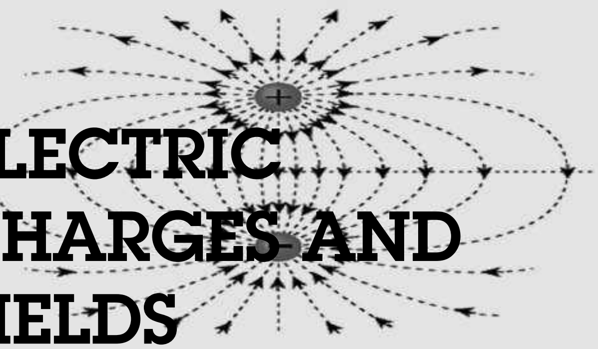
CHAPTER

1 ELECTRIC CHARGES AND FIELDS



Chapter Objectives

This chapter will help you understand :

- ***Electric charge and force :*** *Introduction; Electric charges; Conductors and insulators; Charging by induction; Basic properties of electric charge; Coulomb's Law; and Forces between multiple charges.*
- ***Electric field :*** *Introduction; Electric field; Electric field lines; Electric flux; Electric dipole; Dipole in a uniform external electric field; Continuous charge distribution; Gauss's Law and Application of Gauss's Law.*

TOPIC-1
Electric Charge and Force

Quick Review

- Electrostatics deals with the study of forces, fields and potentials arising from static charges. The name electricity is coined from the Greek word ***electron*** meaning *amber*.
- The **triboelectric effect** (also known as tribo-electric charging) is a type of contact electrification on which certain materials become electrically charged after they come into frictional contact with a different material.
- Coulomb's law should be used for point charges in vacuum at rest. It is not valid for charges in motion.
- The electrostatic force acts along the line joining the two charges. It obeys Newton's third law of motion.
- Coulomb's force is not affected by the presence of other charges in the neighbourhood; hence, the principle of superposition is valid.
- When a body is charged, the charge is distributed equally over the body. The distribution of charge may be one, two or three dimensional. Depending upon that it can have charge.
  - **Linear charge density :** When a charge is distributed along a line (straight or curved), the charge distribution is called linear charge distribution and the charge per unit length is called linear charge density, q/L.
  - **Surface charge density :** When a charge is distributed over a surface (plane or curved), the charge distribution is called surface charge distribution and the charge per unit area is called surface charge density, q/A.
  - **Volume charge density :** When a charge is distributed over the entire volume of the body, the charge distribution is called volume charge distribution and charge per unit volume is called volume charge density, q/V.

TIPS...

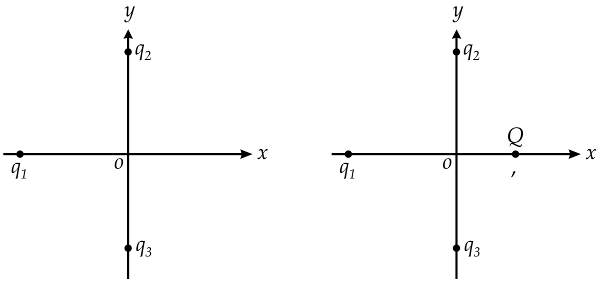
- *Study the Coulomb's law.*
- *Draw diagrams of the charges in the solution.*
- *Learn formulae and values of constants.*

TRICKS...

- *Sum all the x-components, getting the x-component of the resultant electric force.*
- *Sum all the y-components, getting the y-component of the resultant electric force.*
- *Use the Pythagorean Theorem and trigonometry to find the magnitude and direction of the resultant force if desired.*

Multiple Choice Question **(1 mark)**

Q. 1. In given figure, two positive charges q_2 and q_3 fixed along the y axis, exert a net electric force in the + x direction on a charge q_1 fixed along the x axis. If a positive charge Q is added at (x, 0), the force on q_1



(a) **(b)**

(a) shall increase along the positive x-axis.
(b) shall decrease along the positive x-axis.
(c) shall point along the negative x-axis.
(d) shall increase but the direction changes because of the intersection of Q with q_2 and q_3.

[NCERT Exemp. Q. 1.1, Page 1]

Ans. Correct option : (a)

Explanation : Net force on charge q_1, by other charges q_2 and q_3 is along the + x-direction, so nature of force between q_1 and q_2 and q_1 and q_3 is attractive. This is possible when charge q_1 is negative. Now, if a positive charge Q is placed at (x, 0), then, the force on q_1 shall increase. The direction will be along positive x-axis.

Short Answer Type Questions **(3 marks each)**

Q. 1. A paisa coin is made up of Al-Mg alloy and weighs 0.75g. It has a square shape and its diagonal measures 17 mm. It is electrically neutral and contains equal amounts of positive and negative charges.
Treating the paisa coins made up of only Al, find the magnitude of equal number of positive and negative charges. What conclusion do you draw from this magnitude?

[NCERT Exemp. Q. 20, Page 6]

Ans. Given that,
The coin is made of aluminium
Diagonal of the square coin = 17 mm
Atomic mass of Al = 27 g
Atomic number of Al = 13
As we know,
One molar mass of substance has 6.023×10^{23} number of atoms. **[1]**
27 g Al contains 6.023×10^{23} atoms

$$1 \text{ g of } Al \text{ contains} = \frac{6.023\times10^{23}}{27} \text{ atoms}$$

$$0.75 \text{ g of } Al \text{ contains} = \frac{[6.023\times10^{23}\times0.75]}{27}$$

$$=1.67\times10^{23} \text{ atoms}$$

And we know that,
One atom of Al contains 13 electrons 13 protons. **[1]**
1.67×10^{22} atom of Al contains = $13\times1.67\times10^{22}$
= 21.71×10^{22} electron
Hence number of proton = 21.71×10^{22}
The charge on one electron = 1.6×10^{-19} Coulomb
Charge on 21.71×10^{22} electrons
= $1.6\times10^{-19}\times21.71\times10^{22}$
= 34.736×10^{3} C
So, the amount of positive and negative charge on the coin = 34.736×10^{3} C
q = 34.8 kC of ±ve charge. This is a very large amount of charge.
This concludes that all the coins contain a very large amount of positive and negative charges. **[1]**

Q. 2. Consider a coin of above question It is electrically neutral and contains equal amounts of positive and negative charge of magnitude 34.8 kC. Suppose that these equal charges were concentrated in two-point charges separated by

(i) 1 cm ($\sim \frac{1}{2}$ x diagonal of the one paisa coin),
(ii) 100 m (~ length of a long building), and
(iii) 10^6 m (radius of the earth).
Find the force on each such point charge in each of the three cases. What do you conclude from these results?

[NCERT Exemp. Q. 21, Page 6]

Ans. It is clear that charges are equal and opposite. So, by Coulomb's law, force of attraction between charges,

$$F = \frac{1}{4\pi\varepsilon_0}\times\frac{q_1q_2}{r^2}$$

$$q_1 = q_2 = 34.8 \text{ kC}$$

$$\frac{1}{4\pi\varepsilon_0} = 9\times10^9 \text{ Nm}^2/\text{C}^2$$

$$F = \frac{34.8\times10^3\times34.8\times10^3\times9\times10^9}{r^2}$$

$$= \frac{34.8\times34.8\times9\times10^{15}}{r^2}$$

$$= \frac{10899.36\times10^{15}}{r^2}\text{N}$$

$$\cong \frac{1.1\times10^{19}}{r^2}\text{N}$$

$$\therefore F = \frac{1.1\times10^{19}}{r^2}\text{N}$$

(i) Given that,
r_1 = 1 cm = 0.01 m

$$F_1 = \frac{1.1\times10^{19}}{0.01\times0.01} = 1.1\times10^{23}$$

$= 1.1\times10^{23}$ N towards the charges **[1]**

(ii) Given that,

$r_2 = 10^6$ m

$$\therefore F = \frac{1.1\times10^{19}}{100\times100} = 1.1\times10^{15}$$

$= 1.1\times10^{15}$ N towards the charges. **[1]**

(iii) Given that,

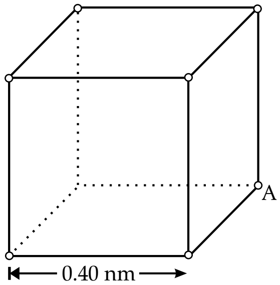
$r_3 = 10^6$ m

$$\therefore F_3 = \frac{1.1\times10^{19}}{10^6\times10^6} = 1.1\times10^{7}$$

$= 1.1\times10^{7}$ *N* towards the charges.

This electrostatic force varies from order 10^7 N to 10^{23} N. The minimum force 10^7 is equivalent to the force of attraction between the earth and 1 million kg body which is too much high. So electrostatic force is so many times larger than the gravitational force. **[1]**

Q. 3. Figure represents a crystal unit of caesium chloride, CsCl. The caesium atoms, represented by open circles are situated at the corners of a cube of side 0.40nm, whereas a Cl atom is situated at the centre of the cube. The Cs atoms are deficient in one electron while the Cl atom carries an excess electron.



(i) What is the net electric field on the Cl atom due to eight Cs atoms?

(ii) Suppose that the Cs atom at the corner *A* is missing. What is the net force now on the Cl atom due to seven remaining Cs atoms?

[NCERT Exemp. Q. 22, Page 6]

Ans. **(i)** It is clear from the given figure that Cl^-atom is at the centre whereas Cs^+ atoms are at eight corners at equal distance. So, by symmetry net force on Cl^- atom due to other Cs^+ atoms will be zero. So by the net force on Cl^- due to Cs^+ ion will be zero, as atoms of Cs^+ attracts the Cl^- equally in opposite direction with pairs diagonally, *i.e.*, (*B*, *H*), (*C*, *E*), (*D*, *F*). **[1½]**

(ii) In the given question, removing Cs atom at the corner *A* is equivalent to adding a singly charged negative *Cs* ion at point *A*. so,

$$\text{Net force} = \frac{q^2}{(4\pi\varepsilon_0 r^2)}$$

Where, q = charge on electron, r = distance between *Cl* and *Cs* atoms

Applying Pythagoras theorem, we have

$$r = \sqrt{(0.02)^2 + (0.20)^2 + (0.20)^2} \times 10^{-19} \text{ m}$$

$$= 0.346\times10^{-9}$$

Now,

$$\text{Net force} = \frac{1}{4\pi\varepsilon_0 r^2}$$

$$= \frac{9\times10^9\times(1.6\times10^{-19})^2}{(0.346\times10^{-9})^2}$$

$$= 1.92\times10^{-9} \text{ N}$$

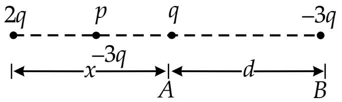
The direction is the force is from *A* to Cl^-.

Force on Cl^- ion,

$F = 1.92\times10^{-9}$ N towards Cl^- ion. **[1½]**

Q. 4. Two charges q and $-3q$ are placed fixed on x-axis separated by distance 'd'. Where should a third charge $2q$ be placed such that it will not experience any force? [NCERT Exemp. Q. 23, Page 6]

Ans. Where, let us keep the charge $2q$ at a distance x from *A*.



So, charge $2q$ will not experience any force.

When force of repulsion on it due to q is balanced by force of attraction on it due to $-3q$, at *B*, where $AB = d$.

So that,

Force of attraction by $-3q$ = Force of repulsion by q **[1½]**

$$\Rightarrow \quad \frac{2q\times q}{4\pi\varepsilon_0 x^2} = \frac{2q\times3q}{4\pi\varepsilon_0 (x+d)^2}$$

$$\Rightarrow \quad (x+d)^2 = 3x^2$$

$$\Rightarrow x^2 + d^2 + 2xd = 3x^2$$

$$\Rightarrow 2x^2 - 2dx - d^2 = 0$$

$$\Rightarrow \quad x = \frac{d}{2} \pm \frac{\sqrt{3}d}{2}$$

[Negative (–) sign be between q and $-3q$ and hence is unadaptable.]

$$x = \frac{d}{2} + \frac{\sqrt{3}d}{2}$$

$$= \frac{d}{2}(1+\sqrt{3}) \text{ to the left of } q$$

[1½]

So, x will be : $x = \dfrac{d}{2} + \dfrac{\sqrt{3}d}{2}$ to the left of q.

Long Answer Type Questions (5 marks each)

Q. 1. There is another useful system of units, besides the SI/mks, a system called the cgs (centimetre-gram-second) system. In this system Coulomb's law is given by : $F = \frac{Qq}{r^2}\hat{r}$ where the distance r is measured in cm (= 10^{-2} m), F in dynes (= 10^{-5} N) and the charges in electrostatic units (esu units), where 1 esu unit of charge = $\frac{1}{|3|} \times 10^{-9}$ C. The number $|3|$ actually arises from the speed of light in vacuum which is now taken to be exactly given by $c = 2.99792458 \times 10^8$ m/s. An approximate value of c then is $c = |3| \times 10^8$ m/s.

(i) Show that the coulomb law in CGS units yields. 1 esu of charge = 1 (dyne)$^{1/2}$ cm. Obtain the dimensions of units of charge in terms of mass M, length L and time T. Show that it is given in terms of fractional powers of M and L.

(ii) Write 1 esu of charge = x C, where x is a dimensionless number. Show that this gives

$$\frac{1}{4\pi\varepsilon_0} = \frac{10^{-9}}{x^2}\frac{\text{N.m}^2}{C^2}$$

With $x = \frac{1}{3} \times 10^9$ we have,

$$\frac{1}{4\pi\varepsilon_0} = 3^2 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}$$

$$\text{or, } \frac{1}{4\pi\varepsilon_0} = (2.99792458)^2 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \text{(exactly)}$$

[NCERT Exemp. Q. 1.29, Page 8]

Ans. (i) From the relation,

$$F = \frac{Qq}{r^2} = 1 \text{ dyne} = \frac{[1 \text{ esu of charge}]^2}{[1 \text{ cm}]^2}$$

So, 1 esu of charge

$$= (1 \text{ dyne})^{\frac{1}{2}} \times 1 \text{ cm} = F^{\frac{1}{2}} \cdot L = [\text{MLT}^{-2}]^{\frac{1}{2}} L$$

So, 1 esu of charge = $\text{M}^{\frac{1}{2}}\text{L}^{\frac{3}{2}}\text{T}^{-1}$

Thus, esu of charge is represented in terms of fractional powers $\frac{1}{2}$ of M and $\frac{3}{2}$ of L. **[2½]**

(ii) Let 1 esu of charge = x C. where x is a dimensionless number Coulomb force on two charges, each of magnitude 1 esu separated by 1 cm is dyne = 10^{-5} N. This situation is equivalent to two charges of magnitude x C separated by 10^{-2} m.

$$\therefore F = \frac{1}{4\pi\varepsilon_0}\frac{x^2}{(10^{-2})^2} = 1 \text{ dyne} = 10^{-5}\text{N}$$

Put the value of constant:

$$\therefore \frac{1}{4\pi\varepsilon_0} = \frac{10^{-9}}{x^2}\frac{\text{Nm}^2}{C^2}$$

Taking, $x = \frac{1}{|3| \times 10^9}$

We get,

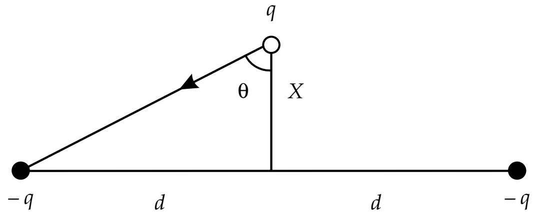
$$\frac{1}{4\pi\varepsilon_0} = |3| \times 10^9 \times 10^{18}\frac{\text{Nm}^2}{\text{C}^2}$$

$$= 9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}$$

If $|3| \rightarrow 2.99792458$,

we get, $\frac{1}{4\pi\varepsilon_0} = 8.98755 \times 10^9 \text{ Nm}^2\text{C}^{-2}$. **[2½]**

Q. 2. Two charges $-q$ each are fixed separated by distance $2d$. A third charge q of mass m placed at the mid-point is displaced slightly by x ($x << d$) perpendicular to the line joining the two-fixed charged as shown in Figure. Show that q will perform simple harmonic oscillation of time period. $T = [(8\pi^3\varepsilon_0 md^3)/q^2]^{\frac{1}{2}}$



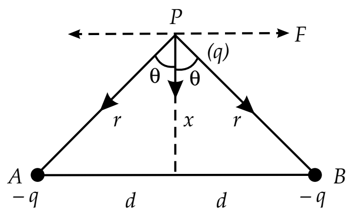
[NCERT Exemp. Q. 1.30, Page 9]

Ans. From the figure,

Given that,

Two charge $-q$ at A and B

$AB = AO + OB = 2d$



x = Small distance perpendicular to O.

i.e., $x << d$ mass of charge q is. So, force of attraction at P towards A and B are each

$$F = \frac{q(q)}{4\pi\varepsilon_0 r^2}$$

where $AP = BP = r$

Horizontal components of these forces F_n are cancel out. Vertical components along PO are added up. **[2½]**

If $\angle APO = \theta$, the net force on q along PO is

$F' = 2 F \cos\theta$

$$= \frac{2q^2}{4\pi\varepsilon_0 r^2}\left(\frac{x}{r}\right)$$

$$= \frac{2q^2 x}{4\pi\varepsilon_0 (d^2 + x^2)^{\frac{3}{2}}}$$

When, $x << d$, $F' = \frac{2q^2 x}{4\pi\varepsilon_0 d^3} = Kx$

where, $K = \frac{2q^2}{4\pi\varepsilon_0 d^3}$

$\Rightarrow F \propto x$

So that, force on charged components is proportional to its displacement from the centre O and it is directed towards O.

So it is clear that, the motion of charge g would be simple harmonic, where

$$\omega = \sqrt{\frac{K}{m}}$$

and $T = \frac{2\pi}{\omega}$

$$= 2\pi\sqrt{\frac{m}{K}}$$

$$= 2\pi\sqrt{\frac{m.4\pi\varepsilon_0 d^3}{2q^2}}$$

$$= \left[\frac{8\pi^3\varepsilon_0 m d^3}{q^2}\right]^{\frac{1}{2}}$$

[2½]

TOPIC-2
Electric Field

Quick Review

❖ When there is more than one charge in a region, the electric field lines will not be straight lines. The electric field lines will curve in response to the different charges. In every case, though, the field is highest where the field lines are close together and decreases as the lines get further apart.

❖ Electric field lines of force have a tendency to get separated from each other in the direction perpendicular to their lengths. They repel if they are of like charges.

❖ These lines of force are like elastic string, they come to contract in length, that is, attract each other with respect to the opposite charges.

❖ Closeness of lines of forces symbolises more strength of electric field and vice versa.

❖ Parallel lines indicate uniform field.

❖ Two lines of forces never intersect each other.

❖ Lines of force never pass through a conductor, that is, field inside a conductor is always zero.

❖ The tangential direction at any point on the lines of forces indicates the direction of the force acting on the positive charge at that point.

❖ Gauss' Law is a powerful method of calculating electric fields.

❖ The permittivity of free space : It is a constant related to the constant k that appears in Coulomb's law. The relationship between the two is this :

$$k = \frac{1}{4\pi\varepsilon_0}$$

or, equivalently,

$$\varepsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12}\ \text{C}^2/\text{Nm}^2$$

TIPS...

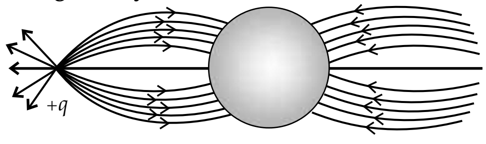
- *Study the properties of Electric field.*
- *Understand the Gauss's Law.*
- *Study the application of Gauss's law.*

TRICKS...

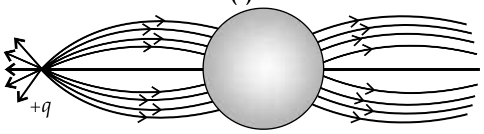
- *Practice a lot in Gauss's law.*
- *Make flow chart to revise quickly.*

Multiple Choice Questions **(1 mark each)**

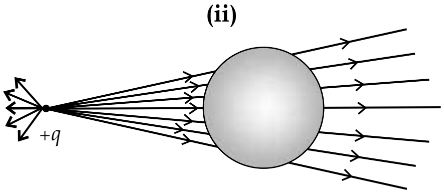
Q. 1. A point positive charge is brought near an isolated conducting sphere in Figure. The electric field is best given by :



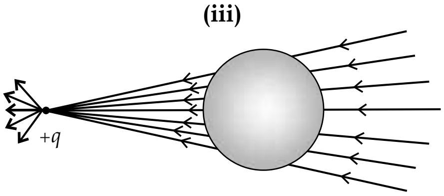
(i)



(ii)



(iii)



(iv)

(a) Fig (i) **(b) Fig (iii)**
(c) Fig (ii) **(d) Fig (iv)**

[NCERT Exemp. Q. 1.2, Page 2]

Ans. Correct option : (a)

Explanation : As given charge is + q and lines of forces in positive charge must be outwards from positive charge q. Now, as the positive charge is kept near an isolated conducting sphere, due to induction left part of sphere gets accumulated negative charge and right part positive, and lines of force from right part of sphere must emerge outwards normally.

So, verifies the answer (a).

As lines of forces are not perpendicular to the surface of sphere, so options (iii) and (iv) are not true again.

Q. 2. A point charge $+q$, is placed at a distance d from an isolated conducting plane. The field at a point P on the other side of the plane is

(a) directed perpendicular to the plane and away from the plane.

(b) directed perpendicular to the plane but towards the plane.

(c) directed radially away from the point charge.

(d) directed radially towards the point charge.

[NCERT Exemp. Q. 1.6, Page 3]

Ans. Correct option : (a)

Explanation : When you place a positive charge near a conducting plane, then electric field lines from positive charges will enter into the conducting plane (from the side where positive charge is kept) and emerge from opposite side of the plane.

In both cases, the direction of electric field lines will always be perpendicular to the surface of the plane.

Q. 3. A hemisphere is uniformly charged positively. The electric field at a point on a diameter away from the centre is directed

(a) perpendicular to the diameter.

(b) parallel to the diameter.

(c) at an angle tilted towards the diameter.

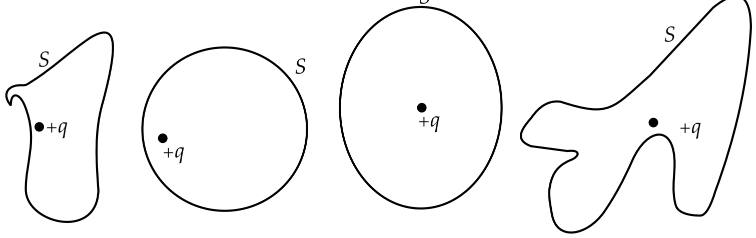
(d) at an angle tilted away from the diameter.

[NCERT Exemp. Q. 1.7, Page 3]

Ans. Correct option : (a)

Explanation : As the side or diameter of hemi-sphere is plane surface, and whole hemi-sphere is charged with positive charge so, the electric field line of forces emerging outward will be perpendicular to the plane surface or diameter.

Q. 4. The electric flux through the surface :



(i) (ii) (iii) (iv)

(a) in Figure (iv) is the largest.

(b) in Figure (iii) is the least.

(c) in Figure (ii) is same as Figure (iii) but is smaller than Figure (iv).

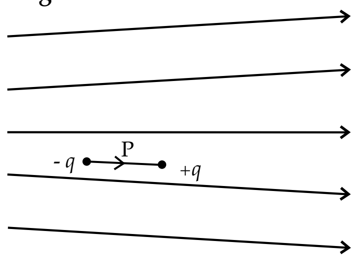
(d) is the same for all the figures.

[NCERT Exemp. Q. 1.3, Page 2]

Ans. Correct option : (d)

Explanation : Electric flux, through the closed surface (or space) depends only on the charge enclosed inside the surface. Here, charges inside all figures are same. So, electric flux will remain same.

Q. 5. Figure shows electric field lines in which an electric dipole p is placed as shown. Which of the following statements is correct?



(a) The dipole will not experience any force.

(b) The dipole will experience a force towards right.

(c) The dipole will experience a force towards left.

(d) The dipole will experience a force upwards.

[NCERT Exemp. Q. 1.5, Page 3]

Ans. *Correct op***tion :** (c)

Explanation : We know electric field emerges radially outward from positive point charge.

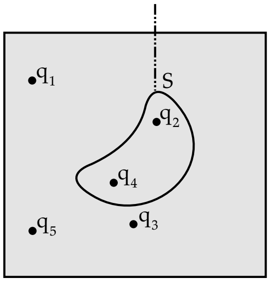
In the figure given above, space between field lines is increasing (or density of electric field line is decreasing). In other words, the electric force is decreasing while moving from left to right.

Thus, the force on charge $-q$ is greater than the force on charge $+q$ in turn dipole will experience a force towards left direction.

Q. 6. Five charges q_1, q_2, q_3, q_4, and q_5 are fixed at their positions as shown in Figure. *S* is a Gaussian surface. The Gauss's law is given by:

$$\oint_s \mathbf{E}.\,\mathbf{ds} = \frac{q}{\varepsilon_0}$$

Gaussian Surface



Which of the following statements is correct?

(a) *E* on the LHS of the above equation will have a contribution from q_1, q_5 and q_3 while q on the RHS will have a contribution from q_2 and q_4 only.

(b) *E* on the LHS of the above equation will have a contribution from all charges while q on the RHS will have a contribution from q_2 and q_4 only.

(c) *E* on the LHS of the above equation will have a contribution from all charges while q on the RHS will have a contribution from q_1, q_3 and q_5 only.

(d) Both *E* on the LHS and q on the RHS will have contributions from q_2 and q_4 only.

[NCERT Exemp. Q. 1.4, Page 3]

Ans. *Correct op***tion :** (b)

Explanation : As all charges are positive (or of same signs) so electric field lines on R.H.S. of Gaussian surface will be due to q_2 and q_4 only.

On L.H.S. of Gaussian surface, the electric field lines on '*E*' will be due to q_1, q_2, q_3, q_4 and q_5. So, answer (*b*) is verified.

Tick Two or More Options

Q. 7. The Electric field at a point is

(a) always continuous.

(b) continuous if there is no charge at that point.

(c) discontinuous only if there is a negative charge at that point.

(d) discontinuous if there is a charge at that point.

[NCERT Exemp. Q. 1.9, Page 4]

Ans. Correct options : (b) and (d)

Explanation : Either positive or negative charges will interact the lines of electric field so makes the electric field discontinuous.

If there is no other charge inside the electric field then the lines will not be affected. So electric field becomes continuous. So, answers (b) and (d) are verified.

Q. 8. Consider a region inside which there are various types of charges but the total charge is zero. At points outside the region

(a) the electric field is necessarily zero.

(b) the electric field is due to the dipole moment of the charge distribution only.

(c) the dominant electric field is $\propto \frac{1}{r^3}$, for large r, where r is the distance from a origin in this region.

(d) the work done to move a charged particle along a closed path, away from the region, will be zero.

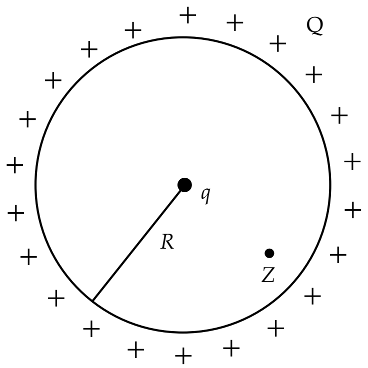
[NCERT Exemp. Q. 1.11, Page 4]

Ans. Correct options : (c) and (d)

Explanation : Although net charge in a dipole is zero but its electric field is proportional to $\frac{1}{r^3}$

Work done again electric field is conservative, so net work done in a closed loop is always zero. So, answer (c) and (d) are verified.

Q. 9. A positive charge Q is uniformly distributed along a circular ring of radius R. A small test charge q is placed at the centre of the ring. Then :



(a) If $q > 0$ and is displaced away from the centre in the plane of the ring, it will be pushed back towards the centre.

(b) If $q < 0$ and is displaced away from the centre in the plane of the ring, it will never return to the centre and will continue moving till it hits the ring.

(c) If $q < 0$, it will perform SHM for small displacement along the axis.

(d) q at the centre of the ring is in an unstable equilibrium within the plane of the ring for $q > 0$.

[NCERT Exemp. Q. 1.13, Page 5]

Ans. Correct options : (a), (b), (c) and (d)

Explanation : For '*d*', charge is uniformly distributed along the ring. It is not sphere in which charge is only outside. So, positive charge of ring will interact equally a charge placed at centre of ring but will be in unstable equilibrium.

For '*c*', if '*q*' is displaced slightly (or small), it will perform SHM and stops at centre. Options (a) and (b) are verified in a similar way.

Q. 10. If $\oint_s$ E. dS = 0 over a surface, then

(a) the electric field inside the surface and on it is zero.

(b) the electric field inside the surface is necessarily uniform.

(c) the number of flux lines entering the surface must be equal to the number of flux lines leaving it.

(d) all charges must necessarily be outside the surface.

[NCERT Exemp. Q. 1.8, Page 4]

Ans. Correct options : (c) and (d)

Explanation : Flux in Gaussian surface is zero. So, the net charge inside the closed surface either is zero or charges are outside the surface. If charge or charges are outside the Gaussian surface, then entering leaving lines of electric field will be equal so net flux (lines of electric filed) is zero verifies answers (c) and (d).

Q. 11. If there were only one type of charge in the universe, then

(a) $\oint_s$ E. dS $\neq 0$ on any surface.

(b) $\oint_s$ E. dS = 0 if the charge is outside the surface.

(c) $\oint_s$ E. dS could not be defined.

(d) $\oint_s$ E. dS = $\frac{q}{\varepsilon_0}$ if charges of magnitude q were inside the surface.

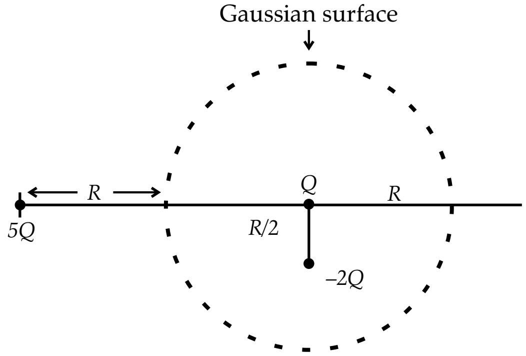
[NCERT Exemp. Q. 1.10, Page 4]

Ans. Correct options : (b) and (d)

Explanation : If a charge q is enclosed inside Gaussian surface then (d) is true. If Gaussian surface (or space) is outside the charge and (c) and (a) are not true.

So, answers (b) and (d) are true.

Q. 12. Refer to the arrangement of charges in Figure and a Gaussian surface of radius R with Q at the centre. Then



(a) total flux through the surface of the sphere is $\frac{-Q}{\varepsilon_0}$.

(b) field on the surface of the sphere is $\frac{-Q}{4\pi\varepsilon_0 R^2}$

(c) flux through the surface of sphere due to 5Q is zero.

(d) field on the surface of sphere due to –2Q is same everywhere. **[NCERT Exemp. Q. 1.12, Page 4]**

Ans. Correct options : (a) and (c)

Explanation :

(a) It is true by Gauss's law. Here net charge = –2Q + Q = –Q.

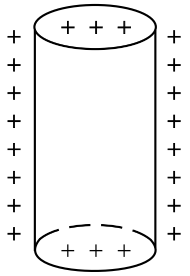
(b) It is the electric field on a conducting sphere. There is no any conducting sphere but only surface or spherical space is there.

(c) 5Q charge outside the Gaussian surface will not contribute to electric flux in Gaussian surface (or space).

(d) For the distance of Gaussian surface from –2Q is different, so field will not be same on the surface.

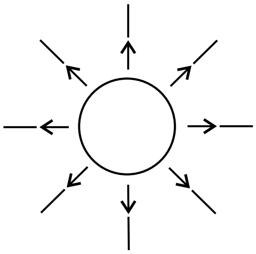
Very Short Answer Type Questions **(1 or 2 marks each)**

Q. 1. Sketch the electric field lines for a uniformly charged hollow cylinder shown in Figure.

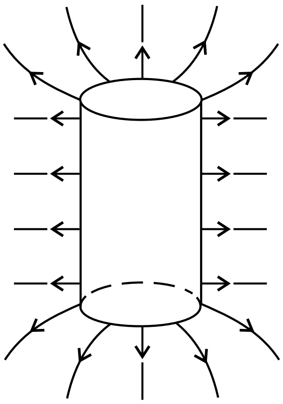


[NCERT Exemp. Q. 18, Page 5]

Ans. As there is no charge inside the hollow cylinders, so no negative charge will be induced. Only positive charge is spreading uniformly on the surface and lines of force emerge away perpendicularly from the surface to infinity.



Top view



Side view

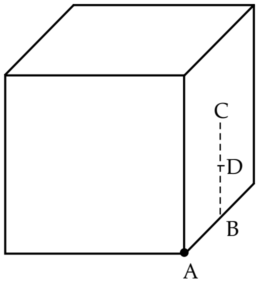
[1]

Q. 2. The dimensions of an atom are of the order of an Angstrom. Thus, there must be large electric fields between the protons and electrons. Why, then is the electrostatic field inside a conductor zero? [NCERT Exemp. Q. 16, Page 5]

Ans. In an atom, number of electrons and protons are equal and the electric fields bind the atoms to neutral entity. Electrostatic fields are caused by excess charges. **[½]**

But there can be no excess charge on the inner surface of an isolated conductor. So, the electrostatic fields inside a conductor are zero despite the fact that the dimensions of an atom are of the order of an Angstrom. **[½]**

Q. 3. What will be the total flux through the faces of the cube in figure with side of length a if *a* charge *q* is placed at :



(a) A : a corner of the cube.

(b) B : mid-point of an edge of the cube.

(c) C : centre of a face of the cube.

(d) D : mid-point of B and C.

[NCERT Exemp. Q. 19, Page 6]

Ans. **(a)** If a charge is placed at corner A of the cube then only $\frac{1}{8}^{th}$ portion of the charge lies inside the Gaussian surface. So, total flux through the faces of the given cube $= \frac{q}{8\varepsilon_0}$. **[½]**

(b) If a charge is placed at point B of the cube then only $\frac{1}{4}^{th}$ portion of the charge lies inside the Gaussian surface. So, total flux through the faces of the given cube $= \frac{q}{4\varepsilon_0}$. **[½]**

(c) If a charge is placed at point *C* of the cube then only $\frac{1}{2}$ portion of the charge lies inside the Gaussian surface. So, total flux through the faces of the given cube $= \frac{q}{2\varepsilon_0}$. **[½]**

(d) If a charge is placed at point *D* of the cube then only $\frac{1}{2}$ portion of the charge lies inside the Gaussian surface. So, total flux through the faces of the given cube $= \frac{q}{2\varepsilon_0}$. **[½]**

Q. 4. A metallic spherical shell has an inner radius R_1 and outer radius R_2. A charge *Q* is placed at the centre of the spherical cavity. What will be surface charge density on (i) the inner surface, and (ii) the outer surface? [NCERT Exemp. Q. 15, Page 5]

Ans. When positive charge is placed at the centre of the spherical cavity then an equal amount of negative charge (–*Q*) appears on inner surface of the sphere due to induction.

This charge is distributed uniformly on the inner surface. So, an equal amount of positive charge ($+Q$) also appears on outer surface of the sphere.

Now surface charge density on inner surface =

$$\frac{\text{Total charge on inner surface}}{\text{Area of the inner surface}} = \frac{-Q}{(4\pi R_1^2)} \quad \textbf{[1]}$$

Surface charge density on outer surface =

$$\frac{\text{Total charge on outer surface}}{\text{Area of the outer surface}} = \frac{+Q}{(4\pi R_2^2)} \quad \textbf{[1]}$$

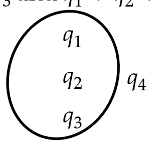
Q. 5. If the total charge enclosed by a surface is zero, does it imply that the electric field everywhere on the surface is zero? Conversely, if the electric field everywhere on a surface is zero, does it imply that net charge inside is zero.

[NCERT Exemp. Q. 17, Page 5]

Ans. According to Gauss's law of electrostatics,

$$\phi = \oint_s E.\,dS = \frac{q}{\varepsilon_0}$$

Now, Consider a Gaussian surface enclosing charges q_1, q_2, q_3 and $q_1 + q_2 + q_3 = 0$ **[1]**



But the term q enclosed on the R.H.S of the equation includes the sum of all charges enclosed by the surface called gaussian surface which is equal to zero. But, on L.H.S., it will be not zero due to q_4. So, first statement is not correct. And,

If electric field on Gaussian surface is zero, then, in above case, it is possible only when $q_4 = 0$,

i.e., if everywhere, on Gaussian surface, electric field is zero then net charge will be zero. **[1]**

Q. 6. An arbitrary surface encloses a dipole. What is the electric flux through this surface?

[NCERT Exemp. Q. 14, Page 5]

Ans. By Gauss' theorem of electrostatics, the flux crossing a closed surface is equal to the net charge enclosed by the surface divided by ε_0.

$$\phi = \oint_s E.\,dS = \frac{q}{\varepsilon_0} \quad \textbf{[1]}$$

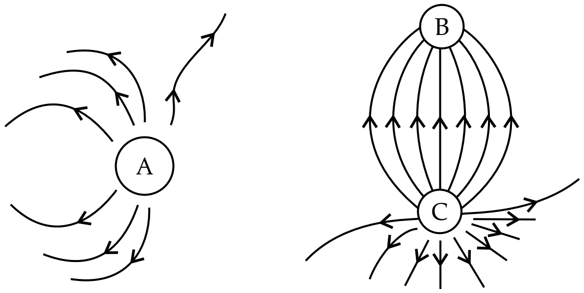
Net charge inside the Gaussian surface due to a dipole, $+q - q = 0$

$$\phi = \oint_s E.\,dS = \frac{q}{\varepsilon_0} = 0$$

Since the net charge of a dipole is zero, the total flux crossing the surface is also zero. **[1]**

Short Answer Type Questions **(3 marks each)**

Q. 1. Figure shows the electric field lines around three-point charges A, B and C.



(a) Which charges are positive?

(b) Which charge has the largest magnitude? Why?

(c) In which region or regions of the picture could the electric field be zero? Justify your answer.

(d) (i) near A, (ii) near B, (iii) near C, (iv) nowhere.

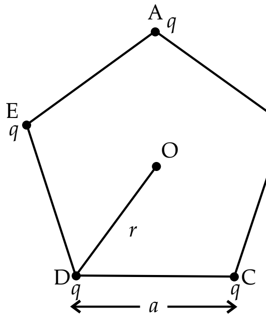
[NCERT Exemp. Q. 1.24, Page 7]

Ans. (a) Electric lines of forces diverge out from positive charge. So, A and C are positive charges. **[1]**

(b) As the density of electric lines forces from charge increases, the intensity of electric field or magnitude of charge will increase. Hence, maximum numbers of field lines are associated with C. **[1]**

(c) Point between two like charges where electrostatic force is zero is called neutral point. So, the neutral point lies between A and C only. Now the position of neutral point depends on the strength of the forces of charges. Here, more number of electric lines of forces shows higher strength of charge C than A. So, neutral point lies near A. As the magnitude of charge C is larger than A so lines of force of C will be stronger than of A. So, the neutral point lies somewhere near A. So, option (a) is the correct answer. **[1]**

Q. 2. Five charges, q each are placed at the corners of a regular pentagon of side 'a' (in figure).



(a) (i) What will be the electric field at O, the centre of the pentagon?

(ii) What will be the electric field at O if the charge from one of the corners (say A) is removed?

(iii) What will be the electric field at O if the charge q at A is replaced by $-q$?

(b) How would your answer to (a) be affected if pentagon is replaced by *n*-sided regular polygon with charge *q* at each of its corners?

[NCERT Exemp. Q. 1.25, Page 7]

Ans. **(a) (i)** The point *O* is equidistant from all the charges at the end point of pentagon. So, due to symmetry, the forces due to all the charges are cancel out each other. So, electric field at *O* is zero.

(ii) When charge *q* is removed a negative charge will develop at A giving electric field

$$E = \frac{(q \times 1)}{4\pi\varepsilon_0 r^2} \text{ along } OA.$$

(iii) If charge *q* at *A* is replaced by $-q$, then two negative charges $-2q$ will develop there. Thus, the value of electric field,

$$E = \frac{2q}{4\pi\varepsilon_0 r^2} \text{ along } OA.$$ **[1½]**

(b) Pentagon is replaced by the *n*-sided polygon :
When pentagon is replaced by *n*-sided regular polygon with charge *q* at each of its corners, the electric field at *O* would continue to be zero as symmetry of the charges is due to the regularity of the polygon. It doesn't depend on the number of sides or the number of charges. **[1½]**

Long Answer Type Questions **(5 marks each)**

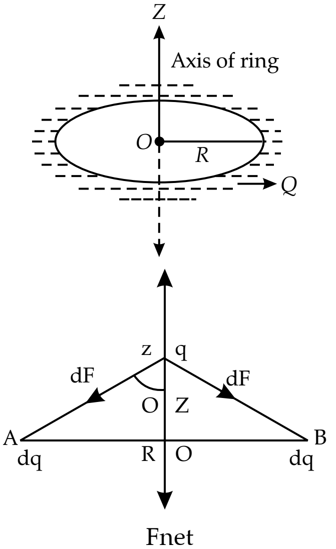
Q. 1. Total charge –*Q* is uniformly spread along length of a ring of radius *R*. A small test charge +*q* of mass m is kept at the centre of the ring and is given a gentle push along the axis of the ring.

(a) Show that the particle executes a simple harmonic oscillation.

(b) Obtain its time period.

[NCERT Exemp. Q. 1.31, Page 9]

Ans. Slight push on *q* along the axis of the ring gives rise to the situation shown in Figure. *A* and *B* are two points on the ring at the end of a diameter.



[1]

Plane of the ring

Let the charge *q* is displaced slightly by *z* (z<<*R*) along the axis of ring. Let force on the charge *q* will be towards *O*. The motion of charge *q* to be simple harmonic, if the force on charge *q* must be proportional to *z* and is directed towards *O*.

Electric field at axis of the ring at a distance *z* from the centre of ring

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Qz}{(R^2+z^2)^{\frac{3}{2}}}; \quad \text{towards O}$$

Net force on the charge $F_{net} = qE$ **[1½]**

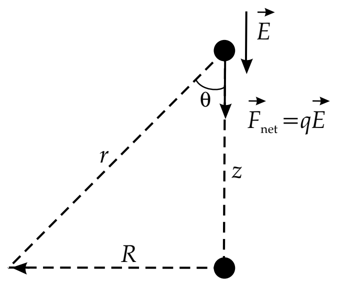
$$F_{net} = \frac{1}{4\pi\varepsilon_0}\frac{qQz}{(R^2+z^2)^{\frac{3}{2}}}$$

$$\Rightarrow \quad F_{net} = \frac{1}{4\pi\varepsilon_0}\frac{qQz}{R^3\left(1+\frac{z^2}{R^2}\right)^{\frac{3}{2}}}$$

As $z << R$ then, $F_{net} = \frac{1}{4\pi\varepsilon_0}\frac{qQz}{R^3}$

or, $\quad \overrightarrow{F_{net}} = -K\vec{z}$

where $\quad K = \frac{Qq}{4\pi\varepsilon_0 R^3} = \text{constant}$



[1]

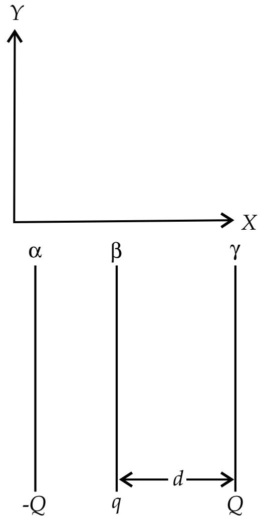
Clearly, force on *q* is proportional to negative of its displacement. Therefore, motion, of *q* is simple harmonic.

$$\omega = \sqrt{\frac{K}{m}} \text{ and } T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{K}}$$

$$T = 2\pi\sqrt{\frac{m4\pi\varepsilon_0 R^3}{Qq}}$$

$$T = 2\pi\sqrt{\frac{4\pi\varepsilon_0 mR^3}{Qq}}$$ **[1½]**

Q. 2. Two fixed, identical conducting plates (α and β), each of surface area *S* are charged to –*Q* and *q*, respectively, where *Q* > *q* > 0. A third identical plate (γ), free to move is located on the other side of the plate with charge *q* at a distance *d* (in Figure). The third plate is released and collides with the plate β. Assume the collision is elastic and the time of collision is sufficient to redistribute charge amongst β and γ.



(a) Find the electric field acting on the plate γ before collision.

(b) Find the charges on β and γ after the collision.

(c) Find the velocity of the plate γ after the collision and at a distance *d* from the plate β.

[NCERT Exemp. Q. 1.28, Page 8]

Ans. **(a)** Net electric field at plate *y* before collision is equal to the sum of electric field at plate γ due to plate α and β

The electric field at plate γ due to plate α is $E_1 = \frac{-Q}{S(2\varepsilon_0)}$ to the left.

The electric field at plate γ due to plate β is $E_2 = \frac{q}{S(2\varepsilon_0)}$ to the right.

So, the net electric field at plate *y* before collision,

$$E = E_1 + E_2 = \frac{q - Q}{S(2\varepsilon_0)} \text{ to the left, if } Q > q. \quad \textbf{[1½]}$$

(b) During collision, plates β and γ are together. Their potentials become same.

Suppose charge on plate β is q_1, and charge on plate γ is q_2. At any point *O*. in between the two plates, the electric field must be zero.

Electric field at *O* due to plate $\alpha = \frac{-Q}{S(2\varepsilon_0)}$ to the left

Electric field at *O* due to plate $\beta = \frac{-q_1}{S(2\varepsilon_0)}$ to the right

Electric field at *O* due to plate $\gamma = \frac{-q_2}{S(2\varepsilon_0)}$ to the left

As the electric field at *O* is zero, therefore

$$\frac{Q+q_2}{S(2\varepsilon_0)} = \frac{q_1}{S(2\varepsilon_0)}$$

$$\therefore Q + q_2 = q_1$$

$$Q = q_1 - q_2 \quad \text{...(i)}$$

As there is no loss of charge on collision,

$$Q + q = q^1 + q^2 \quad \text{...(ii)}$$

On solving Equations (i) and (ii), we get

$$q_1 = \left(Q + \frac{q}{2}\right) = \text{ Charge on plate } \beta$$

$$q_2 = \left(\frac{q}{2}\right) = \text{ Charge on plate } \gamma$$

[1½]

(c) After collision, at a distance *d* from plate β.

Let the velocity of plate γ be *v*. After the collision, electric field at plate γ is

$$E_2 = \frac{-Q}{2\varepsilon_0 S} + \frac{(Q+\frac{q}{2})}{2\varepsilon_0 S} = \frac{\frac{q}{2}}{2\varepsilon_0 S} \text{ to the right}$$

Just before collision, electric field at plate γ is $E_1 = \frac{Q-q}{2\varepsilon_0 S}$

If *F*; is force on plate γ before collision, then $F_1 = E_1 Q = \frac{(Q-q)Q}{2\varepsilon_0 s}$

Similarly, Force F_2 on plate *r* after collision,

$$F_2 = E_2, \frac{q}{2} = \frac{(q/2)^2}{2\varepsilon_0 S}$$

Total work done by the electric field in round trip movement of plate γ

$$w = (F_1 + F_2)d$$

$$= \frac{[(Q-q)Q + (\frac{q}{2})^2]d}{2\varepsilon_0 S} = \frac{(Q-\frac{q}{2})^2 d}{2\varepsilon_0 S}$$

If *m* is mass of plate γ, the K.E. gained by plate $\gamma = \frac{1}{2}mv^2$

According to work-energy principle,

$$\frac{1}{2}mv^2 = W = \frac{\left(Q - \frac{q}{2}\right)^2 d}{2\varepsilon_0 S}$$

$$v^2 = \frac{2d\left(Q - \frac{q}{2}\right)^2}{2\varepsilon_0 Sm}$$

$$v = \left(Q - \frac{q}{2}\right)\left(\frac{d}{m\varepsilon_0 S}\right)^{\frac{1}{2}} \quad \textbf{[2]}$$

Q. 3. In 1959, Lyttleton and Bondi suggested that the expansion of the Universe could be explained if matter carried a net charge. Suppose that the Universe is made up of hydrogen atoms with a number density *N*, which is maintained a constant. Let the charge on the proton be :

$e_p = -(1 + y)e$ where '*e*' is the electronic charge.

(a) Find the critical value of *y* such that expansion may start.

(b) Show that the velocity of expansion is proportional to the distance from the centre.

[NCERT Exemp. Q. 1.26, Page 7]

Ans. **(a)** Let us assume that the universe is a perfect sphere of radius '*R*' and its constituent hydrogen atoms which is distributed uniformly in the sphere.

As we know that the hydrogen atom contains one proton and one electron, charge on each hydrogen atom.

$$e_H = e_P + e$$
$$= -(1+y)e + e$$
$$= -ye = |ye|$$

If E is electric field intensity at distance R, on the surface of the sphere, then according to Gauss' theorem,

$$\oint E.dS = \frac{q}{\varepsilon_0} \quad i.e., E(4\pi R^2) = \frac{4}{3}\frac{\pi R^3 N|ye|}{\varepsilon_0}$$

$$E = \frac{1}{3}\frac{N|ye|R}{\varepsilon_0} \quad \text{...(i)}$$

Now, suppose, mass of each hydrogen atom $\simeq m_p$ = Mass of a proton,

G_R = Gravitational field at distance R on the sphere. Then,

$$-4\pi R^2 G_R = 4\pi G m_P (\frac{4}{3}\pi R^3)N$$

$$\Rightarrow \quad G_R = \frac{-4}{3}\pi G m_P N R \quad \text{...(ii)}$$

Gravitational force on this atom is

$$F_G = m_P \times G_R = \frac{-4\pi}{3} G m_P{}^2 N R \quad \text{...(iii)}$$

Coulomb force on hydrogen atom at R is

$$F_C = (ye)E = \frac{1}{3}\frac{Ny^2e^2R}{\varepsilon_0} \quad \text{[From equation (i)]}$$

Now, to start expansion $F_C > F_G$ and critical value of γ to start expansion would be when

$$F_C = F_G$$

$$\Rightarrow \frac{1}{3}\frac{Ny^2e^2R}{\varepsilon_0} = \frac{4\pi}{3} G m_P^2 N R$$

$$\Rightarrow \quad y^2 = (4\pi\varepsilon_0)G\left(\frac{m_P}{e}\right)^2$$

$$= \frac{1}{9\times 10^9} \times (6.67\times 10^{-11})(\frac{(1.66\times 10^{-27})^2}{(1.6\times 10^{-19})^2}) = 79.8\times 10^{-38}$$

$$\Rightarrow y = \sqrt{79.8\times 10^{-38}} = 8.9\times 10^{-19} \approx 10^{-18}$$

Thus, 10^{-18} is the required critical value of y corresponding to which expansion of universe would start. **[2½]**

(b) Net force experience by the hydrogen atom is given by

$$F = F_C - F_G = \frac{1}{3}\frac{Ny^2e^2R}{\varepsilon_0} - \frac{4\pi}{3} G m_p^2 N R$$

If acceleration of hydrogen atom is representing by $\frac{d^2R}{dt^2}$, then

$$m_p \frac{d^2R}{dt^2} = F = \frac{1}{3}\frac{Ny^2e^2R}{\varepsilon_0} - \frac{4\pi}{3} G m_p^2 N R$$

$$= (\frac{1}{3}\frac{Ny^2e^2}{\varepsilon_0} - \frac{4\pi}{3} G m_p^2 N)R$$

$$\therefore \frac{d^2R}{dt^2} = \frac{1}{m_p}[\frac{1}{3}\frac{Ny^2e^2}{\varepsilon_0} - \frac{4\pi}{3} G m_p^2 N]R = \alpha^2 R \quad \text{...(iv)}$$

$$where,\ \alpha^2 = \frac{1}{m_p}[\frac{1}{3}\frac{Ny^2e^2}{\varepsilon_0} - \frac{4\pi}{3} G m_p^2 N]$$

The general solution of equation (iv) is given by $R = Ae^{\alpha t} + Be^{-\alpha t}$ We are looking for expansion, here, so, $B = 0$ and $R = Ae^{\alpha t}$.

Velocity of expansion, $v = \frac{dR}{dt} = Ae^{\alpha t}(\alpha) = \alpha Ae^{\alpha t} = \alpha R$

So, $v \propto R$, *i.e.*, velocity of expansion is proportional to the distance from the centre. **[2½]**

Q. 4. Consider a sphere of radius R with charge density distributed as

$\rho(r) = kr$ for $r \le R$

$= 0$ for $r > R$.

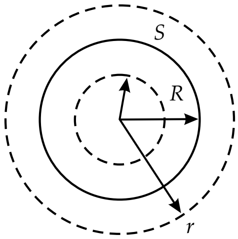
(a) Find the electric field at all points r.

(b) Suppose the total charge on the sphere is $2e$ where e is the electron charge. Where can two protons be embedded such that the force on each of them is zero. Assume that the introduction of the proton does not alter the negative charge distribution.

[NCERT Exemp. Q. 1.27, Page 8]

Ans. **(a)** Let us consider a sphere 'S' of radius 'R' and two hypothetic sphere of radius $r < R$ and $r > R$.

Now, for point $r < R$. electric field intensity will be given by.



$$\oint E.dS = \frac{1}{\varepsilon_0}\int \rho dV$$

$$\text{[For } dV,\ V = \frac{4}{3}\pi r^3 \Rightarrow dV = 3\times\frac{4}{3}\pi r^2 dr = 4\pi r^2 dr]$$

$$\Rightarrow \oint E.dS = \frac{1}{\varepsilon_0} 4\pi k \int_0^r r^3 dr \quad [\because \rho(r) = kr]$$

$$\Rightarrow (E)4\pi r^2 = \frac{4\pi k}{\varepsilon_0}\frac{r^4}{4}$$

$$\Rightarrow \quad E = \frac{1}{4\varepsilon_0} k r^2$$

Here, charge density is positive

So, direction of E is radially outwards.

For points $r > R$, electric field intensity will be given by,

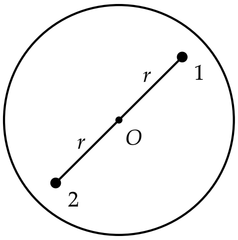
$$\oint E.dS = \frac{1}{\varepsilon_0}\int \rho dV$$

$$\Rightarrow E(4\pi r^2) = \frac{4\pi k}{\varepsilon_0}\int_0^R r^3 dr = \frac{4\pi k}{\varepsilon_0}\frac{R^4}{4}$$

$$\Rightarrow \qquad E = \frac{k}{4\varepsilon_0}\frac{R^4}{r^2}$$

Charge density is again positive. So, the direction of E is radially outward. **[2½]**

(b) The two protons must be on the opposite sides of the centre along a diameter following the rule of symmetry. This can be shown by the figure given below. Charge on the sphere,



$$q = \int_0^R \rho dV = \int_0^R (kr)4\pi r^2 dr$$

$$q = 4\pi k\frac{R^4}{4} = 2e$$

$$\therefore\ k = \frac{2e}{\pi R^4}$$

If protons 1 and 2 are embedded at distance r from the centre of the sphere as shown, then attractive force on proton 1 due to charge distribution is,

$$F_1 = -eE = \frac{-ekr^2}{4\varepsilon_0}$$

Repulsive force on proton 1 due to proton 2 is,

$$F_2 = \frac{e^2}{4\pi\varepsilon_0(2r)^2}$$

Net force on proton 1,

$$F = F_1 + F_2$$

$$F = \frac{-ekr^2}{4\varepsilon_0} + \frac{e^2}{16\pi\varepsilon_0 r^4}$$

So,

$$F = \left[\frac{-er^2}{4\varepsilon_0}\frac{2e}{\pi R^4} + \frac{e^2}{16\pi\varepsilon_0 r^4}\right]$$

Thus, net force on proton 1 will be zero, when

$$\frac{er^2 2e}{4\varepsilon_0\pi R^4} = \frac{e^2}{16\pi\varepsilon_0 r^2}$$

$$\Rightarrow \qquad r^4 = \frac{R^4}{8}$$

$$\Rightarrow \qquad r = \frac{R}{(8)^{\frac{1}{4}}}$$

This is the distance of each of the two protons from the centre of the sphere. **[2½]**