2)]}{(m_2 + m_1)}$$

When mass of the body is same in both the cases the velocity of the body interchanges *i.e.*,

$$u_1 = v_2 \text{ and } u_2 = v_1$$

In simple words we can say that, if mass of two objects is same then the mass exchanges their velocity.

If $m_1 = m_2$, then $v_1 = 0$, $v_2 = u_1$

If $m_1 < m_2$, then $v_1 = -u_1$, and $v_2 = 0$

If $m_1 > m_2$, then $v_1 = u_1$, and $v_2 = 2u_1$

B. Inelastic One-Dimensional Collision :

In inelastic one-dimensional collision, the colliding masses stick together and move in the same direction at same speeds. The momentum is conserved, and kinetic energy is changed to different forms of energies. For inelastic collisions the equation for conservation of momentum is,

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

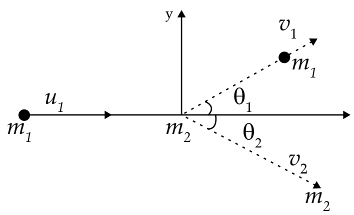
Final velocity after collision (as the object sticks) will be,

$$v = \frac{(m_1u_1 + m_2u_2)}{(m_1 + m_2)}$$

KE lost during the phenomena will be, $\Delta KE = \frac{1}{2}m_1u_2^2 - \frac{1}{2}(m_1 + m_2)v^2$

$$\Delta KE = \frac{1}{2}\frac{m_1m_2}{(m_1 + m_2)}(u_1 - u_2)^2$$

Two-Dimensional Collision :



The above figure signifies collision in two dimensions, where the masses move in different directions after colliding. Here the moving mass m_1 collides with stationary mass m_2. The linear momentum is conserved in the two-dimensional interaction of masses. In this case, we see the masses moving in *xy* plane. The *x* and *y* components equations are :

$$m_1u_1 = m_1u_2 \cos\theta_1 + m_2v_2 \cos\theta_2$$

$$0 = m_1u_2 \sin\theta_1 - m_2v_2 \sin\theta_2$$

For spherical objects with smooth surfaces, the collision happens only when the objects touch each other.

II. Important Formulae

1. Work in Rectangular Component (W) = $xF_x + yF_y + zF_z$
2. Work Done by variable force (W) = $\int_{x_A}^{x_B} F(dx)$
3. Relation between KE and Linear momentum = $KE = \frac{1}{2}mv^2 = \frac{1}{2m}(m^2v^2) = \frac{p^2}{2m}$
4. Work Energy theorem (W) = $KE_f - KE_i$
5. Work done by spring (W) = $\int_{x=0}^{x=x} kx\,dx = k\left(\frac{x^2}{2}\right)_{x=0}^{x=x} = \frac{1}{2}kx^2$
6. Time Period of a conical pendulum $(T) = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l\cos\theta}{g}}$
7. Motion in a vertical circle
   - At bottom : $(v_b)_{max} = \sqrt{5gr}$
   - At top : $(v_t)_{min} = \sqrt{rg}$
8. Condition for one-dimension elastic collision
   - If $m_1 = m_2$, then $v_1 = 0$, $v_2 = u_1$
   - If $m_1 < m_2$, then $v_1 = -u_1$, and $v_2 = 0$
   - If $m_1 > m_2$, then $v_1 = u_1$, and $v_2 = 2u_1$
9. Final velocity – One-dimensional inelastic collision $(v) = \frac{(m_1u_1 + m_2u_2)}{(m_1 + m_2)}$
10. KE Change – One-dimensional inelastic collision $(\Delta KE) = \frac{1}{2}\frac{m_1m_2}{(m_1 + m_2)}(u_1 - u_2)^2$

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