CHAPTER 9

RAY OPTICS AND OPTICAL INSTRUMENTS

Reflection of Light

- When light is incident on the interface of two homogeneous medium of different densities, then a portion of light comes back to the first medium. This phenomenon is known as reflection of light.

➢ **Types of reflection:**

- **Regular:** When parallel rays of light incident on a reflecting surface, remains parallel after reflection.
- **Diffused:** When parallel rays of light incident on a reflecting surface, does not remain parallel after reflection.

➢ **Laws of reflection:**

- **Law-I:** Incident ray, reflected ray and normal drawn at the point of incidence on the reflecting surface lie on the same plane.
- **Law-II:** Angle of incidence = angle of reflection.

➢ **Image:**

- **Real:** When rays of light after reflection or refraction meet at a point.
- **Virtual:** When rays of light after reflection or refraction without meeting at a point, appear to diverge from a point.

Reflection of Light by Spherical Mirrors

➢ **Spherical Mirror:** Any reflecting surface which is the part of a hollow sphere is called the spherical mirror.

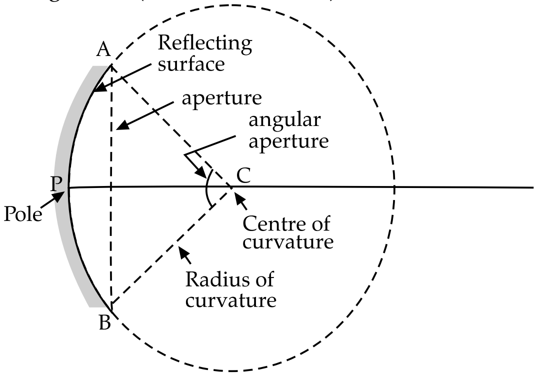
➢ **Types of spherical mirrors:** (i) Concave (ii) Convex.

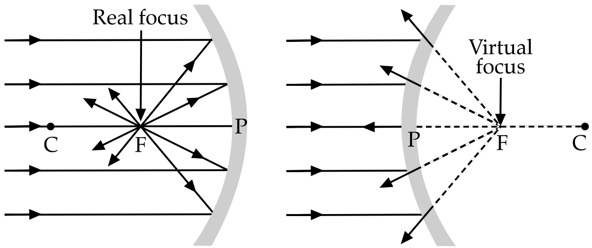
➢ **Concave Mirror:** A spherical mirror, whose reflecting surface is curved inwards, i.e., the reflecting surface faces the center of the sphere of which it is a part.

➢ **Convex Mirror:** A spherical mirror whose reflecting surface is curved outwards i.e., the reflecting surface is away from the center of the sphere of which it is a part.

➢ **Important parts of a spherical mirror:**

- **Pole:** Mid point of the spherical mirror.
- **Centre of curvature:** Centre of the sphere of which the mirror is a part.
- **Principal axis:** The imaginary line joining the pole and the centre of curvature.
- **Radius of curvature:** Radius of the sphere of which the mirror is a part.
- **Aperture:** Line joining the two end of the reflecting surface.
- **Angular aperture:** Angle made by the two ends of the reflecting surface at the centre of curvature.
- **Principal focus:** A point on the principal axis at which the rays parallel to principal axis, after reflection, meet (for concave mirror) or appear to diverge from (for convex mirror).
- **Focal length:** Distance of focus from the pole.
- **Secondary focus:** A point at which the parallel rays, making an angle with the principal axis, after reflection by the mirror, meet (for concave mirror) or appear to diverge from (for convex mirror).

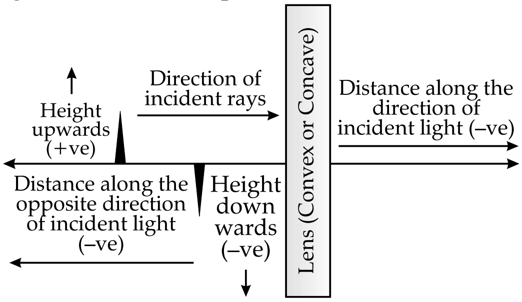




- Relation between focal length (f) and radius of curvature (R):

$$f = \frac{R}{2}$$

➢ **Sign conventions in spherical mirrors:**



➢ **Mirror formula:**

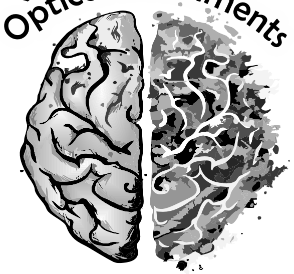
- Relation between u, v and f

$$\frac{1}{v}+\frac{1}{u} = \frac{1}{f}$$

- **Linear Magnification:**

$$m = \frac{\text{Height of image}}{\text{Height of object}} = -\frac{v}{u}$$

Ray Optics & Optical Instruments

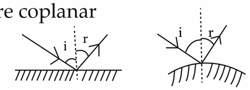


Sign Conventions
- Pole in case of mirror and optical centre in case of lenses are taken as origin
- Principle axis as the *X*-axis
- All distances are measured from origin (or pole)
- All distances measured in the direction of incident ray are taken + ve.
- All distances measured in the direction opposite to the incident ray are taken - ve.

Spherical Mirror
$\frac{1}{u}+\frac{1}{v}=\frac{2}{R}=\frac{1}{f}$

Lateral Magnification $=\frac{h_2}{h_1}=-\frac{v}{u}$

Reflection of Light
- $\angle i = \angle r$
- Incident ray, reflected ray and normal to the reflecting surface are coplanar

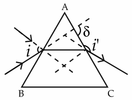


Prism
Dispersion
Splitting of polychromatic light into its constituent colours.

Angle of deviation $\delta = i + i' - A$

$\delta_{minimum} = 2i - A \quad [i = i']$

$\delta_{minimum} = (\mu - 1)A$, if A is small



Lens Marker's Formula
$\frac{1}{v}-\frac{1}{u}=\frac{\mu_2-\mu_1}{\mu_1}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

$\frac{1}{f}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ [If $\mu_2=\mu$, $\mu_1 = 1$ (air)]

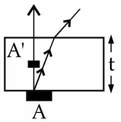
$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$ (lens formula)

Lateral Magnification $=\frac{h_2}{h_1}=\frac{v}{u}$

Refraction of light
$\frac{\sin i}{\sin r}=\frac{v_1}{v_2}=\frac{\mu_2}{\mu_1}$

$\mu=\frac{\text{real depth}}{\text{apparent depth}}$

$\Delta t=\left(1-\frac{1}{\mu}\right)t$ = image shift



Total Internal reflection
When ray passes from optically denser to rarer medium, if incident angle (i) greater than critical angle, entire light is then reflected back to the denser medium again, this process is called T.I.R. It is used in optical fibre.

Critical Angle
- Incident angle (θ_c) for which angle of refraction is 90°

i.e., $\sin\theta_c=\frac{1}{\mu}$

$\theta_c=\sin^{-1}\left(\frac{1}{\mu}\right)$

When ray passes from optically denser to rarer medium.

Refraction on Spherical surface
$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$

Lateral Magnification

$m=\frac{h_2}{h_1}=\frac{\mu_1 v}{\mu_2 u}=\frac{R-v}{R-u}$

Power of a lens
$P=\frac{1}{f}$

[For combination of lens when they are in contact]

$P=\frac{1}{f_1}+\frac{1}{f_2}$

[For combination of lens when they are separated]

$P=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2}$

Optical Instruments
Microscope
$M=1+\frac{D}{f}$ [image at near point]

$M=D/f$ [image at infinity]

Compound Microscope
$M=\frac{v}{u}\left[\frac{D}{f_e}\right]$ [normal adjustment]

$M=\frac{v}{u}\left(1+\frac{D}{f_e}\right)=-\frac{1}{f_o}\left(1+\frac{D}{f_e}\right)$

For final image at least distance

Telescope
$M=\frac{f_o}{f_e}\left(1+\frac{f_e}{D}\right)$ [image at near point]

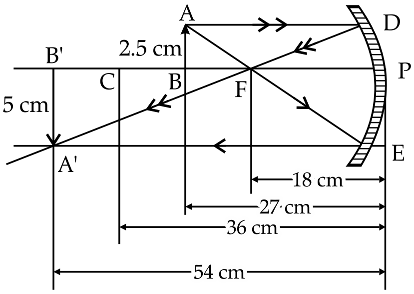
$M=-\frac{fo}{fe}$ [image at infinity]

Trace the Mind Map
- First Level
- Second Level
- Third Level

➢ **Image formation by spherical mirrors:**

Type of mirror	Object distance (u)	Image distance (v)	Nature of image	Size of image compared to object
Concave	$u = \infty$	$v = f$	Real and inverted	Point sized
	$\infty > u > 2f$	$R > v > f$		Reduced
	$u = 2f$	$v = R$		Equal
	$2f > u > f$	$\infty > v > R$		Magnified
	$u = f$	$v = \infty$		Highly magnified
	$f > u > 0$	Behind the mirror	Virtual and erect	Magnified
Convex	$u = \infty$	$v = f$	Virtual and erect	Point sized
	$\infty > u > 0$	$f > v > 0$ Behind the mirror		Smaller

Example 1: A candle of 2.5 cm in size is placed at a distance 27 cm in front of a concave mirror of radius of curvature 36 cm. At what distance from the mirror should a screen be placed in order to obtain a sharp image? Describe the nature and size of the image. Draw a ray diagram of the same.



Sol.

Object size, $h = 2.5$ cm

Image size, $h' = ?$

Object distance, $u = -27$ cm

Image distance, v

Radius of curvature, $R = -36$ cm

Focal length, $f = \frac{R}{2}$

$$= -\frac{36}{2} = -18 \text{ cm}$$

Using mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

Or, $$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

Or, $$\frac{1}{v} = -\frac{1}{18} + \frac{1}{27}$$

Or, $$\frac{1}{v} = -\frac{1}{54}$$

$\therefore$ $v = -54$ cm

The screen is to be placed at a distance 54 cm in front of the mirror.

Magnification, $$m = \frac{h'}{h} = -\frac{v}{u}$$

Or, $$h' = h \times -\frac{v}{u}$$

Or, $$h' = 2.5 \times -\frac{-54}{-27}$$

$\therefore$ $h' = -5$ cm

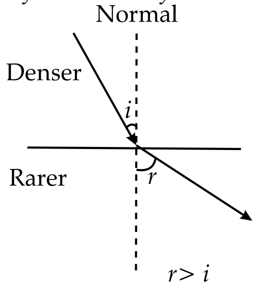
So, the image is real and inverted of height 5 cm.

Refraction of Light

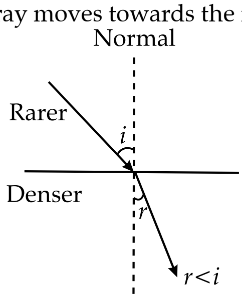
- Change of direction of a ray of light when it enters obliquely from one medium to another.

➢ **Change of direction of light ray when refracted:**

- **From denser to rarer medium:**
  The refracted ray moves away from the normal.



- **From rarer to denser medium:**
  The refracted ray moves towards the normal.



➢ **Laws of refraction:**

- **Law-I**
  The incident ray, the refracted ray and the normal drawn at the point of incidence on the surface of separation lie on the same plane.
- **Law-II (Snell's Law)**
  The ratio of sine of angle of incidence to the sine of angle of refraction is a constant, for the light of a given colour and for the given pair of media. This constant value is called the refractive index of the second medium with respect to the first medium.

$$\frac{\sin i}{\sin r} = {}_1\mu_2$$

$$\sin i = {}_1\mu_2 \sin r$$

If $\sin i = 0$, then $\sin r = 0$

Light rays do not deviate when they travel normally from one medium to another.

➢ **Absolute refractive index:** If the first medium is air or free space, then the refractive index is known as the absolute refractive index of the second medium. The absolute refractive index of a medium is expressed as

$$\mu_2 = \frac{\text{velocity of light in air or free space}}{\text{velocity of light in the medium}}$$

$$= \frac{c}{v}$$

since $c > v$,

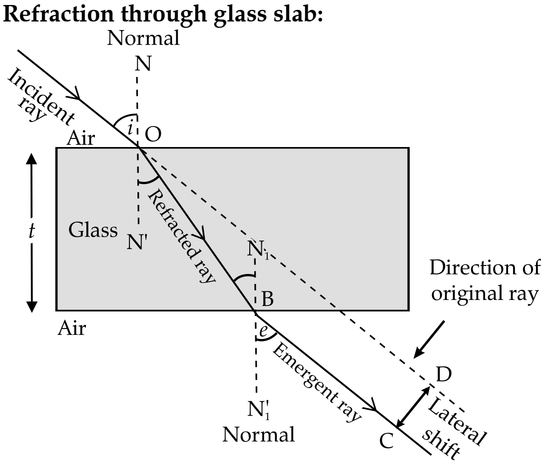
so, $\mu > 1$

➢ **Principle of Reversibility:**

- According to the principle of reversibility, the path of light is reversible even if it is going through several media.
  It means light follows exactly the same path when its direction is reversed.
- Applying this rule, we may find that if light travels through several media say medium 1 to medium 2, medium 2 to medium 3 and then medium 3 to medium 1, then

$${}_1\mu_2 \times {}_2\mu_3 \times {}_3\mu_1 = 1$$

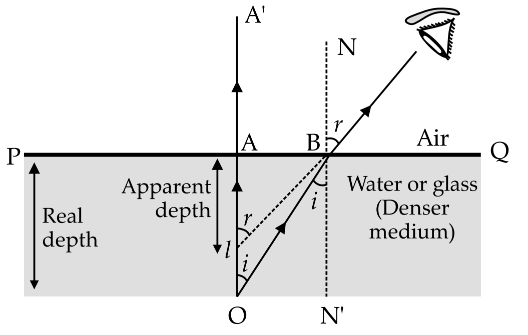
➢ **Refraction through glass slab:**



Emergent ray is parallel to the incident ray but it suffers lateral shift.

$$\text{Lateral shift} = t \sin i\left(1-\frac{\cos i}{\sqrt{\mu^2-\sin^2 i}}\right)$$

➢ **Apparent depth of an object in a denser medium when viewed from rarer medium:**



O is an object inside a denser medium. When it is viewed from a rarer medium,

Apparent rise of image = OI

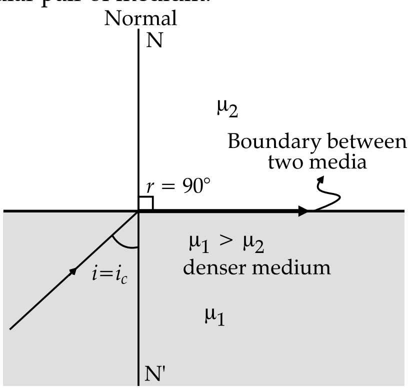
$$= \text{Real depth} \times \left(1-\frac{1}{\mu}\right)$$

➢ **Critical angle:** When light travels from denser to rarer medium,

$$\angle r > \angle i$$

As the angle of incidence increases, the angle of refraction also increases. For certain angle of incidence the angle of refraction becomes equal to 90°.

So, when light travels from denser to rarer medium, then the angle of incidence for which the angle of refraction is 90° is known as critical angle for that particular pair of medium.



$$\frac{\sin i}{\sin r} = \frac{1}{{}_2\propto_1}$$

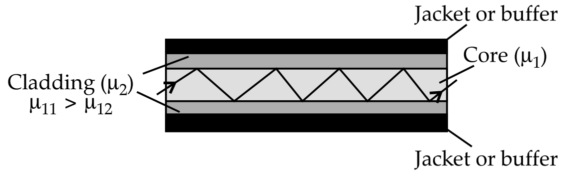
$$\therefore \quad \sin i_C = \frac{1}{{}_2\propto_1}$$

Total Internal Reflection

- If the angle of incidence in the denser medium is greater than the critical angle, for the pair of media, then the angle of refraction cannot be greater than 90° and so it reflects back to the denser medium. This phenomenon is known as total internal reflection.
- **Conditions for total internal reflection:**
  (a) Light should travel from denser to rarer medium.
  (b) Angle of incidence should be greater than critical angle for the pair of media.
- Why "total"?
  During normal reflection certain portion of incident light is refracted. But in case of total internal reflection no portion of incident light is refracted. Only certain portion of light is absorbed and rest portion is totally reflected.

➢ **Application of total internal reflection:**

Total internal reflection is used for communication through optical fibre. Optical fibre has 3 layers:



- **Core:** The innermost fibre made of transparent glass or plastic of high refractive index.
- **Cladding:** A layer surrounding the core. This layer has lower refractive index compared that of core.
- **Outer jacket or buffer:** A protective layer for protection.

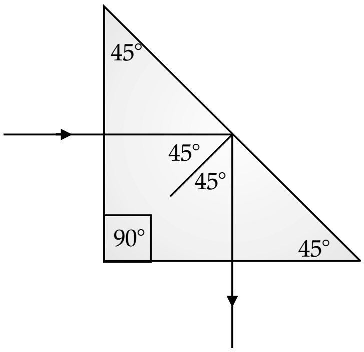
  The angle of incidence in the fibre is so adjusted that it is greater than the critical angle for the core cladding pair of media. So, the total internal reflections take place and the signal travels from one end to other.

➢ **Total internal reflection through prism:**

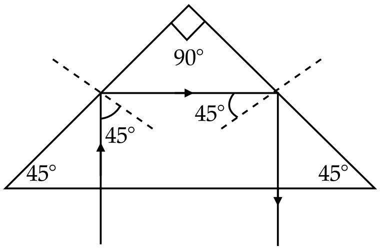
Prisms are designed for 90° or 180° deviation by making use of total internal reflection.

Critical angle for Glass-air media pair is 420 approximately. So by using a 90° isosceles prism, the incident may be deviated by 90° or 180°.

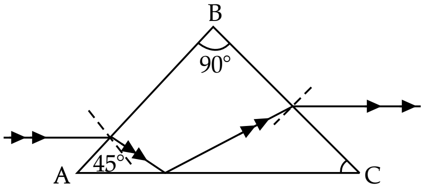
- **90° deviation:**



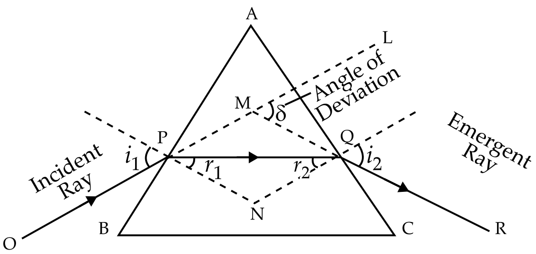
- **180° deviation:**



- **No angular deviation:**



➢ **Refraction through prism:**



Angle of prism $= A = r_1 + r_2$

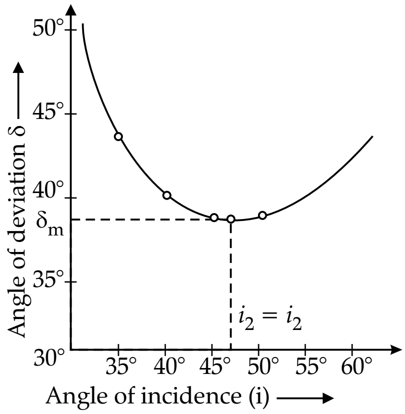
Angle of deviation $= \delta = i_1 + i_2 - A$

For minimum deviation,

$$i_1 = i_2 \text{ and } r_1 = r_2$$

$$\mu = \frac{\sin \frac{\delta_m + A}{2}}{\sin \frac{A}{2}}$$

➢ **Angle of deviation vs. angle of incidence graph for a prism:**



➢ **Dispersion of white light through Prism:** Splitting of white light into its constituent colours is known as dispersion of light. This is due to the different colours having different deviations.

- The seven colours are violet, indigo, blue, green, yellow, orange and red. The acronym of this colour band is VIBGYOR.
- Different colours of light have different wavelengths and different frequencies in medium. This is the cause of dispersion.

➢ **Angular dispersion:** Difference of deviations of two different colours after passing through a prism.

$$\delta_V - \delta_R = A(\mu_V - \mu_R)$$

➢ **Dispersive power:** Ratio of difference of deviations of red and violet colours and the deviation due to yellow light (mean colour) after passing through a prism.

$$\omega = \frac{\delta_V - \delta_R}{\delta}$$

$$= \frac{\mu_V - \mu_R}{\mu - 1}$$

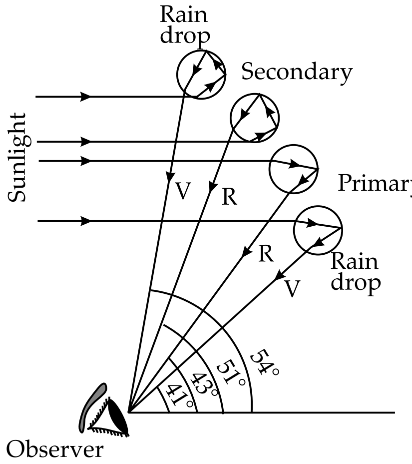
➢ **Formation of rainbow:** Rainbow is the natural phenomenon of dispersion of light. After rain when sky is clear and sunny, we may observe a rainbow in a direction opposite to the direction of sun. It is caused due to the combined effect of refraction, total internal reflection and dispersion of sunlight by the raindrops suspended in the air.

- **Primary rainbow:** There is only one total internal reflection before different colours reach observer's eye. In this rainbow, observer watches red colour at top and violet at bottom.

  Primary rainbow is seen at an angle 41° – 43° on the sky.
- **Secondary rainbow:** In it there are two total internal reflections before different colours reach observer's

eye. In this rainbow, observer watches violet colour at top and red at bottom.

Secondary rainbow is seen at a higher (51° – 54°) angle on the sky. Its Intensity is lower than the primary rainbow.



➢ **Refraction at spherical surface:**

- When two media have a spherical interface and ray of light is incident from medium of refractive index μ_1, to medium of refractive index μ_2, then

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \qquad (\mu_2 > \mu_1)$$

R= Radius of curvature
u= Object distance
v= image distance

- When two media have a spherical interface and ray of light is incident from medium of refractive index μ_2, to medium of refractive index μ_1, then

$$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R} \qquad (\mu_2 > \mu_1)$$

R= Radius of curvature
u= Object distance
v= image distance

Lens

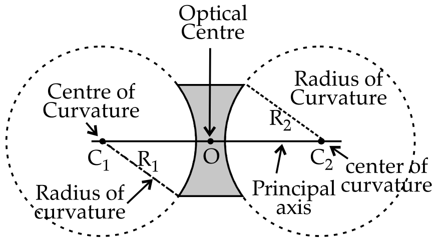
- A piece of transparent medium which is bounded by two surfaces out of which at least one surface is spherical.

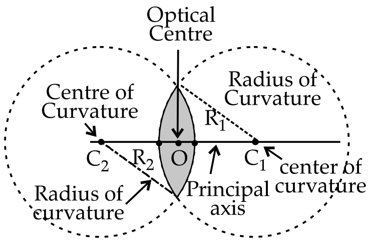
➢ **Types of lens:**

- **Convex lens:** A convex lens is one which is thinner at the edges and thicker at centre.
- **Concave lens:** A concave lens is one which is thicker at edges and thinner at centre.

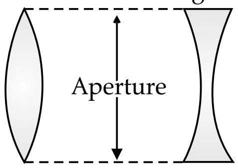
➢ **Important parts of a lens:**

- **Centre of curvature:** Centres of the spheres of which it is a part.
- **Radius of curvature:** Radius of the spheres of which it is a part.
- **Principal axis:** The imaginary line joining the two centres of curvature.
- **Optical centre:** It is a point on the principal such that when light ray passes through the point, the incident ray and the refracted ray becomes parallel.
(For thin lens, it is a point on the principal such that light ray passes through it undeviated)

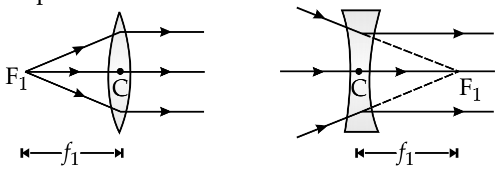




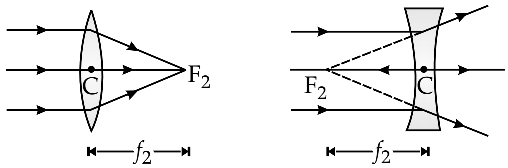
- **Aperture:** Effective diameter of light transmitting area.



- **First principal focus:** It is the point on the principal axis from which diverging rays of light (for convex lens) or rays of light approaching the point (for concave lens), after refraction by the lens, become parallel to the principal axis.



- **Second principal focus:** It is the point on the principal axis where the rays of light parallel to the principal axis, after refraction by the lens, converge (for convex lens) or appear to diverge from (for concave lens).



- **Focal length:** The distance between the optical centre and focus.
- **Secondary focus:** A point at which the parallel rays, making an angle with the principal axis, after refraction by the lens, meet (for convex lens) or appear to diverge from (for concave lens).

➢ **Relation between radii of curvature, focal length and refractive index (Lens Maker's formula):**

$$\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Or,
$$\frac{1}{f} = ({}_1\mu_2 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

where, f = focal length

$_1\mu_2$ = Refractive index

R_1 and R_2 = radii of curvature of two spherical surfaces.

➢ **Lens formula:**

- $$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

where, u = object distance

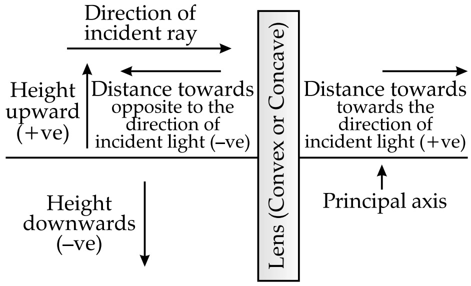
v = image distance

f = focal length

- **Linear magnification by lens:**

$$m = \frac{\text{Image height}}{\text{object height}} = \frac{v}{u}$$

➢ **Sign convention in lens:**



➢ **Image formation by lens for different object positions:**

Type of mirror	Object distance (u)	Image distance (v)	Nature of image	Size of image compared to object
Convex	$u = \infty$	$v = f$	Real and inverted	Point sized
	$\infty > u > 2f$	$2f > v > f$		Reduced
	$u = 2f$	$v = 2f$		Equal
	$2f > u > f$	$\infty > v > 2f$		Magnified
	$u = f$	$v = \infty$		Highly magnified
	$f > u > 0$	Same side of the lens	Virtual and erect	Magnified
Concave	$u = \infty$	$v = f$	Virtual and erect	Point sized
	$\infty > u > 0$	$f > v > 0$ Same side of the lens		Smaller

Power of Lens

- Ability of a lens to diverge (for concave lens) or converge (for convex lens) parallel rays of light.
It is the reciprocal of the focal length expressed in metre.
SI unit of power is dioptre (D).

$$P = \frac{1}{f}$$

➢ **Combination of lens:** When lens are in contact:

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} + ...$$

$$P_{eq} = P_1 + P_2 + ...$$

Example 2: An object is placed at a distance 25 cm from a lens. Image is formed at a distance 100 cm on other side of the lens. If the object is moved 10 cm towards the lens what will be the shift of the image?

Sol. In 1st case,

$$u = -25 \text{ cm}$$
$$v = 100 \text{ cm}$$

Applying lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Or, $$\frac{1}{100} + \frac{1}{25} = \frac{1}{f}$$

Or, $$\frac{1}{20} = \frac{1}{f}$$

$\therefore$ $$f = 20 \text{ cm}$$

So, the lens is convex lens.

In 2nd case,

$$u = -15 \text{ cm}$$
$$f = 15 \text{ cm}$$

Applying lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Or, $$\frac{1}{v} + \frac{1}{15} = \frac{1}{20}$$

Or, $$\frac{1}{v} = -\frac{1}{60}$$

$\therefore$ $$v = -60 \text{ cm}$$

So, the image is formed on the object side of the lens.

So, the shift of image is 100 + 60 = 160 cm

Example 3: Refractive index of the material of a equiangular prism is 1.5. A ray of light so passes through the prism so that the angle of incidence = angle of emergence = 3/4th of the angle of deviation. Find the angle of deviation.

Sol. Given, $$i_1 = e = \frac{3}{4}\delta$$

$$A = 60°$$ (since, the prism is equiangular)

$$\delta = i_1 + e - A$$

or, $$\delta = \frac{3}{4}\delta + \frac{3}{4}\delta - 60°$$

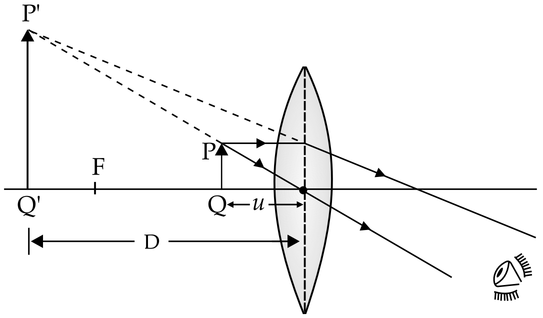
or, $\frac{\delta}{2} = 60°$

$\therefore$ $\delta = 120°$

Optical Instruments

- **Microscope** is an optical instrument which helps us to see and study micro objects or organisms. It forms magnified image of the object.
- **Telescope** is an optical instrument which helps us to see and study far off objects magnified & resolved.
- Microscopes and telescopes may generally be set at two different image vision positions:
- **Image at least distance of distinct vision:** When the image is formed at a distance of 25 cm from our eye.
- **Image at relaxed vision:** When the image is formed at infinite distance from our eye.
- Magnification at distinct vision is always greater than magnification at relaxed vision.
- **Simple Microscope:** Convex lens behaves as simple microscope.
- **The magnifying power of the simple microscope:**

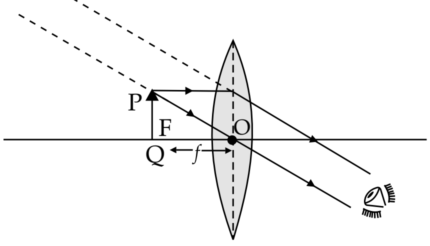
(i) Magnification at least distance of distinct vision:



$$m = \frac{1 + D}{f}$$

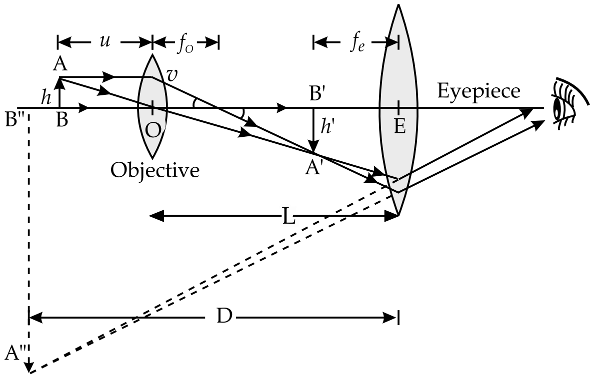
where, D is the least distance of distinct vision of the eye and f is focal length of the lens.

(ii) Magnification for relaxed eye:



$$m = \frac{D}{f}$$

- **Compound Microscope:** For much larger magnification, compound microscope is used. It is a combination of two convex lenses when the magnification of each lens is compounded.
- The two lenses are placed co-axially and the distance between them is adjustable.
- The lens towards the object is called objective and towards the eye is called eyepiece.
- The final image formed by the compound microscope is magnified and inverted.
- **Compound Microscope (image at least distance of distinct vision):**



Magnification of objective, $m_o = \frac{v}{u} = \frac{v}{f_o} - 1$

Magnification of eyepiece, $m_e = \frac{1 + D}{f_e}$

Total magnification, $m = m_0 \times m_e$

$$= \left(\frac{v}{f_o} - 1\right)\left(\frac{1 + D}{f_e}\right)$$

Since $v \approx L$

$$m = \left(\frac{L}{f_o} - 1\right)\left(\frac{1 + D}{f_e}\right)$$

since $L >> f_o$

$$m = \left(\frac{L}{f_o} - 1\right)\left(\frac{1 + D}{f_e}\right)$$

where,

f_o = Focal length of objective

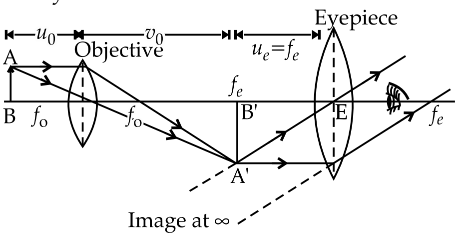
f_e = focal length of eyepiece

L = Length of tube

= Distance between objective and eyepiece.

- **Compound Microscope (image at infinity)**

This is also called normal adjustment or adjustment for relaxed eye.



Here,

$$m_e = \frac{D}{f_e}$$

So, magnification, $m = \left(\frac{L}{f_o}\right)\left(\frac{D}{f_e}\right)$

➢ **Characteristics of compound microscope:**

- If the length of tube increases, magnification increases.

$$m \propto L$$

- If focal length of the objective decreases, magnification increases.

$$m \propto \frac{1}{f_o}$$

- If focal length of the eyepiece decreases, magnification increases.

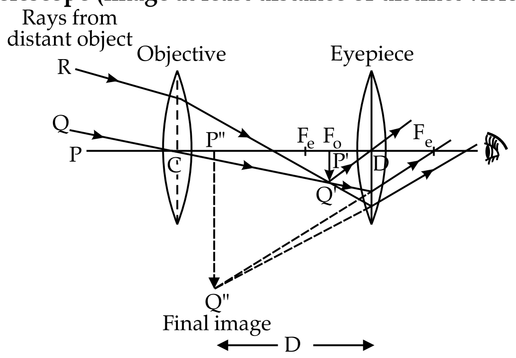
$$m \propto \frac{1}{f_e}$$

(Normally $f_o > f_e$)

Telescope

- Telescope is an instrument to magnify and resolve far off objects.
- Far off objects make much smaller angle at our eye. Telescope makes that angle larger without much intensity loss.
- To maximise the intensity, aperture size of objective lens is quite large. It focuses a bright point size image at its focal plane.
- Now with eyepiece, this point size image is finally observed as final inverted magnified image. This type of telescope is known as astronomical telescope.

➢ **Telescope (image at least distance of distinct vision):**



Magnification, $m = \left(\frac{f_o}{f_e}\right)\left(1+\frac{f_e}{D}\right)$

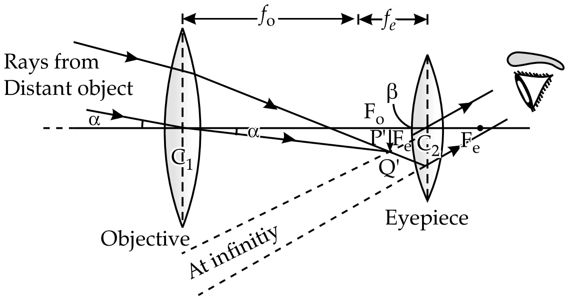
➢ **Telescope (image at infinity)**

This is also called normal adjustment or adjustment for relaxed eye.

Magnification, $m = \frac{\beta}{\alpha} = \frac{f_o}{f_e} = \frac{d_o}{d_e}$

where, d_o = Diameter of the objective

and d_e = Diameter of the eyepiece.



➢ **Characteristics of astronomical telescope:**

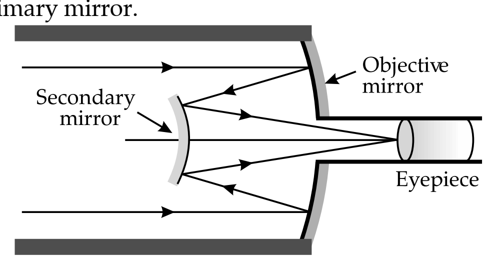
- For larger magnifying power, f_o should be large and f_e should be small.
- The length of the tube of an astronomical telescope is $L = f_o + f_e$ for relaxed vision adjustment.
- When the length of the tube of the telescope increases, f_o increases and magnifying power also increases.

➢ **Limitations of refractive telescope:**

- Manufacturing of large aperture objective lens is costly and its handling is also difficult.
- It has spherical and chromatic aberrations.

➢ **Modern Telescope (Reflecting Telescope):**

- Reflecting telescope consists of a concave mirror of large radius of curvature in place of objective lens.
- A secondary convex mirror is used to focus the incident light, which now passes through a hole in the objective primary mirror.



➢ **Magnifying power of the reflecting telescope:**

$$m = \frac{f_o}{f_e}$$

➢ **Advantages of reflecting telescope:**

- Very sharp image
- No spherical or chromatic aberrations.
- Cost effective
- Light and easy to handle.

➢ **Disadvantages of reflecting telescope:**

- The secondary mirror obstructs the incoming rays.

NCERT CORNER

Exercise Questions

Q. 1. A small candle, 2.5 cm in size is placed at 27 cm in front of a concave mirror of radius of curvature 36 cm. At what distance from the mirror should a screen be placed in order to obtain a sharp image? Describe the nature and size of the image. If the candle is moved closer to the mirror, how would the screen have to be moved?

[NCERT Ex. Q. 9.1, Page 249]

Ans. Given that,

Size of the candle, $h = 2.5$ cm

Image size = h'

Object distance, $u = -27$ cm

Radius of curvature of the concave mirror, $R = -36$ cm

As we know that the focal length of the concave mirror,

$$f = \frac{R}{2}$$

$$= -18 \text{ cm}$$

Let the image distance = v

The image distance can be obtained using the mirror formula :

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$= \frac{1}{-18} - \frac{1}{-27}$$

$$= \frac{-3+2}{54}$$

$$= -\frac{1}{54}$$

Hence, $v = -54$ cm

So, the screen should be placed 54 cm away from the mirror to obtain a sharp image.

The magnification of the image is:

$$m = \frac{h'}{h} = -\frac{v}{u}$$

Therefore, $h' = -\frac{v}{u} \times h$

$$= -\left(\frac{-54}{-27}\right) \times 2.5 = -5 \text{ cm}$$

The height of the candle's image is 5 cm. The negative sign indicates that the image is inverted and real.

If the candle is moved closer to the mirror, then the screen will have to be moved away from the mirror in order to obtain the image.

Q. 2. A 4.5 cm needle is placed 12 cm away from a convex mirror of focal length 15 cm. Give the location of the image and the magnification. Describe what happens as the needle is moved farther from the mirror.

[NCERT Ex. Q. 9.2, Page 249]

Ans. Given that, Height of the needle, $h_1 = 4.5$ cm

Object distance, $u = -12$ cm

Focal length of the convex mirror, $f = 15$ cm

Let the image distance = v

As we know the mirror formula,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$= \frac{1}{15} + \frac{1}{12} = \frac{4+5}{60} = \frac{9}{60}$$

Therefore, $v = \frac{60}{9} = 6.7$ cm

So, the image of the needle is 6.7 cm away from the mirror. Also, it is on the other side of the mirror.

The image size is given by the magnification formula:

$$m = \frac{h_2}{h_1} = -\frac{v}{u}$$

$$\therefore \quad h_2 = -\frac{v}{u} \times h_1 = \frac{-6.7}{-12} \times 4.5 = +2.5 \text{ cm}$$

Hence, magnification of the image,

$$m = \frac{h_2}{h_1} = \frac{2.5}{4.5} = 0.56$$

The height of the image is 2.5 cm. The positive sign indicates that the image is erect and virtual.

If the needle is moved farther from the mirror, the image will also move away from the mirror, and the size of the image will reduce gradually.

Q. 3. A tank is filled with water to a height of 12.5 cm. The apparent depth of a needle lying at the bottom of the tank is measured by a microscope to be 9.4 cm. What is the refractive index of water? If water is replaced by a liquid of refractive index 1.63 up to the same height, by what distance would the microscope have to be moved to focus on the needle again? [NCERT Ex. Q. 9.3, Page 249]

Ans. Given that,

Actual depth of the needle in water, $h_1 = 12.5$ cm

Apparent depth of the needle in water, $h_2 = 9.4$ cm

Refractive index of water

$$\mu = \frac{h_1}{h_2}$$

$$= \frac{12.5}{9.4} \approx 1.33$$

So, the refractive index of water is about 1.33.

Water is replaced by a liquid of refractive index, $\mu' = 1.63$

As we know that the actual depth of the needle remains the same, but its apparent depth changes. So, let y be the new apparent depth of the needle.

So, we can write the relation:

$$\mu' = \frac{h_1}{y}$$

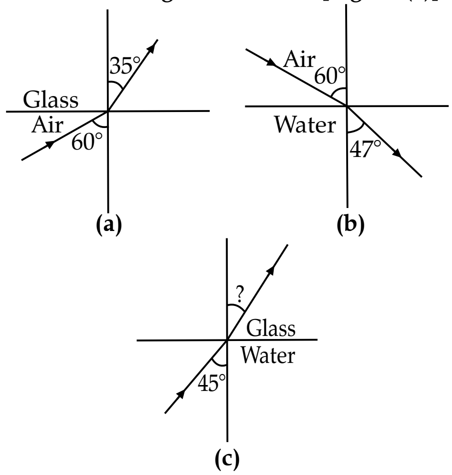
Therefore, $y = \frac{h_1}{\mu'}$

$$= \frac{12.5}{1.63} = 7.67 \text{ cm}$$

Thus, the new apparent depth of the needle is 7.67 cm. It is less than h_2. Therefore, to focus the needle again, the microscope should be moved up.

So, distance by which the microscope should be moved up = 9.4 − 7.67 = 1.73 cm.

Q. 4. Figures (a) and (b) show refraction of a ray in air incident at 60° with the normal to a glass-air and water-air interface, respectively. Predict the angle of refraction in glass when the angle of incidence in water is 45° with the normal to a water-glass interface [Figure (c)].



[NCERT Ex. Q. 9.4, Page 249]

Ans. Given that,

For the glass-air interface:

Angle of incidence, $i = 60°$

Angle of refraction, $r = 35°$

The relative refractive index of glass with respect to air,

$$^{a}\mu_{g} = \frac{\sin i}{\sin r} = \frac{\sin 60°}{\sin 35°} = \frac{0.8660}{0.5736} = 1.51 \quad \text{...(i)}$$

Given that,

For the air-water interface:

Angle of incidence, $i = 60°$

Angle of refraction, $r = 47°$

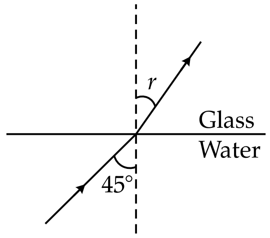
The relative refractive index of water with respect to air,

$$^{a}\mu_{w} = \frac{\sin i}{\sin r} = \frac{\sin 60°}{\sin 47°} = \frac{0.8660}{0.7314} = 1.184 \quad \text{...(ii)}$$

Using (i) and (ii), the relative refractive index of glass with respect to water,

$$\therefore \quad ^{w}\mu_{g} = \frac{^{a}\mu_{g}}{^{a}\mu_{w}} = \frac{1.51}{1.184} = 1.275$$

The following figure shows the situation involving the glass-water interface.



Angle of incidence, $i = 45°$

Angle of refraction $= r$

From Snell's law,

$$\frac{\sin i}{\sin r} = {^{w}\mu_{g}}$$

$$\frac{\sin 45°}{\sin r} = 1.275$$

$$\sin r = \frac{\frac{1}{\sqrt{2}}}{1.275} = 0.5546$$

$$\therefore \quad r = \sin^{-1}(0.5546) = 38.68°$$

Hence, the angle of refraction at the water-glass interface is 38.8°.

Q. 5. A small bulb is placed at the bottom of a tank containing water to a depth of 80 cm. What is the area of the surface of water through which light from the bulb can emerge out? Refractive index of water is 1.33. (Consider the bulb to be a point source.)

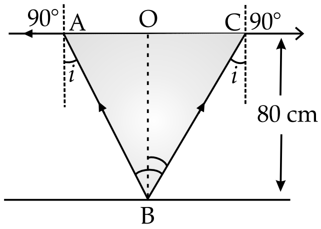
[NCERT Ex. Q. 9.5, Page 249]

Ans. Given that,

Actual depth of the bulb in water, $d = 80$ cm $= 0.8$ m

Refractive index of water, $\mu = 1.33$

The given situation is shown in the below figure:



where,

i = Angle of incidence

r = Angle of refraction = 90°

Since the bulb is a point source, the emergent light can be considered as a circle of radius,

$$R = \frac{AC}{2} = AO = OB$$

$$\mu = \frac{\sin r}{\sin i}$$

$$1.33 = \frac{\sin 90°}{\sin i}$$

$$\therefore \quad i = \sin^{-1}\left(\frac{1}{1.33}\right) = 48.75°$$

Using the given figure, we have the relation:

$$\tan i = \frac{OC}{OB} = \frac{R}{d_1}$$

$$\therefore \quad R = \tan 48.75° \times 0.8 = 0.91 \text{ m}$$

$\therefore$ Area of the surface of water $= \pi R^2$

$= \pi (0.91)^2$

$= 2.61$ m^2

Thus, the area of the surface of water through which the light from the bulb can emerge is approximately 2.61 m^2.

Q. 6. A prism is made of glass of unknown refractive index. A parallel beam of light is incident on a face of the prism. The angle of minimum deviation is measured to be 40°. What is the refractive index of the material of the prism? The refracting angle of the prism is 60°. If the prism is placed in water (refractive index 1.33), predict the new angle of minimum deviation of a parallel beam of light. **[NCERT Ex. Q. 9.6, Page 249]**

Ans. Given that,

Angle of minimum deviation, $\delta_m = 40°$

Angle of the prism, $A = 60°$

Refractive index of water, $\mu = 1.33$

Refractive index of the material of the prism $= \mu'$

The angle of deviation is related to refractive index as :

$$\mu' = \frac{\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}} = \frac{\sin\left(\frac{60° + 40°}{2}\right)}{\sin\frac{60°}{2}} = \frac{\sin 50°}{\sin 30°} = 1.532$$

So, the refractive index of the material of the prism is 1.532.

Since the prism is placed in water, let δ'_m be the new angle of minimum deviation for the same prism.

The refractive index of glass with respect to water is given by the relation:

$$^{W}\mu_{g} = \frac{\mu'}{\mu} = \frac{\sin\left(\frac{A+\delta'_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\sin\left(\frac{A+\delta'_m}{2}\right) = \frac{\mu'}{\mu}\sin\left(\frac{A}{2}\right)$$

$$\sin\left(\frac{A+\delta'_m}{2}\right) = \frac{1.532}{1.33} \times \sin\left(\frac{60°}{2}\right) = 0.5759$$

$$\left(\frac{A+\delta'_m}{2}\right) = \sin^{-1}\ 0.5759 = 35.16°$$

$$60° + \delta'_m = 70.32°$$

$$\therefore \quad \delta'_m = 70.32° - 60° = 10.32°$$

So, the new minimum angle of deviation is 10.32°.

Q. 7. Double-convex lenses are to be manufactured from a glass of refractive index 1.55, with both faces of the same radius of curvature. What is the radius of curvature required if the focal length is to be 20 cm?

[NCERT Ex. Q. 9.7, Page 346]

Ans. Given that,

Refractive index of glass, $\mu = 1.55$

Focal length of the double-convex lens, $f = 20$ cm

Radius of curvature of one face of the lens = R_1

Radius of curvature of the other face of the lens = R_2

Radius of curvature of the double-convex lens = R

Therefore, $R_1 = R$ and $R_2 = -R$

The value of R can be calculated as:

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\frac{1}{20} = (1.55 - 1)\left[\frac{1}{R} + \frac{1}{R}\right]$$

$$\frac{1}{20} = 0.55 \times \frac{2}{R}$$

$$\therefore \quad R = 0.55 \times 2 \times 20 = 22 \text{ cm}$$

Hence, the radius of curvature of the double-convex lens is 22 cm.

Q. 8. A beam of light converges at a point P. Now a lens is placed in the path of the convergent beam 12 cm from P. At what point does the beam converge if the lens is (a) a convex lens of focal length 20 cm, and (b) a concave lens of focal length 16 cm? [NCERT Ex. Q. 9.8, Page 249]

Ans. Given that,

According to the given situation, the object is virtual and the image formed is real.

Object distance, $u = +12$ cm

(a) Focal length of the convex lens, $f = 20$ cm

Image distance = v

According to the lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{12} = \frac{1}{20}$$

$$\frac{1}{v} = \frac{1}{20} + \frac{1}{12} = \frac{3+5}{60} = \frac{8}{60}$$

So, $$v = \frac{60}{8} = 7.5 \text{ cm}$$

Hence, the image is formed 7.5 cm away from the lens, towards its right.

(b) Focal length of the concave lens, $f = -16$ cm

Image distance = v

According to the lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} = -\frac{1}{16} + \frac{1}{12}$$

$$\frac{1}{v} = \frac{-3+4}{48} = \frac{1}{48}$$

So, $$v = 48 \text{ cm}$$

So, the image is formed 48 cm away from the lens, towards its right.

Q. 9. An object of size 3.0 cm is placed 14 cm in front of a concave lens of focal length 21 cm. Describe the image produced by the lens. What happens if the object is moved further away from the lens?

[NCERT Ex. Q. 9.9, Page 249]

Ans. Given that,

Size of the object, $h_1 = 3$ cm

Object distance, $u = -14$ cm

Focal length of the concave lens, $f = -21$ cm

Let the Image distance = v

According to the lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} = -\frac{1}{21} - \frac{1}{14}$$

$$= \frac{-2-3}{42} = \frac{-5}{42}$$

$$\therefore \quad v = -\frac{42}{5} = -8.4 \text{ cm}$$

So, the image is formed on the other side of the lens, 8.4 cm away from it. The negative sign shows that the image is erect and virtual.

The magnification of the image is:

$$m = \frac{\text{Image height } (h_2)}{\text{Object height } (h_1)} = \frac{v}{u}$$

$$\therefore \quad h_2 = \frac{-8.4}{-14} \times 3 = 0.6 \times 3 = 1.8 \text{ cm}$$

Thus, the height of the image is 1.8 cm.

If the object is moved further away from the lens, then the virtual image will move towards the focus of the lens, but not beyond it. So, the size of the image will decrease with the increase in the object distance.

Q. 10. What is the focal length of a convex lens of focal length 30 cm in contact with a concave lens of focal length 20 cm? Is the system a converging or a diverging lens? Ignore thickness of the lenses.

[NCERT Ex. Q. 9.10, Page 249]

Ans. Given that,

Focal length of the convex lens, $f_1 = 30$ cm

Focal length of the concave lens, $f_2 = -20$ cm

Focal length of the system of lenses = f

The equivalent focal length of a system of two lenses in contact is:

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{f} = \frac{1}{30} - \frac{1}{20}$$

$$= \frac{2-3}{60} = -\frac{1}{60}$$

$$\therefore \quad f = -60 \text{ cm}$$

Thus, the focal length of the combination of lenses is 60 cm. The negative sign indicates that the system of lenses acts as a diverging lens.

Q. 11. A compound microscope consists of an objective lens of focal length 2.0 cm and an eyepiece of focal length 6.25 cm separated by a distance of 15 cm. How far from the objective should an object be placed in order to obtain the final image at (a) the least distance of distinct vision (25 cm), and (b) at infinity? What is the magnifying power of the microscope in each case?

[NCERT Ex. Q. 9.11, Page 250]

Ans. Given that,

Focal length of the objective lens, $f_1 = 2.0$ cm

Focal length of the eyepiece, $f_2 = 6.25$ cm

Distance between the objective lens and the eyepiece, $d = 15$ cm

(a) Least distance of distinct vision, $d' = 25$ cm

Image distance for the eyepiece, $v_2 = -25$ cm

Object distance for the eyepiece $= u_2$

According to the lens formula,

$$\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2}$$

$$\frac{1}{u_2} = \frac{1}{v_2} - \frac{1}{f_2}$$

$$= \frac{1}{-25} - \frac{1}{6.25}$$

$$= \frac{-1-4}{25} = \frac{-5}{25}$$

$$\therefore \quad u_2 = -5 \text{ cm}$$

Image distance for the objective lens,

$$v_1 = d + u_2 = 15 - 5 = 10 \text{ cm}$$

Object distance for the objective lens $= u_1$

According to the lens formula,

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$$

$$\frac{1}{u_1} = \frac{1}{v_1} - \frac{1}{f_1} = \frac{1}{10} - \frac{1}{2}$$

$$= \frac{1-5}{10} = \frac{-4}{10}$$

$$\therefore \quad u_1 = -2.5 \text{ cm}$$

Magnitude of the object distance, $|u_1| = 2.5$ cm

The magnifying power of a compound microscope is:

$$m = \frac{v_1}{|u_1|}\left(1 + \frac{d'}{f_2}\right)$$

$$= \frac{10}{2.5}\left(1 + \frac{25}{6.25}\right)$$

$$= 4(1+4) = 20$$

(b) The final image is formed at infinity.

Therefore, image distance for the eyepiece, $v_2 = \infty$

Object distance for the eyepiece $= u_2$

According to the lens formula,

$$\frac{1}{v_2} - \frac{1}{v_2} = \frac{1}{f_2}$$

$$\frac{1}{\infty} - \frac{1}{u_2} = \frac{1}{6.25}$$

$$u_2 = -6.25 \text{ cm}$$

Image distance for the objective lens, $v_1 = d + u_2$

$$= 15 - 6.25 = 8.75 \text{ cm}$$

Object distance for the objective lens $= u_1$

According to the lens formula,

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$$

$$\frac{1}{u_1} = \frac{1}{v_1} - \frac{1}{f_1} = \frac{1}{8.75} - \frac{1}{2.0} = \frac{2-8.75}{17.5}$$

$$\therefore \quad u_1 = -\frac{17.5}{6.75} = -2.59 \text{ cm}$$

Magnitude of the object distance, $|u_1| = 2.59$ cm

The magnifying power of a compound microscope,

$$m = \frac{v_1}{|u_1|}\left(\frac{d'}{|u_2|}\right) = \frac{8.75}{2.59} \times \frac{25}{6.25} = 13.51$$

Thus, the magnifying power of the microscope is 13.51.

Q. 12. A person with a normal near point (25 cm) using a compound microscope with objective of focal length 8.0 mm and an eyepiece of focal length 2.5 cm can bring an object placed at 9.0 mm from the objective in sharp focus. What is the separation between the two lenses? Calculate the magnifying power of the microscope. **[NCERT Ex. Q. 9.12, Page 250]**

Ans. Given that,

Focal length of the objective lens, $f_o = 0.8$ cm

Focal length of the eyepiece, $f_e = 2.5$ cm

Object distance for the objective lens,

$$u_o = -9.0 \text{ mm}$$
$$= -0.9 \text{ cm}$$

Least distance of distant vision, $d = 25$ cm

Image distance for the eyepiece, $v_e = -d = -25$ cm

Object distance for the eyepiece $= u_e$

Using the lens formula,

$$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$$

$$\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e}$$

$$= \frac{1}{-25} - \frac{1}{2.5}$$

$$= \frac{-1-10}{25}$$

$$= \frac{-11}{25}$$

$$\therefore \quad u_e = -\frac{25}{11} = -2.27 \text{ cm}$$

We can also obtain the value of the image distance for the objective lens, v_o using the lens formula:

$$\frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{f_o}$$

$$\frac{1}{v_o} = \frac{1}{f_o} - \frac{1}{u_o}$$

$$= \frac{1}{0.8} - \frac{1}{0.9}$$

$$= \frac{0.9-0.8}{0.72} = \frac{0.1}{0.72}$$

$$\therefore \quad v_o = 7.2 \text{ cm}$$

The distance between the objective lens and the eyepiece $= |u_e| + v_o$

$= 2.27 + 7.2$

$= 9.47$ cm

The magnifying power of microscope,

$$m = \frac{v_o}{|u_o|}\left(1 + \frac{d}{f_e}\right)$$

$$= \frac{7.2}{0.9}\left(1 + \frac{25}{2.5}\right) = 8(1 + 10) = 88$$

So, the magnifying power of the microscope is 88.

Q. 13. A small telescope has an objective lens of focal length 144 cm and an eyepiece of focal length 6.0 cm. What is the magnifying power of the telescope? What is the separation between the objective and the eyepiece?

[NCERT Ex. Q. 9.13, Page 250]

Ans. Given that,

Focal length of the objective lens, $f_o = 144$ cm

Focal length of the eyepiece, $f_e = 6.0$ cm

The magnifying power of the telescope,

$$m = \frac{f_o}{f_e}$$

$$= \frac{144}{6} = 24$$

The separation between the objective lens and the eyepiece,

$$f_0 + f_e = 144 + 6 = 150 \text{ cm}$$

Thus, the magnifying power of the telescope is 24 D and the separation between the objective lens and the eyepiece is 150 cm.

Q. 14. (a) A giant refracting telescope at an observatory has an objective lens of focal length 15 m. If an eyepiece of focal length 1.0 cm is used, what is the angular magnification of the telescope?

(b) If this telescope is used to view the moon, what is the diameter of the image of the moon formed by the objective lens? The diameter of the moon is 3.48 × 10^6 m, and the radius of lunar orbit is 3.8 × 10^8 m.

[NCERT Ex. Q. 9.14, Page 346]

Ans. Given that,

Focal length of the objective lens,

$f_0 = 15$ m $= 15 \times 100$ cm $= 1500$ cm

Focal length of the eyepiece, $f_e = 1.0$ cm

(a) The angular magnification of a telescope,

$$\alpha = \frac{f_0}{f_e}$$

$$= \frac{15 \times 10^2}{1.0} = 1500$$

Thus, the angular magnification of the given refracting telescope is 1500.

(b) Given that,

Diameter of the moon, $d = 3.48 \times 10^6$ m

Radius of the lunar orbit, $r_o = 3.8 \times 10^8$ m

Let 'd'' is the diameter of the image of the moon formed by the objective lens.

The angle subtended by the diameter of the moon is equal to the angle subtended by the image.

$$\frac{d}{r_0} = \frac{d'}{f_0}$$

$$\frac{3.48 \times 10^6}{3.8 \times 10^8} = \frac{d'}{15}$$

So, $$d = \frac{3.48}{3.8} \times 10^{-2} \times 15$$

$$= 13.74 \times 10^{-2} \text{ m}$$

$$= 13.74 \text{ cm}$$

Thus, the diameter of the moon's image formed by the objective lens is 13.74 cm.

Q. 15. Use the mirror equation to deduce that

(a) an object placed between *f* and 2*f* of a concave mirror produces a real image beyond 2*f*.

(b) a convex mirror always produces a virtual image independent of the location of the object.

(c) the virtual image produced by a convex mirror is always diminished in size and is located between the focus and the pole.

(d) an object placed between the pole and focus of a concave mirror produces a virtual and enlarged image. **[NCERT Ex. Q. 9.15, Page 250]**

Ans. (a) For a concave mirror, the focal length (f) is negative.

So, $f < 0$

When the object is placed on the left side of the mirror, the object distance (u) is negative.

So, $u < 0$

For image distance v, according to the lens formula:

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} \quad \text{...(i)}$$

So the object lies between f and $2f$.

$\therefore \quad 2f < u < f \quad (\because u \text{ and } f \text{ are negative})$

$$\frac{1}{2f} > \frac{1}{u} > \frac{1}{f}$$

$$-\frac{1}{2f} < -\frac{1}{u} < -\frac{1}{f}$$

$$\frac{1}{f} - \frac{1}{2f} < \frac{1}{f} - \frac{1}{u} < 0 \quad \text{...(ii)}$$

Using equation (i), we get:

$$\frac{1}{2f} < \frac{1}{v} < 0$$

Since, $\frac{1}{v}$ is negative, *i.e.*, v is negative.

$$\frac{1}{2f} < \frac{1}{v}$$

$$2f > v$$

$$-v > -2f$$

Therefore, the image lies beyond $2f$.

(b) For a convex mirror, the focal length (f) is positive.

$\therefore \quad f > 0$

When the object is placed on the left side of the mirror, the object distance (u) is negative.

$\therefore \quad u < 0$

For image distance v, according to the mirror formula:

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}$$

Using equation (ii), we can conclude that:

$$\frac{1}{v}<0$$

$$v>0$$

Thus, the image is formed on the back side of the mirror.

So, a convex mirror always produces a virtual image, regardless of the object distance.

(c) For a convex mirror, the focal length (f) is positive.

So, $f > 0$

When the object is placed on the left side of the mirror, the object distance (u) is negative,

Therefore, $u < 0$

For image distance v, for this the mirror formula:

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}$$

But we have $u < 0$

Therefore, $$\frac{1}{v}>\frac{1}{f}$$

$$v<f$$

So, the image formed is diminished and is located between the focus (f) and the pole.

(d) For a concave mirror, the focal length (f) is negative.

So, $f < 0$

When the object is placed on the left side of the mirror, the object distance (u) is negative.

So, $u < 0$

It is placed between the focus (f) and the pole.

Therefore,

$$f>u>0$$

$$\frac{1}{f}<\frac{1}{u}<0$$

$$\frac{1}{f}-\frac{1}{u}<0$$

For image distance v, we have the mirror formula,

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}$$

Therefore, $$\frac{1}{v}<0$$

$$v>0$$

The image is formed on the right side of the mirror. So that, it is a virtual image.

For $u<0$ and $v>0$,

$$\frac{1}{u}>\frac{1}{v}$$

So, $v > u$

Magnification, $m=\frac{v}{u}>1$

Therefore, the formed image is enlarged.

Q. 16 A small pin fixed on a table top is viewed from above from a distance of 50 cm. By what distance would the pin appear to be raised if it is viewed from the same point through a 15 cm thick glass slab held parallel to the table? Refractive index of glass = 1.5. Does the answer depend on the location of the slab?

[NCERT Ex. Q. 9.16, Page 250]

Ans. Given that,

Actual depth of the pin, d = 15 cm

Apparent depth of the pin = d'

Refractive index of glass, μ = 1.5

Ratio of actual depth to the apparent depth is equal to the refractive index of glass,

$$\mu=\frac{d}{d'}$$

$$d'=\frac{d}{\mu}$$

$$=\frac{15}{1.5}$$

$$=10 \text{ cm}$$

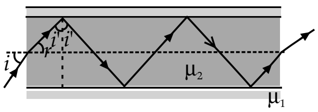
The distance at which the pin appears to be raised $= d' - d$

$= 15 - 10 = 5$ cm

For a small angle of incidence, this distance does not depend on the location of the slab.

Q. 17. (a) Figure shows a cross-section of a 'light pipe' made of a glass fibre of refractive index 1.68. The outer covering of the pipe is made of a material of refractive index 1.44. What is the range of the angles of the incident rays with the axis of the pipe for which total reflections inside the pipe take place, as shown the figure.

(b) What is the answer if there is no outer covering of the pipe?



[NCERT Ex. Q. 9.17, Page 251]

Ans. (a) Given that,

Refractive index of the glass fibre, μ_1 = 1.68

Refractive index of the outer covering of the pipe, μ_2 = 1.44

Let the,

Angle of incidence = i

Angle of refraction = r

Angle of incidence at the interface = i'

So, the refractive index (μ) of the inner core-outer core interface is:

$$\mu=\frac{\mu_2}{\mu_1}$$

$$=\frac{1}{\sin i'}$$

$$\sin i'=\frac{\mu_1}{\mu_2}$$

$$=\frac{1.44}{1.68}$$

$$=0.8571$$

So, $i' = 59^\circ$

For the critical angle, total internal reflection (TIR) takes place only when $i > i'$,

So, $i > 59°$

Maximum angle of reflection,

$$r_{max} = 90^\circ - i'$$
$$= 90° - 59° = 31°$$

Let, i_{max} be the maximum angle of incidence.

The refractive index at the air-glass interface, $\mu_1 = 1.68$

We have the relation for the maximum angles of incidence and reflection as:

$$\mu_1 = \frac{\sin i_{max}}{\sin r_{max}}$$
$$\sin i_{max} = \mu_1 \sin r_{max}$$
$$= 1.68 \sin 31°$$
$$= 1.68 \times 0.5150$$
$$= 0.8652$$

Therefore, $i_{max} = \sin^{-1} 0.8652 \approx 60^\circ$

So that, all the rays incident at angles lying in the range $0 < i < 60°$ will suffer total internal reflection.

(b) If the outer covering of the pipe is not present, then:

Refractive index of the outer pipe, μ_1 = refractive index of air = 1

For the angle of incidence $i = 90°$,

As we now the Snell's law at the air-pipe interface as:

$$\frac{\sin i}{\sin r} = \mu_2 = 1.68$$
$$\sin r = \frac{\sin 90^\circ}{1.68} = \frac{1}{1.68}$$
$$r = \sin^{-1}(0.5952) = 36.5^\circ$$
$$\therefore \quad i' = 90^\circ - 36.5^\circ = 53.5^\circ$$

Since $i' > r$, So that all incident rays will suffer total internal reflection.

Q. 18. The image of a small electric bulb fixed on the wall of a room is to be obtained on the opposite wall 3 m away by means of a large convex lens. What is the maximum possible focal length of the lens required for the purpose? [NCERT Ex. Q. 9.18, Page 251]

Ans. Given that,

Distance between the object and the image, $d = 3$ m

Maximum focal length of the convex lens = f_{max}

For real images, the maximum focal length is given as:

$$f_{max} = \frac{d}{4} = \frac{3}{4} = 0.75 \text{ m}$$

So, for the required purpose, the maximum possible focal length of the convex lens is 0.75 m.

Q. 19. A screen is placed 90 cm from an object. The image of the object on the screen is formed by a convex lens at two different locations separated by 20 cm. Determine the focal length of the lens. [NCERT Ex. Q. 9.19, Page 251]

Ans. Given that,

Distance between the image (screen) and the object, $D = 90$ cm

Distance between two locations of the convex lens, $d = 20$ cm

Let the focal length of the lens = f

Focal length is related to d and D as:

$$f = \frac{D^2 - d^2}{4D}$$
$$= \frac{(90)^2 - (20)^2}{4 \times 90}$$
$$= \frac{770}{36}$$
$$= 21.39 \text{ cm}$$

So, the focal length of the convex lens is 21.39 cm.

Q. 20. (a) Determine the 'effective focal length' of the combination of the two lenses in Exercise 9.10, if they are placed 8.0 cm apart with their principal axes coincident. Does the answer depend on which side of the combination a beam of parallel light is incident? Is the notion of effective focal length of this system useful at all?

(b) An object 1.5 cm in size is placed on the side of the convex lens in the arrangement (a) above. The distance between the object and the convex lens is 40 cm. Determine the magnification produced by the two-lens system, and the size of the image. [NCERT Ex. Q. 9.20, Page 251]

Ans. Given that,

Focal length of the convex lens, $f_1 = 30$ cm

Focal length of the concave lens, $f_2 = -20$ cm

Distance between the two lenses, $d = 8.0$ cm

(a) (i) When the parallel beam of light is incident on the convex lens:

According to the lens formula,

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$$

where,

u_1 = Object distance = ∞

v_1 = Image distance

$$\frac{1}{v_1} = \frac{1}{30} - \frac{1}{\infty} = \frac{1}{30}$$

$\therefore v_1 = 30$ cm

The image will act as a virtual object for the concave lens.

Applying lens formula to the concave lens,

$$\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2}$$

where,

u_2 = Object distance

$= (30 - d) = 30 - 8 = 22$ cm

v_2 = Image distance

$$\frac{1}{v_2} = \frac{1}{22} - \frac{1}{20} = \frac{10-11}{220} = \frac{-1}{220}$$
$$\therefore \quad v_2 = -220 \text{ cm}$$

The parallel incident beam appears to diverge from a point that is

$$\left(220 - \frac{d}{2} = 220 - 4\right) = 216 \text{ cm}$$

from the centre of the combination of the two lenses.

(ii) When the parallel beam of light is incident, from the left, on the concave lens:

According to the lens formula,

$$\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2}$$
$$\frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{u_2}$$

where,
u_2 = Object distance = ∞
v_2 = Image distance

$$\frac{1}{v_2} = \frac{1}{-20} + \frac{1}{-\infty} = -\frac{1}{20}$$

$$\therefore \quad v_2 = -20 \text{ cm}$$

The image will act as a real object for the convex lens. Applying lens formula to the convex lens,

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$$

where,
u_1 = Object distance
= −(20 + d) = −(20+8) = −28 cm
And,
v_1 = Image distance

$$\frac{1}{v_1} = \frac{1}{30} + \frac{1}{-28} = \frac{14-15}{420} = \frac{-1}{420}$$

$$\therefore \quad v_1 = -420 \text{ cm}$$

So, the parallel incident beam appears to diverge from a point that is (420 – 4 cm) = 416 cm from the left of the centre of the combination of the two lenses.

The answer does depend on the side of the combination at which the parallel beam of light is incident. The notion of effective focal length does not seem to be useful for this combination.

(b) Height of the image, h = 1.5 cm
Object distance from the side of the convex lens, u_1 = −40 cm

$$|u_1| = 40 \text{ cm}$$

According to the lens formula:

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$$

where,
v_1 = Image distance

$$\frac{1}{v_1} = \frac{1}{30} + \frac{1}{-40} = \frac{4-3}{120} = \frac{1}{120}$$

$$\therefore \quad v_1 = 120 \text{ cm}$$

Magnification, $m = \left|\frac{v_1}{u_1}\right|$

$$= \frac{120}{40} = 3$$

So, the magnification due to the convex lens is 3.

The image formed by the convex lens acts as an object for the concave lens.

According to the lens formula:

$$\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2}$$

where,
u_2 = Object distance
= +(120 − 8) = 112 cm.
v_2 = Image distance

$$\frac{1}{v_2} = \frac{1}{-20} + \frac{1}{112} = \frac{-112+20}{2240} = \frac{-92}{2240}$$

$$\therefore v_2 = \frac{-2240}{92} \text{ cm}$$

Magnification, $m' = \left|\frac{v_2}{u_2}\right| = \frac{2240}{92} \times \frac{1}{112} = \frac{20}{92}$

The magnification produced by the combination of the two lenses is calculated as:

$$= m \times m' = 3 \times \frac{20}{92} = \frac{60}{92} = 0.652$$

The magnification of the combination is given as :

$$\frac{h_2}{h_1} = 0.652$$

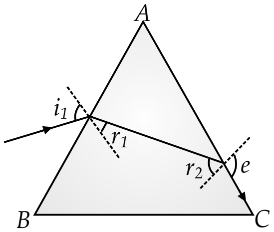
$$h_2 = 0.652 \times h_1$$

where,
h_1 = Object size = 1.5 cm
h_2 = Size of the image
$\therefore h_2 = 0.652 \times 1.5 = 0.98$ cm

So, the height of the image is 0.98 cm.

Q. 21. At what angle should a ray of light be incident on the face of a prism of refracting angle 60° so that it just suffers total internal reflection at the other face? The refractive index of the material of the prism is 1.524. [NCERT Ex. Q. 9.21, Page 251]

Ans. The incident, refracted and emergent rays associated with a glass prism ABC are shown in the below figure.



Given that,
Angle of prism, ∠A = 60°
Refractive index of the prism, μ = 1.524
i_1 = Incident angle
r_1 = Refracted angle
r_2 = Angle of incidence at the face AC
e = Emergent angle = 90°
According to Snell's law, for side AC,

$$\frac{\sin e}{\sin r_2} = \mu$$

$$\sin r_2 = \frac{1}{\mu} \times \sin 90° = \frac{1}{1.524} = 0.6562$$

$$\therefore \quad r_2 = \sin^{-1} 0.6562 \approx 41°$$

It is clear from the figure that angle,

$$A = r_1 + r_2$$

$$\therefore \quad r_1 = A - r_2 = 60 - 41 = 19°.$$

According to Snell's law,

$$\mu = \frac{\sin i_1}{\sin r_1}$$

$$\sin i_1 = \mu \sin r_1 = 1.524 \times \sin 19° = 0.496$$

$$i_1 = 29.75°$$

So, the angle of incidence is 29.75°.

Q. 22. A card sheet divided into squares each of size 1 mm^2 is being viewed at a distance of 9 cm through a

magnifying glass (a converging lens of focal length 9 cm) held close to the eye.

(a) What is the magnification produced by the lens? How much is the area of each square in the virtual image?

(b) What is the angular magnification (magnifying power) of the lens?

(c) Is the magnification in (a) equal to the magnifying power in (b)? Explain. [NCERT Ex. Q. 9.22, Page 251]

Ans. (a) Given that,

Area of each square, $A = 1\ \text{mm}^2$

Object distance, $u = -9$ cm

Focal length of a converging lens, $f = 9$ cm

For image distance v, the lens formula can be written as:

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{10} = \frac{1}{v} + \frac{1}{9}$$

$$\frac{1}{v} = -\frac{1}{90}$$

$$\therefore \quad v = -90 \text{ cm}$$

$$\text{Magnification, } m = \frac{v}{u}$$

$$= \frac{-90}{-9} = 10$$

$\therefore$ Area of each square in the virtual image $=(\text{Magnification})^2 \times$ Area of each square

$$= 10^2 \times 1 = 100 \text{ mm}^2 = 1 \text{ cm}^2$$

(b) Magnifying power of the lens

$$= \frac{d}{|u|} = \frac{25}{9} = 2.8$$

(c) The magnification in (a) is not the same as the magnifying power in (b).

The magnification magnitude is, $\left(\left|\frac{v}{u}\right|\right)$ and the magnifying power is, $\left(\frac{d}{|u|}\right)$.

The two quantities will be equal when the image is formed at the near point (25 cm).

Q. 23. (a) At what distance should the lens be held from the figure in Question 22 in order to view the squares distinctly with the maximum possible magnifying power?

(b) What is the magnification in this case?

(c) Is the magnification equal to the magnifying power in this case? Explain. [NCERT Ex. Q. 9.23, Page 251]

Ans. (a) Given that,

The maximum possible magnification is obtained when the image is formed at the near point (d = 25 cm).

Image distance, $v = -d = -25$ cm

Focal length, $f = 10$ cm

Let the object distance $= u$

According to the lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{u} = \frac{1}{v} - \frac{1}{f}$$

$$= \frac{1}{-25} - \frac{1}{10} = \frac{-2-5}{50} = -\frac{7}{50}$$

$$\therefore \quad u = -\frac{50}{7} = -7.14 \text{ cm}$$

So, to view the squares distinctly, the lens should be kept at 7.14 cm away from them.

(b) Magnification $= \left|\frac{v}{u}\right| = \frac{25}{\frac{50}{7}} = 3.5$

(c) Magnifying power $= \frac{d}{u} = \frac{25}{\frac{50}{7}} = 3.5$

Since the image is formed at the near point (25 cm), the magnifying power is equal to the magnitude of magnification.

Q. 24. What should be the distance between the object in Question 23 and the magnifying glass if the virtual image of each square in the figure is to have an area of 6.25 mm^2. Would you be able to see the squares distinctly with your eyes very close to the magnifier?

[NCERT Ex. Q. 9.24, Page 251]

Ans. Given that,

Area of the virtual image of each square, $A = 6.25\ \text{mm}^2$

Area of each square, $A = 1$ mm

So, the linear magnification of the object can be calculated as,

$$m = \sqrt{\frac{A}{A_0}} = \sqrt{\frac{6.25}{1}} = 2.5$$

$$\text{But} \quad m = \frac{\text{Image distance } (v)}{\text{Object distance } (u)}$$

$$\therefore \quad v = mu = 2.5\,u \quad \text{...(i)}$$

Focal length of the magnifying glass, $f = 10$ cm

According to the lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{10} = \frac{1}{2.5u} - \frac{1}{u} = \frac{1}{u}\left(\frac{1}{2.5} - \frac{1}{1}\right) = \frac{1}{u}\left(\frac{1-2.5}{2.5}\right)$$

$$\therefore \quad u = \frac{1.5 \times 10}{2.5} = -6 \text{ cm}$$

And $v = 2.5\,u$

$$= 2.5 \times 6 = -15 \text{ cm}$$

The virtual image is formed at a distance of 15 cm, which is less than the near point (i.e., 25 cm) of a normal eye. Thus, it cannot be seen by the eyes distinctly.

Q. 25. Answer the following questions:

(a) The angle subtended at the eye by an object is equal to the angle subtended at the eye by the virtual image

produced by a magnifying glass. In what sense then does a magnifying glass provide angular magnification?

(b) In viewing through a magnifying glass, one usually position's one's eyes very close to the lens. Does angular magnification change if the eye is moved back?

(c) Magnifying power of a simple microscope is inversely proportional to the focal length of the lens. What then stops us from using a convex lens of smaller and smaller focal length and achieving greater and greater magnifying power?

(d) Why must both the objective and the eyepiece of a compound microscope have short focal lengths?

(e) When viewing through a compound microscope, our eyes should be positioned not on the eyepiece but a short distance away from it for best viewing. Why? How much should be that short distance between the eye and eyepiece? [NCERT Ex. Q. 9.25, Page 252]

Ans. (a) Though the image size is bigger than the object, the angular size of the image is equal to the angular size of the object. A magnifying glass helps one see the objects placed closer than the least distance of distinct vision (i.e., 25 cm). A closer object causes a larger angular size. A magnifying glass provides angular magnification. Without magnification, the object cannot be placed closer to the eye. With magnification, the object can be placed much closer to the eye.

(b) Yes, the angular magnification changes. When the distance between the eye and a magnifying glass is increased, the angular magnification decreases a little. This is because the angle subtended at the eye is slightly less than the angle subtended at the lens. Image distance does not have any effect on angular magnification.

(c) The focal length of a convex lens cannot be decreased by a greater amount. This is because making lenses having very small focal lengths is not easy. Spherical and chromatic aberrations are produced by a convex lens having a very small focal length.

(d) The angular magnification produced by the eyepiece of a compound microscope is $\left[\left(\frac{25}{f_e}\right)+1\right]$.

where,

f_e = Focal length of the eyepiece

It can be inferred that if f_e is small, then angular magnification of the eyepiece will be large.

The angular magnification of the objective lens of a compound microscope is given as:

$$=\frac{1}{[|u_o|f_o]}$$

where,

u_o = Object distance for the objective lens

f_o = Focal length of the objective

The magnification is large when $u_o > f_o$.

In the case of a microscope, the object is kept close to the objective lens. So, the object distance is very little. Since u_o is small, f_o will be even smaller. Therefore, f_e and f_o are both small in the given condition.

(e) When we place our eyes too close to the eyepiece of a compound microscope, we are unable to collect much refracted light. As a result, the field of view decreases substantially.

So, the clarity of the image gets blurred.

The best position of the eye for viewing through a compound microscope is at the eye-ring attached to the eyepiece. The precise location of the eye depends on the separation between the objective lens and the eyepiece.

Q. 26. An angular magnification (magnifying power) of 30× is desired using an objective of focal length 1.25 cm and an eyepiece of focal length 5 cm. How will you set up the compound microscope? [NCERT Ex. Q. 9.26, Page 252]

Ans. Given that,

Focal length of the objective lens, f_o = 1.25 cm

Focal length of the eyepiece, f_e = 5 cm

Least distance of distinct vision, d = 25 cm

Angular magnification of the compound microscope = 30 ×

Total magnifying power of the compound microscope, m = 30

The angular magnification of the eyepiece is given by the relation,

$$m_e = \left(1+\frac{d}{f_e}\right) = \left(1+\frac{25}{5}\right) = 6$$

The angular magnification of the objective lens (m_o) is related to m_e as:

$$m = m_o \cdot m_e$$

$$m_e = \frac{m}{m_e} = \frac{30}{6} = 5$$

And we know that,

$$m_e = \frac{\text{Image distance for the objective lens } (v_o)}{\text{Object distance for the objective lens } (-u_o)}$$

$$5 = \frac{v_o}{-u_o}$$

$$\therefore v_o = -5u_o \qquad \text{...(i)}$$

Applying the lens formula for the objective lens,

$$\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o}$$

$$\frac{1}{1.25} = \frac{1}{-5u_o} - \frac{1}{u_o} = \frac{-6}{5u_o}$$

$$\therefore \quad u_o = \frac{-6}{5} \times 1.25 = -1.5 \text{ cm}$$

And $v_o = -5u_o$

$$= -5 \times (-1.5) = 7.5 \text{ cm}$$

The object should be placed 1.5 cm away from the objective lens to obtain the desired magnification.

Applying the lens formula for the eyepiece:

$$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$$

where,

v_e = Image distance for the eyepiece

$= -d = -25$ cm

u_e = Object distance for the eyepiece

$$\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e}$$

$$= \frac{-1}{25} - \frac{1}{5} = -\frac{6}{25}$$

So,

$u_e = -4.17$ cm

Separation between the objective lens and the eyepiece, $= |u_e| + |v_o|$

$= 4.17 + 7.5$

$= 11.67$ cm

So, the separation between the objective lens and the eyepiece should be 11.67 cm.

Q. 27. A small telescope has an objective lens of focal length 140 cm and an eyepiece of focal length 5.0 cm. What is the magnifying power of the telescope for viewing distant objects when :

(a) the telescope is in normal adjustment (i.e., **when the final image is at infinity)?**

(b) the final image is formed at the least distance of distinct vision (25 cm)? [NCERT Ex. Q. 9.27, Page 252]

Ans. Given that,

Focal length of the objective lens, $f_0 = 140$ cm

Focal length of the eyepiece, $f_e = 5$ cm

Least distance of distinct vision, $d = 25$ cm

(a) When the telescope is in normal adjustment, its magnifying power is given as:

$$m = \frac{f_0}{f_e}$$

$$= \frac{140}{5} = 28$$

(b) When the final image is formed at d, the magnifying power of the telescope is given as:

$$\frac{f_o}{f_e}\left[1 + \frac{f_e}{d}\right] = \frac{140}{5}\left[1 + \frac{5}{25}\right]$$

$$= 28[1 + 0.2]$$

$$= 28 \times 1.2$$

$$= 33.6$$

Q. 28. (a) For the telescope described in Question 27, what is the separation between the objective lens and the eyepiece?

(b) If the telescope is used to view a 100 m tall tower 3 km away, what is the height of the image of the tower formed by the objective lens?

(c) What is the height of the final imge of the tower if it is formed at 25 cm? [NCERT Ex. Q. 9.28, Page 252]

Ans. Focal length of the objective lens, $f_0 = 140$ cm

Focal length of the eyepiece, $f_e = 5$ cm

(a) The separation between the objective lens and the eyepiece $= f_0 + f_e = 140 + 5 = 145$ cm.

(b) Height of the tower, $h_1 = 100$ m

Distance of the tower from the telescope, u = 3 km = 3000 m.

The angle subtended by the tower at the telescope is given as:

$$\theta = \frac{h_1}{\mu}$$

$$= \frac{100}{3000} = \frac{1}{30} \text{rad}$$

The angle subtended by the image produced by the objective lens is given as:

$$\theta = \frac{h_2}{f_0} = \frac{h_2}{140} \text{ rad}$$

where h_2 = height of the image of the tower formed by the objective lens.

$$\frac{1}{30} = \frac{h_2}{140}$$

$$\therefore \quad h_2 = \frac{140}{30} = 4.7 \text{ cm}$$

Therefore, the objective lens forms a 4.7 cm tall image of the tower.

(c) Image is formed at a distance, $d = 25$ cm

The magnification of the eyepiece is given by the relation,

$$m = 1 + \frac{d}{f_e}$$

$$= 1 + \frac{25}{5} = 1 + 5 = 6$$

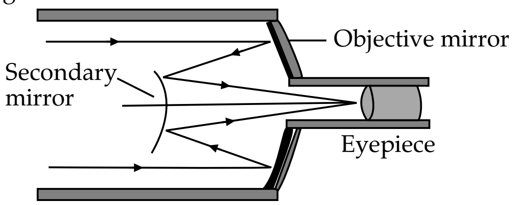
Height of the final image $= mh_2 = 6 \times 4.7 = 28.2$ cm.

Hence, the height of the final image of the tower is 28.2 cm.

Q. 29. A Cassegrain telescope uses two mirrors as shown in Figure. Such a telescope is built with the mirrors 20 mm apart. If the radius of curvature of the large mirror is 220 mm and the small mirror is 140 mm, where will the final image of an object at infinity be?

[NCERT Ex. Q. 9.29, Page 252]

Ans. The following figure shows a Cassegrain telescope consisting of a concave mirror and a convex mirror.



Given that,

Distance between the objective mirror and the secondary mirror, $d = 20$ mm

Radius of curvature of the objective mirror,

$R_1 = 220$ mm

Radius of curvature of the secondary mirror,

$R_2 = 140$ mm

So the focal length of the objective mirror,

$$f_1 = \frac{R_1}{2} = 110 \text{ mm}$$

Radius of curvature of the secondary mirror,

$R_2 = 140$ mm

So the focal length of the secondary mirror,

$$f_2 = \frac{R_2}{2} = 70 \text{ mm}$$

The image of an object placed at infinity, formed by the objective mirror, will act as a virtual object for the secondary mirror.

Thus, the virtual object distance for the secondary mirror, $u = f_1 - d = 110 - 20 = 90$ mm

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f_2}$$

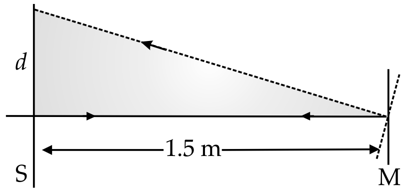
$$\frac{1}{v} = \frac{1}{f_2} - \frac{1}{u} = \frac{1}{70} - \frac{1}{90} = \frac{9-7}{630} = \frac{2}{630}$$

Therefore,

$$v = \frac{630}{2} = 315 \text{ mm}$$

So, the final image will be formed 315 mm away from the secondary mirror.

Q. 30.Light incident normally on a plane mirror attached to a galvanometer coil retraces backwards as shown in Figure. A current in the coil produces a deflection of 3.5° of the mirror. What is the displacement of the reflected spot of light on a screen placed 1.5 m away?



[NCERT Ex. Q. 9.30, Page 252]

Ans. Given that,

Angle of deflection, $\phi = 3.5°$

Distance of the screen from the mirror, $D = 1.5$ m

The reflected rays get deflected by an amount twice the angle of deflection, $2\theta = 7.0°$

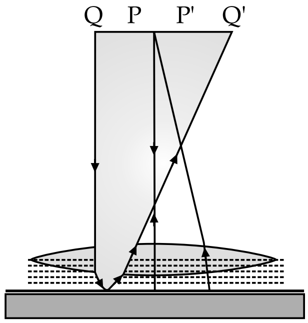
The displacement (d) of the reflected spot of light on the screen is given as:

$$\tan 2\theta = \frac{d}{1.5}$$

$$\therefore \quad d = 1.5 \times \tan 7^\circ = 0.184 \text{ m} = 18.4 \text{ cm}$$

So, the displacement of the reflected spot of light is 18.4 cm.

Q. 31. Figure shows an equi convex lens (of refractive index 1.50) in contact with a liquid layer on top of a plane mirror. A small needle with its tip on the principal axis is moved along the axis until its inverted image is found at the position of the needle. The distance of the needle from the lens is measured to be 45.0 cm. The liquid is removed and the experiment is repeated. The new distance is measured to be 30.0 cm. What is the refractive index of the liquid?



[NCERT Ex. Q. 9.31, Page 253]

Ans. Given that,

Focal length of the convex lens, $f_1 = 30$ cm

The liquid acts as a mirror. So, focal length of the liquid = f_2

Focal length of the system (convex lens + liquid), $f = 45$ cm

For a pair of optical systems placed in contact, the equivalent focal length is given as:

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{f_2} = \frac{1}{f} - \frac{1}{f_1}$$

$$= \frac{1}{45} - \frac{1}{30} = \frac{1}{90}$$

$$\therefore \quad f_2 = -90 \text{ cm}$$

Let the refractive index of the lens be μ_1 and the radius of curvature of one surface be R. Therefore, the radius of curvature of the other surface is −R.

R can be obtained using the relation,

$$\frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{R} - \frac{1}{-R}\right)$$

$$\frac{1}{30} = (1.5 - 1)\left(\frac{2}{R}\right)$$

$$\therefore \quad R = \frac{30}{0.5 \times 2} = 30 \text{ cm}$$

Let μ_2 be the refractive index of the liquid.

Radius of curvature of the liquid on the side of the plane mirror = ∞

Radius of curvature of the liquid on the side of the lens, $R = -30$ cm

The value of μ_2 can be calculated using the relation,

$$\frac{1}{f_2} = (\mu_2 - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right)$$

$$\frac{-1}{90} = (\mu_2 - 1)\left[\frac{1}{+30} - 0\right]$$

$$\mu_2 - 1 = \frac{1}{3}$$

$$\therefore \quad \mu_2 = \frac{4}{3} = 1.33$$

Therefore, the refractive index of the liquid is 1.33.