Chapter 11

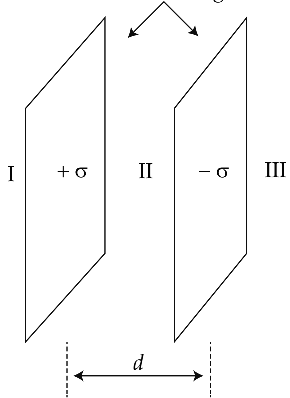
Electrostatics

Topic-1
Electrostatic Force, Electric Field and Electrostatic Potential

JEE (Main) Previous Year Questions

Multiple Choice Questions

1. Let σ be the uniform surface charge density of two infinite thin plane sheets shown in figure. Then the electric fields in three different regions E_I, E_{II} and E_{III} are:



(1) $\vec{E_I} = \frac{2\sigma}{\varepsilon_o}\hat{n}, \vec{E}_{II} = 0, \vec{E}_{III} = \frac{2\sigma}{\varepsilon_o}\hat{n}$

(2) $\vec{E_I} = \frac{2\sigma}{2\varepsilon_o}\hat{n}, \vec{E}_{II} = 0, \vec{E}_{III} = \frac{2\sigma}{2\varepsilon_o}\hat{n}$

(3) $\vec{E_I} = -\frac{\sigma}{\varepsilon_o}\hat{n}, \vec{E}_{II} = 0, \vec{E}_{III} = \frac{\sigma}{\varepsilon_o}\hat{n}$

(4) $\vec{E_I} = 0, \vec{E}_{II} = \frac{\sigma}{\varepsilon_o}\hat{n}, E_{III} = 0$

[JEE (Main) – 1st Feb. 2023 - Shift-1]

2. Considering a group of positive charges, which of the following statements is correct?

(1) Both the net potential and the net electric field cannot be zero at a point.

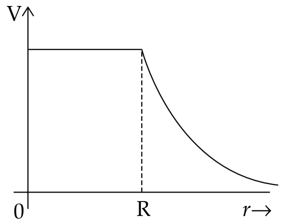
(2) Net potential of the system at a point can be zero but net electric field can't be zero at that point.

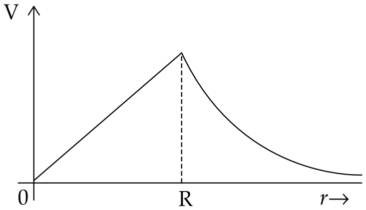
(3) Net potential of the system cannot be zero at a point but net electric field can be zero at that point.

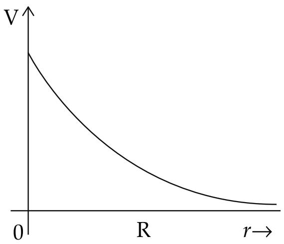
(4) Both the net potential and the net field can be zero at a point.

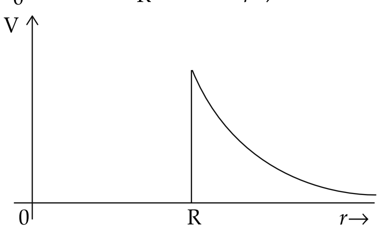
[JEE (Main) – 31st Jan. 2023 - Shift-2]

3. Which of the following correctly represents the variation of electric potential (V) of a charged spherical conductor of radius (R) with radial distance (*r*) from the centre?

(1) 

(2) 

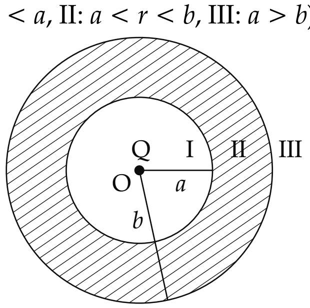
(3) 

(4) 

[JEE (Main) – 31st Jan. 2023 - Shift-1]

4. As shown in the figure, a point charge Q is placed at the centre of conducting spherical shell of inner radius a and outer radius b. The electric field due to charge Q in three different regions I, II and III is given by:

(I: $r < a$, II: $a < r < b$, III: $a > b$)



(1) $E_I = 0, E_{II} = 0, E_{III} = 0$
(2) $E_I = 0, E_{II} = 0, E_{III} \neq 0$
(3) $E_I \neq 0, E_{II} = 0, E_{III} \neq 0$
(4) $E_I \neq 0, E_{II} = 0, E_{III} = 0$

[JEE (Main) – 30th Jan. 2023 - Shift-2]

5. Electric field in a certain region is given by $\vec{E} = \left(\frac{A}{x^2}\hat{i} + \frac{B}{y^2}\hat{j}\right)$. The SI unit of A and B are:

(1) $N\,m^3\,C^{-1}$; $N\,m^2\,C^{-1}$
(2) $N\,m^2\,C^{-1}$; $N\,m^3\,C^{-1}$
(3) $N\,m^3\,C$; $N\,m^2\,C$
(4) $N\,m^2C$; $N\,m^3C$

[JEE (Main) – 30th Jan. 2023 - Shift-1]

6. Two isolated metallic solid spheres of radii R and $2R$ are charged such that both have same charge density σ. The spheres are then connected by a thin conducting wire. If the new charge density of the bigger sphere is σ'. The ratio $\frac{\sigma'}{\sigma}$

(1) $\frac{4}{3}$ **(2)** $\frac{5}{3}$ **(3)** $\frac{5}{6}$ **(4)** $\frac{9}{4}$

[JEE (Main) – 30th Jan. 2023 - Shift-1]

7. A point charge 2×10^{-2} C is moved from P to S in a uniform electric field of 30 NC^{-1} directed along positive x-axis. If co-ordinates of P and S are (1, 2, 0) m and (0, 0, 0) m respectively, the work done by electric field will be:

(1) 1200 m J **(2)** −1200 m J
(3) −600 m J **(4)** 600 m J

[JEE (Main) – 29th Jan. 2023 - Shift-2]

8. In a cuboid of dimension 2L × 2L × L, a charge q is placed at the center of the surface ' S ' having area of $4L^2$. The flux through the opposite surface to ' S ' is given by:

(1) $\frac{q}{12\varepsilon_0}$ **(2)** $\frac{q}{6\varepsilon_0}$ **(3)** $\frac{q}{3\varepsilon_0}$ **(4)** $\frac{q}{2\varepsilon_0}$

[JEE (Main) – 29th Jan. 2023 - Shift-1]

9. A point charge of 10 μC is placed at the origin. At what location on the x-axis should a point charge of 40 μC be placed so that the net electric field is zero at $x = 2$ cm on the x-axis?

(1) $x = -4$ cm **(2)** $x = 6$ cm
(3) $x = 4$ cm **(4)** $x = 8$ cm

[JEE (Main) – 25th Jan. 2023 - Shift-2]

10. The electric field due to a short electric dipole at a large distance (r) from center of dipole on the equatorial plane varies with distance as:

(1) r **(2)** $\frac{1}{r}$ **(3)** $\frac{1}{r^2}$ **(4)** $\frac{1}{r^3}$

[JEE (Main) – 15th April 2023 - Shift-1]

11. A 10 μC charge is divided into two parts and placed at 1 cm distance so that the repulsive force between them is maximum. The charges of the two parts are:

(1) 7 μ C, 3 μ C **(2)** 8 μ C, 2 μ C
(3) 9 μ C, 1 μ C **(4)** 5 μ C, 5 μ C

[JEE (Main) – 13th April 2023 - Shift-2]

12. Two charges each of magnitude 0.01 C and separated by a distance of 0.4 mm constitute an electric dipole. If the dipole is placed in an uniform electric field ' $\vec{E}$ ' of 10 dyne C^{-1} making 30° angle with $\vec{E}$, the magnitude of torque acting on dipole is:

(1) 1.5×10^{-9} N m **(2)** 2.0×10^{-10} N m
(3) 1.0×10^{-8} N m **(4)** 4.0×10^{-10} N m

[JEE (Main) – 13th April 2023 - Shift-1]

13. Given below are two statements. One is labelled as Assertion A and the other is labelled as Reason R.
Assertion A : If an electric dipole of dipole moment 30×10^{-5} C m is enclosed by a closed surface, the net flux coming out of the surface will be zero.
Reason R : Electric dipole consists of two equal and opposite charges.
In the light of above statements, choose the correct answer from the options given below:
(1) Both A and R are true but R is NOT the correct explanation of A.
(2) A is false but R is true.
(3) Both A and R are true and R is the correct explanation of A.
(4) A is true but R is false.

[JEE (Main) – 12th April 2023 - Shift-1]

14. A capacitor of capacitance C is charge d to a potential V. The flux of the electric field through a closed surface enclosing the positive plate of the capacitor is:
(1) Zero **(2)** $\frac{CV}{\varepsilon_0}$ **(3)** $\frac{2CV}{\varepsilon_0}$ **(4)** $\frac{CV}{2\varepsilon_0}$

[JEE (Main) – 11th April 2023 - Shift-2]

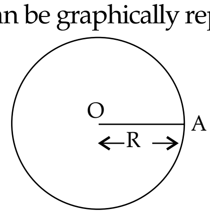
15. A parallel plate capacitor of capacitance 2 F is charged to a potential V. The energy stored in the capacitor is E_1. The capacitor is now connected to another uncharged identical capacitor in parallel combination. The energy stored in the combination is E_2. The ratio E_2/E_1 is:
(1) 2 : 3 **(2)** 1 : 2 **(3)** 1 : 4 **(4)** 2 : 1

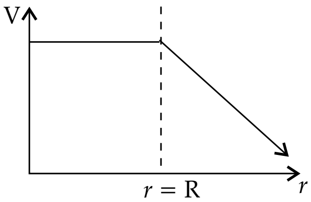
[JEE (Main) – 11th April 2023 - Shift-1]

16. A dipole comprises of two charged particles of identical magnitude q and opposite in nature. The mass 'm' of the positive charged particle is half of the mass of the negative charged particle. The two charges are separated by a distance 'l'. If the dipole is placed in a uniform electric field '$\bar{E}$'; in such a way that dipole axis makes a very small angle with the electric field, '$\bar{E}$'. The angular frequency of the oscillations of the dipole when released is given by:
(1) $\sqrt{\frac{4qE}{3ml}}$ **(2)** $\sqrt{\frac{8qE}{ml}}$ **(3)** $\sqrt{\frac{8qE}{3ml}}$ **(4)** $\sqrt{\frac{4qE}{ml}}$

[JEE (Main) – 6th April 2023 - Shift-2]

17. For a uniformly charged thin spherical shell, the electric potential (V) radially away from the centre (O) of shell can be graphically represented as:



(1)


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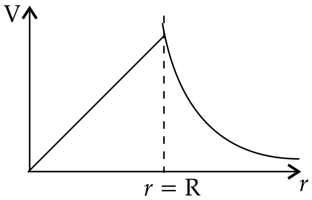
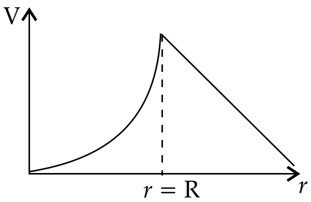


(3)


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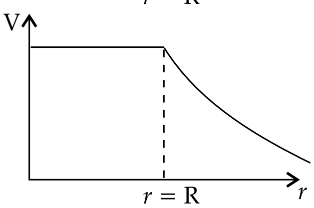


[JEE (Main) – 6th April 2023 - Shift-1]

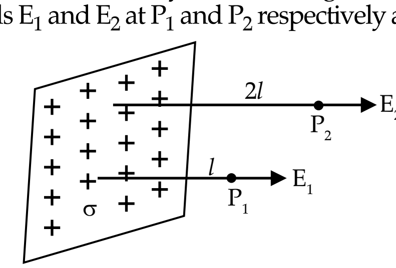
18. Two identical charged particles each having mass of 10 g and charge of 2.0×10^{-7} C are placed on a horizontal table with a separation of L between them such that they stay in limited equilibrium. If the coefficient of friction between each particle and the table is 0.25, find the value of L. [Use $g = 10$ ms^{-2}]
(1) 12 cm **(2)** 10 cm **(3)** 8 cm **(4)** 5 cm

[JEE (Main) – 24th June 2022 - Shift-2]

19. A long cylindrical volume contains uniformly distributed charge of density ρ. The radius of cylindrical volume is R. A charge particle (q) revolves around the cylinder in a circular path. The kinetic energy of the particle is:
(1) $\frac{\rho q R^2}{4\varepsilon_0}$ **(2)** $\frac{\rho q R^2}{2\varepsilon_0}$ **(3)** $\frac{q\rho}{4\varepsilon_0 R^2}$ **(4)** $\frac{4\varepsilon_0 R^2}{q\rho}$

[JEE (Main) – 24th June 2022 - Shift-2]

20. In the figure, a very large plane sheet of positive charge is shown. P_1 and P_2 are two points at distance l and $2l$ from the charge distribution. If σ is the surface density, then the magnitude of electric fields E_1 and E_2 at P_1 and P_2 respectively are:



(1) $E_1 = \sigma/\varepsilon_0,\ E_2 = \sigma/2\varepsilon_0$

(2) $E_1 = 2\sigma/\varepsilon_0,\ E_2 = \sigma/\varepsilon_0$

(3) $E_1 = E_2 = \sigma/2\varepsilon_0$

(4) $E_1 = E_2 = \sigma/\varepsilon_0$

[JEE (Main) – 25th June 2022 - Shift-1]

21. Sixty four conducting drops each of radius 0.02 m and each carrying a charge of 5 μC are combined to form a bigger drop. The ratio of surface charge density of bigger drop to the smaller drop will be:

(1) 1 : 4 **(2)** 4 : 1 **(3)** 1 : 8 **(4)** 8 : 1

[JEE (Main) – 26th June 2022 - Shift-2]

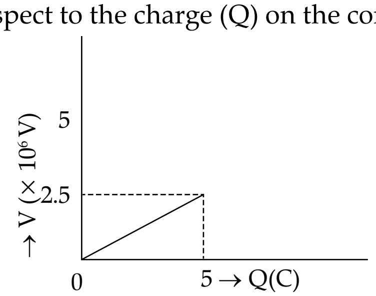
22. Two point charges A and B of magnitude $+8 \times 10^{-6}$ C and -8×10^{-6} C respectively are placed at a distance d apart. The electric field at the middle point O between the charges is 6.4×10^4 NC^{-1}. The distance '*d*' between the point charges A and B is:

(1) 2.0 m **(2)** 3.0 m **(3)** 1.0 m **(4)** 4.0 m

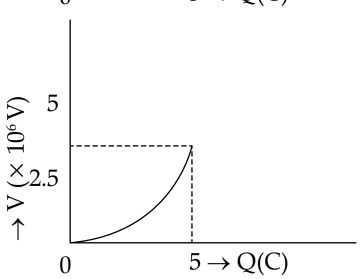
[JEE (Main) – 28th June 2022 - Shift-2]

23. A condenser of 2 μF capacitance is charged steadily from 0 to 5 C. Which of the following graph represents correctly the variation of potential difference (V) across its plates with respect to the charge (Q) on the condenser?

(1)



(2)



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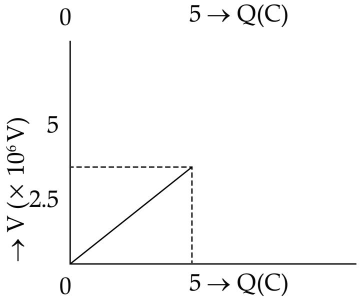

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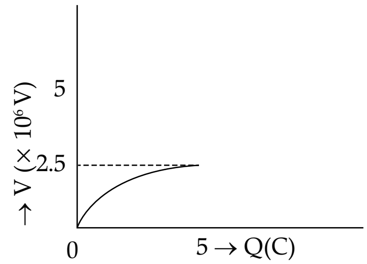


[JEE (Main) – 25th July 2022 - Shift-1]

24. Capacitance of an isolated conducting sphere of radius R_1 becomes n times when it is enclosed by a concentric conducting sphere of radius R_2 connected to earth. The ratio of their radii $\left(\frac{R_2}{R_1}\right)$ is :

(1) $\frac{n}{n-1}$ **(2)** $\frac{2n}{2n+1}$ **(3)** $\frac{n+1}{n}$ **(4)** $\frac{2n+1}{n}$

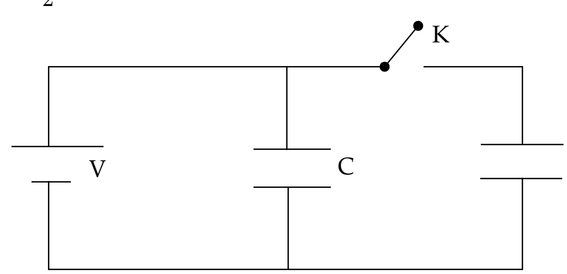
[JEE (Main) – 25th July 2022 - Shift-2]

25. Two uniformly charged spherical conductors A and B of radii 5 mm and 10 mm are separated by a distance of 2 cm. If the spheres are connected by a conducting wire, then in equilibrium condition, the ratio of the magnitudes of the electric fields at the surface of the sphere A and B will be:

(1) 1 : 2 **(2)** 2 : 1 **(3)** 1 : 1 **(4)** 1 : 4

[JEE (Main) – 26th July 2022 - Shift-2]

26. A source of potential difference V is connected to the combination of two identical capacitors as shown in the figure. When key 'K' is closed, the total energy stored across the combination is E_1. Now key 'K' is opened and dielectric of dielectric constant 5 is introduced between the plates of the capacitors. The total energy stored across the combination is now E_2. The ratio E_1/E_2 will be:



(1) $\frac{1}{10}$ **(2)** $\frac{2}{5}$ **(3)** $\frac{5}{13}$ **(4)** $\frac{5}{26}$

[JEE (Main) – 26th July 2022 - Shift-2]

27. Two identical positive charges Q each are fixed at a distance of '$2a$' apart from each other. Another point charge q_0 with mass 'm' is placed at midpoint between two fixed charges. For a small displacement along the line joining the fixed charges, the charge q_0 executes SHM. The time period of oscillation of charge q_0 will be :

(1) $\sqrt{\dfrac{4\pi^3\varepsilon_0 ma^3}{q_0Q}}$ (2) $\sqrt{\dfrac{q_0Q}{4\pi^3\varepsilon_0 ma^3}}$

(3) $\sqrt{\dfrac{2\pi^2\varepsilon_0 ma^3}{q_0Q}}$ (4) $\sqrt{\dfrac{8\pi^2\varepsilon_0 ma^3}{q_0Q}}$

[JEE (Main) – 27th July 2022 - Shift-1]

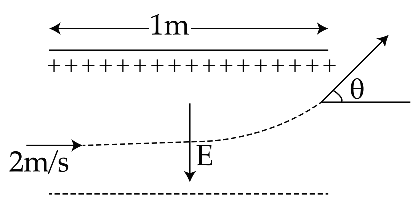
28. A charge 4 μC is to be divided into two parts. The value of two charges such that the force between them is maximum, will be :

(1) 1 μC and 3 μC (2) 2 μC and 2 μC

(3) 0 μC and 4 μC (4) 1.5 μC and 2.5 μC

[JEE (Main) – 27th July 2022 - Shift-2]

29. A uniform electric field E = (8m/e) V/m is created between two parallel plates of length 1 m as shown in figure, (where m = mass of electron and e = charge of electron). An electron enters the field symmetrically between the plates with a speed of 2 m/s. The angle of the deviation (θ) of the path of the electron as it comes out of the field will be ______.



(1) $\tan^{-1}(4)$ (2) $\tan^{-1}(2)$

(3) $\tan^{-1}\left(\dfrac{1}{3}\right)$ (4) $\tan^{-1}(3)$

[JEE (Main) – 28th July 2022 - Shift-2

30. A slab of dielectric constant K has the same cross–sectional area as the plates of a parallel plate capacitor and thickness $\dfrac{3}{4}d$, where d is the separation of the plates. The capacitance of the capacitor when the slab is inserted between the plates will be: (Given Co = capacitance of capacitor with air as medium between plates.)

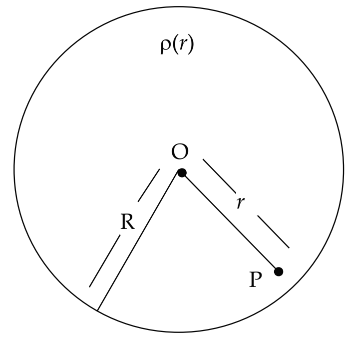
(1) $\dfrac{4KC_0}{3+K}$ (2) $\dfrac{3KC_0}{3+K}$ (3) $\dfrac{3+K}{4KC_0}$ (4) $\dfrac{K}{4+K}$

[JEE (Main) – 28th July 2022 - Shift-2]

31. A spherically symmetric charge distribution is considered with charge density varying as

$$\rho(r)=\begin{cases}\rho_0\left(\dfrac{3}{4}-\dfrac{r}{R}\right) & \text{for } r\le R\\ \text{Zero} & \text{for } r>R\end{cases}$$

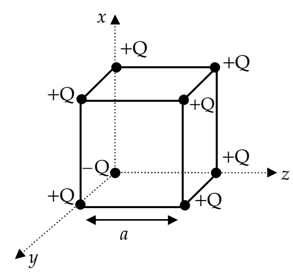
Where, r (r < R) is the distance from the center O (as shown in figure). The electric field at point P will be:



(1) $\dfrac{\rho_0 r}{4\varepsilon_0}\left(\dfrac{3}{4}-\dfrac{r}{R}\right)$ (2) $\dfrac{\rho_0 r}{3\varepsilon_0}\left(\dfrac{3}{4}-\dfrac{r}{R}\right)$

(3) $\dfrac{\rho_0 r}{4\varepsilon_0}\left(1-\dfrac{r}{R}\right)$ (4) $\dfrac{\rho_0 r}{5\varepsilon_0}\left(1-\dfrac{r}{R}\right)$

32. A cube of side 'a' has point charges +Q located at each of its vertices except at the origin where the charge is – Q. The electric field at the centre of cube is:



(1) $\dfrac{2Q}{3\sqrt{3}\pi\varepsilon_0 a^2}\left(\hat{x}+\hat{y}+\hat{z}\right)$ (2) $\dfrac{Q}{3\sqrt{3}\pi\varepsilon_0 a^2}\left(\hat{x}+\hat{y}+\hat{z}\right)$

(3) $\dfrac{-2Q}{3\sqrt{3}\pi\varepsilon_0 a^2}\left(\hat{x}+\hat{y}+\hat{z}\right)$ (4) $\dfrac{-Q}{3\sqrt{3}\pi\varepsilon_0 a^2}\left(\hat{x}+\hat{y}+\hat{z}\right)$

[JEE (Main) – 24th Feb 2021 - Shift-1]

33. Two electrons each are fixed at a distance '$2d$'. A third charge proton placed at the midpoint is displaced slightly by a distance x (x<d) perpendicular to the line joining the two fixed charges. Proton will execute simple harmonic motion having angular frequency:

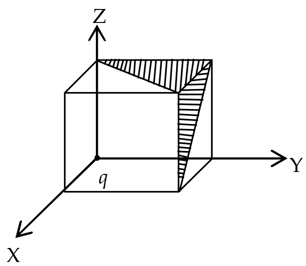
(m = mass of charged particle)

(1) $\left(\dfrac{q^2}{2\pi\varepsilon_0 md^3}\right)^{\frac{1}{2}}$ (2) $\left(\dfrac{\pi\varepsilon_0 md^3}{2q^2}\right)^{\frac{1}{2}}$

(3) $\left(\dfrac{\pi\varepsilon_0 md^3}{q^2}\right)^{\frac{1}{2}}$ (4) $\left(\dfrac{2q^2}{\pi\varepsilon_0 md^3}\right)^{\frac{1}{2}}$

[JEE (Main) – 24th Feb 2021 - Shift-2]

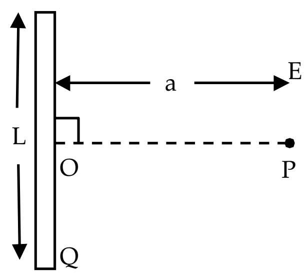
34. A charge 'q' is placed at one corner of a cube as shown in figure. The flux of electrostatic field $\vec{E}$ though the shaded area is:



(1) $\frac{q}{48\varepsilon_0}$ **(2)** $\frac{q}{8\varepsilon_0}$ **(3)** $\frac{q}{24\varepsilon_0}$ **(4)** $\frac{q}{4\varepsilon_0}$

[JEE (Main) – 25th Feb 2021 - Shift-2]

35. Find the electric field at point P (as shown in figure) on the perpendicular bisector of a uniformly charged thin wire of length L carrying a charge Q. The distance of the point P from the centre of the rod is $a = \frac{\sqrt{3}}{2}L$.



(1) $\frac{Q}{2\sqrt{3}\pi\varepsilon_0 L^2}$ **(2)** $\frac{\sqrt{3}Q}{4\pi\varepsilon_0 L^2}$

(3) $\frac{Q}{3\pi\varepsilon_0 L^2}$ **(4)** $\frac{Q}{4\pi\varepsilon_0 L^2}$

[JEE (Main) – 26th Feb 2021 - Shift-1]

36. Given below are two statements :
Statement – I : An electric dipole is placed at the centre of a hollow sphere. The flux of electric field through the sphere is zero but the electric field is not zero anywhere in the sphere.
Statement – II : If R is the radius of a solid metallic sphere and Q be the total charge on it. The electric field at any point on the spherical surface of radius r (< R) is zero but the electric flux passing through this closed spherical surface of radius r is not zero.
In the light of the above statements. Choose the correct answer from the option given below :
(1) Statement I is true but Statement II is false.
(2) Statement I is false but Statement II is true.
(3) Both Statement I and Statement II are true.
(4) Both Statement I and Statement II are false.

[JEE (Main) – 26th Feb 2021 - Shift-2]

37. A certain charge Q is divided into two parts q and (Q - q). How should the charges Q and q be divided so that q and (Q - q) placed at a certain distance apart experience maximum electrostatic repulsion?

(1) $Q = \frac{q}{2}$ **(2)** $Q = 3q$ **(3)** $Q = 2q$ **(4)** $Q = 4q$

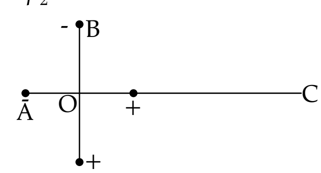
[JEE (Main) – 20th July 2021 - Shift-1]

38. An electric dipole is placed on x-axis in proximity to a line charge of linear charge density 3.0×10^{-6} C/m. Line charge is placed on z - axis and positive and negative charge of dipole is at a distance of 10 mm and 12 mm from the origin respectively. If total force of 4 N is exerted on the dipole, find out the amount of positive or negative charge of the dipole.

(1) 815.1 nC **(2)** 8.8 μC
(3) 0.485 mC **(4)** 4.44 μC

[JEE (Main) – 22nd July 2021 - Shift-2]

39. Two ideal electric dipoles A and B having their dipole moment p_1 and p_2 respectively are placed on a plane with their centres at O as shown in the figure. At point C on the axis of dipole A, the resultant electric field is making an angle of 37° with the axis. The ratio of the dipole moment of A and B, $\frac{p_1}{p_2}$ is: (take sin 37° = $\frac{3}{5}$)



(1) $\frac{3}{2}$ **(2)** $\frac{2}{3}$ **(3)** $\frac{3}{8}$ **(4)** $\frac{4}{3}$

[JEE (Main) – 25th July 2021 - Shift-2]

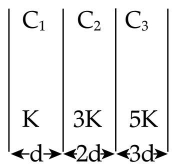
40. Two identical tennis balls each having mass 'm' and charge 'q' are suspended from a fixed point by threads of length 'l'. What is the equilibrium separation when each thread makes a small angle 'θ' with the vertical?

(1) $x = \left(\frac{q^2 l}{2\pi\varepsilon_0 mg}\right)^{\frac{1}{2}}$ **(2)** $x = \left(\frac{q^2 l^2}{2\pi\varepsilon_0 m^2 g^2}\right)^{\frac{1}{3}}$

(3) $x = \left(\frac{q^2 l}{2\pi\varepsilon_0 mg}\right)^{\frac{1}{3}}$ **(4)** $x = \left(\frac{q^2 l^2}{2\pi\varepsilon_0 m^2 g}\right)^{\frac{1}{3}}$

[JEE (Main) – 26th July 2021 - Shift-2]

41. In the reported figure, a capacitor is formed by placing a compound dielectric between the plates of parallel plate capacitor. The expression for the capacity of the said capacitor will be : (Given area of plate = A)

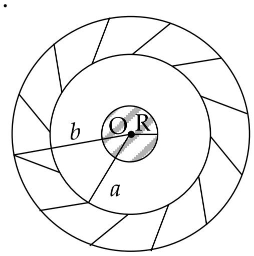


(1) $\frac{25}{6}\frac{K\varepsilon_0 A}{d}$ (2) $\frac{15}{34}\frac{K\varepsilon_0 A}{d}$

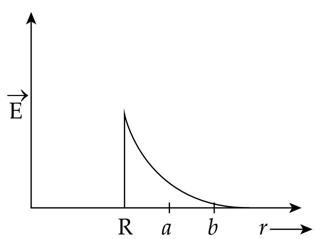
(3) $\frac{15}{6}\frac{K\varepsilon_0 A}{d}$ (4) $\frac{9}{6}\frac{K\varepsilon_0 A}{d}$

[JEE (Main) – 27th July 2021 - Shift-1]

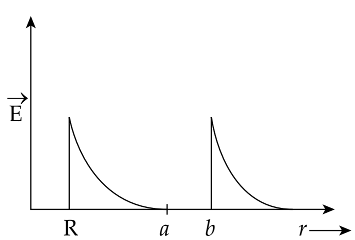
42. A solid metal sphere of radius R having charge q is enclosed inside the concentric spherical shell of inner radius a and outer radius b as shown in figure. The approximate variation of electric field $\vec{E}$ as a function of distance r from centre O is given by:



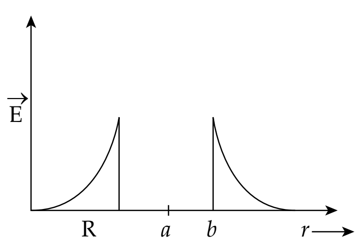
(1)



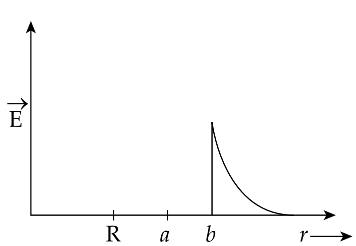
(2)



(3)



(4)



[JEE (Main) – 26th Aug 2021 - Shift-1]

43. The two thin coaxial rings, each of radius 'a' and having charges +Q and –Q respectively are separated by a distance of 's'. The potential difference between the centre of the two rings is:

(1) $\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a}+\frac{1}{\sqrt{s^2+a^2}}\right]$ (2) $\frac{Q}{2\pi\varepsilon_0}\left[\frac{1}{a}-\frac{1}{\sqrt{s^2+a^2}}\right]$

(3) $\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a}+\frac{1}{\sqrt{s^2+a^2}}\right]$ (4) $\frac{Q}{2\pi\varepsilon_0}\left[\frac{1}{a}+\frac{1}{\sqrt{s^2+a^2}}\right]$

[JEE (Main) – 26th Aug 2021 - Shift-2]

44. A uniform charged disc of radius R having surface charge density σ is placed in the xy plane with its center at the origin. Find the electric field intensity along the z-axis at a distance Z from origin:

(1) $E=\frac{\sigma}{2\varepsilon_0}\left(1+\frac{Z}{(Z^2+R^2)^{\frac{1}{2}}}\right)$

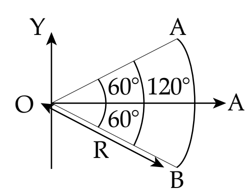
(2) $E=\frac{\sigma}{2\varepsilon_0}\left(1-\frac{Z}{(Z^2+R^2)^{\frac{1}{2}}}\right)$

(3) $E=\frac{2\varepsilon_0}{\sigma}\left(\frac{1}{(Z^2+R^2)^{\frac{1}{2}}}+Z\right)$

(4) $E=\frac{\sigma}{2\varepsilon_0}\left(\frac{Z}{(Z^2+R^2)}+\frac{1}{Z^2}\right)$

[JEE (Main) – 27th Aug 2021 - Shift-1]

45. Figure shows a rod AB, which is bent in a 120° circular arc of radius R. A charge (–Q) is uniformly distributed over rod AB. What is the electric field $\vec{E}$ at the centre of curvature O?

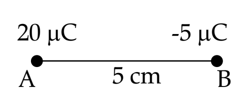


(1) $\frac{3\sqrt{3}Q}{8\pi\varepsilon_0 R^2}(\hat{i})$ (2) $\frac{3\sqrt{3}Q}{8\pi^2\varepsilon_0 R^2}(\hat{i})$

(3) $\frac{3\sqrt{3}Q}{16\pi^2\varepsilon_0 R^2}(\hat{i})$ (4) $\frac{3\sqrt{3}Q}{8\pi^2\varepsilon_0 R^2}(-\hat{i})$

[JEE (Main) – 27th Aug 2021 - Shift-2]

46. Two particles A and B having charges 20 μC and –5 μC respectively are held fixed with a separation of 5 cm. At what position a third charged particle should be placed so that it does not experience a net electric force?



(1) At 5 cm from 20 μC on the left side of system.
(2) At midpoint between two charges.
(3) At 1.25 cm from a – 5 μC between two charges.
(4) At 5 cm from –5 μC on the right side.

[JEE (Main) – 31st Aug 2021 - Shift-1]

47. Choose the incorrect statement:

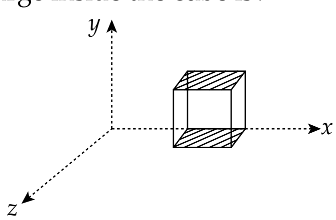
(a) The electric lines of force entering into Gaussian surface provide negative flux.
(b) A charge 'q' is placed at the centre of a cube. The flux through all the faces will be the same.
(c) In a uniform electric field, net flux through a closed Gaussian surface containing no net charge, is zero.
(d) When electric field is parallel to a Gaussian surface, it provides a finite non-zero flux.

Choose the most appropriate answer from the options given below:

(1) (d) Only **(2)** (b) and (d) Only
(3) (a) and (c) Only **(4)** (c) and (d) Only

[JEE (Main) – 31st Aug 2021 - Shift-2]

48. A cube is placed inside an electric field, $\vec{E} = 150y^2\hat{j}$. The side of the cube is 0.5 m and is placed in the field as shown in the given figure. The charge inside the cube is:

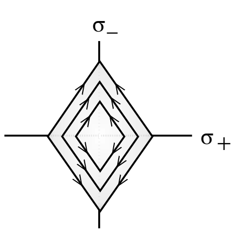


(1) 3.8×10^{-12} C **(2)** 3.8×10^{-11} C
(3) 8.3×10^{-12} C **(4)** 8.3×10^{-11} C

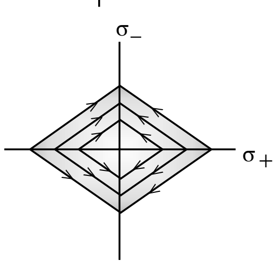
[JEE (Main) – 1st Sep 2021 - Shift-2]

49. Two charged thin infinite plane sheets of uniform surface charge density σ_+ and σ_-, where $|\sigma_+| > |\sigma_-|$, intersect at the right angle. Which of the following best represents the electric field lines for the system :

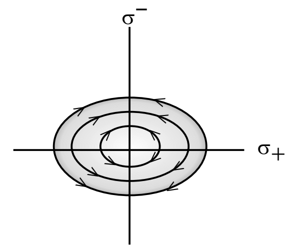
(1)



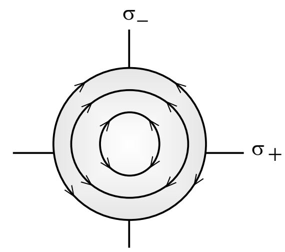
(2)



(3)



(4)



[JEE (Main) – 4th Sep. 2020 - Shift-1]

50. Three charges $+Q, q, +Q$ are placed respectively, at distance, 0, $\frac{d}{2}$ and d from the origin, on the x-axis. If the net force experienced by $+Q$, placed at $x = 0$, is zero, then value of q is :

(1) $-\frac{Q}{4}$ (2) $+\frac{Q}{2}$ **(3)** $+\frac{Q}{4}$ **(4)** $-\frac{Q}{2}$

[JEE (Main) – 9th Jan 2019 - Shift-1]

51. A dipole is formed by two charges of 5μc and –5μc at a distance of 8 mm. The electric field at a point 20 cm away on its, a line perpendicular to the axis and passing through its centre is:

(1) 4.6×10^4 N/C **(2)** -3.5×10^4 N/C

(3) -4.5×10^4 N/C **(4)** None

52. The work done in bringing a 20 coulomb charge from Point A to Point B for distance 0.2 m is 2 Joule. The Potential difference between two points will be (in volt:

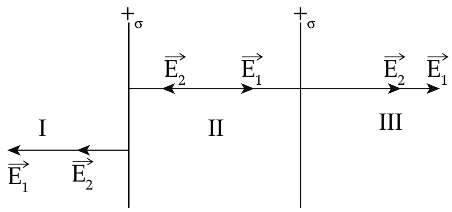
(1) 8 **(2)** 0.2 **(3)** 0.1 **(4)** 0.4

ANSWER – KEY

1. (3)	**2. (3)**	**3. (1)**	**4. (3)**
5. (2)	**6. (3)**	**7. (3)**	**8. (2)**
9. (2)	**10. (4)**	**11. (4)**	**12. (2)**
13. (3)	**14. (2)**	**15. (2)**	**16. (Bonus)**
17. (4)	**18. (1)**	**19. (1)**	**20. (3)**
21. (2)	**22. (2)**	**23. (1)**	**24. (1)**
25. (2)	**26. (3)**	**27. (1)**	**28. (2)**
29. (2)	**30. (1)**	**31. (3)**	**32. (3)**
33. (1)	**34. (3)**	**35. (1)**	**36. (1)**
37. (3)	**38. (4)**	**39. (2)**	**40. (3)**
41. (2)	**42. (2)**	**43. (2)**	**44. (2)**
45. (2)	**46. (4)**	**47. (1)**	**48. (4)**
49. (2)	**50. (1)**	**51. (3)**	**52. (3)**

ANSWERS WITH EXPLANATIONS

1. Option (3) is correct.



Let $\hat{n}$ is unit vector along Normal towards right of sheet.

At region I, $E_1 = \frac{\sigma}{2\varepsilon_0} \Rightarrow E_2 = \frac{\sigma}{2\varepsilon_0}$

$$E_I = E_1 + E_2 = \frac{(\sigma)}{\varepsilon_0}$$

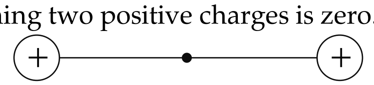
$$\vec{E}_I = \frac{\sigma}{\varepsilon_0}(-\hat{n})$$

$$E_{II} = E_1 - E_2 = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$

$$E_{III} = E_1 + E_2 = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0} \Rightarrow \vec{E}_{III} = \frac{\sigma}{\varepsilon_0}\hat{n}$$

2. Option (3) is correct.

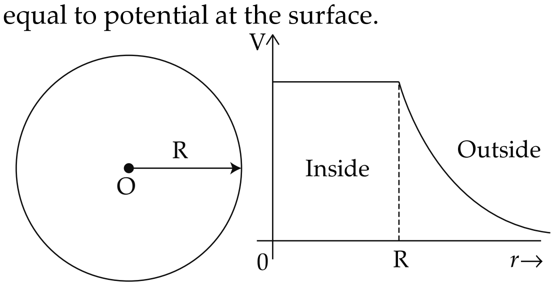
Electric field is a vector quantity and its value could be zero because of group of +ve charges. For example, Electric field at the mid point of line joining two positive charges is zero.



But the potential is a scalar quantity and due to group of +ve charges it would be never zero as it would be added algebraically.

3. Option (1) is correct.

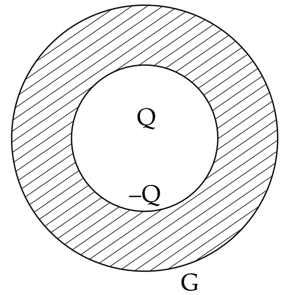
Potential inside the sphere is constant which is equal to potential at the surface.



$$V_{inside} = V_{surface} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R} \text{ and } V_{outside} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

4. Option (3) is correct.

Applying Gauss's law for all the regions, we get Note that and –Q and +Q appear on the inner and the outer walls of the conductor due to induction.



For $r < a$, $E_1(4\pi r^2) = \frac{Q}{\varepsilon_0}$

$$E_I = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_0^2} \neq 0$$

For $a < r < b E_{II}(4\pi r^2) = 0$

$$E_{II} = 0$$

For $r > b$, $E_{III}(4\pi r^2) = \frac{Q}{\varepsilon_0}$

$$E_{III} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \neq 0$$

5. Option (2) is correct.

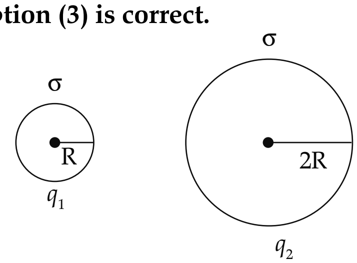
$$\overline{E} = \frac{A}{x^2}\hat{\imath} + \frac{B}{y^2}\hat{j}$$

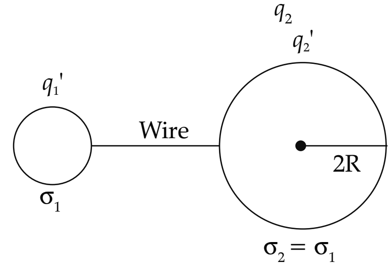
Unit of $\vec{E}$ is N C^{-1}

Unit of A is N m^2 C^{-1}

Unit of B is N m^3 C^{-1}

6. Option (3) is correct.





Let q, and q_2 be the charges before connection, and q_1' and q_2' be the charges after connection.

Since total charge on the system is conserved

$q_1 + q_2 = \rho 4\pi(R^2 + 4R^2) = 20\pi\rho R^2 = q_1' + q_2'$ .. (i)

Also after connection, the potentials of both spheres become identical

$$\frac{Kq_1'}{R} = \frac{Kq_2'}{2R}$$

$$\Rightarrow \quad q_1^1 = \frac{q_2^1}{2} \quad \ldots\text{(ii)}$$

Putting value from eq. (ii) in (i), we get

$$\frac{q_2^1}{2} + q_2^1 = 20\rho R^2 \Rightarrow q_2^1 = \frac{40}{3}\pi\sigma R^2$$

$$\rho_2 = \frac{q_2^1}{4\pi(2R)^2}$$

$$= \frac{\frac{40}{3}\pi\sigma R^2}{16\pi R^2}$$

$$= \frac{5}{6}\sigma = \sigma'$$

$$\frac{\sigma^1}{\sigma} = \frac{5}{6}$$

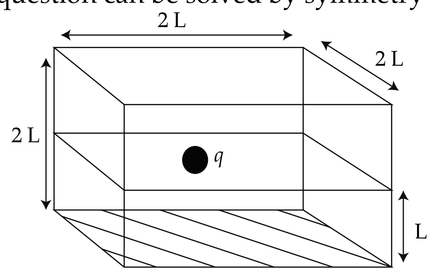
7. **Option (3) is correct.**
Work done by electric field

$$W = \vec{F}.\vec{S} = q\vec{E}.\vec{S}$$

$$= 2 \times 10^{-2}(30\,\hat{i}).(-\hat{i} - 2\hat{j})$$

$$= -60 \times 10^{-2} = -600 \text{ mJ}$$

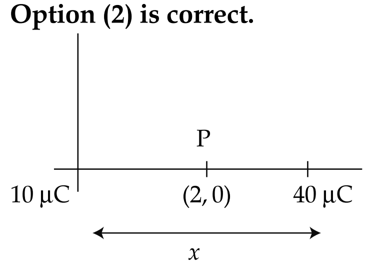
8. **Option (2) is correct.**
This question can be solved by symmetry



Now, $Q = \dfrac{\frac{q}{\varepsilon_0}}{6}$

So, the flux passes through the shaded face $= \dfrac{q}{6\varepsilon_0}$

9. **Option (2) is correct.**



Let $q = 40\ \mu C$ be placed at a distance x from the origin for $\overrightarrow{E_p} = 0, \overrightarrow{E_{10}} = -\overrightarrow{E_{40}}$

$$|\overrightarrow{E_{10}}| = |\overrightarrow{E_{40}}|$$

$$\frac{K(10)}{(2)^2} = \frac{K(40)}{(x-2)^2}$$

$$16 = (x-2)^2$$

$$x - 2 = 4$$

$$x = 6 \text{ cm}$$

10. **Option (4) is correct.**
At an equatorial point,

$$E = \frac{kp}{r^3} \Rightarrow E \propto \frac{1}{r^3}$$

11. **Option (4) is correct.**
If Q is divided into q and $Q - q$ and kept at a separation r, $F = \dfrac{kq(Q-q)}{r^2}$

for F_{max}, $\dfrac{dF}{dq} = 0 = Q - 2q$

$$q = \frac{10}{2} = 5\ \mu C$$

$$Q - q = 10\ \mu C - 5\ \mu C = 5\ \mu C$$

12. **Option (2) is correct.**
Torque on a dipole in a uniform electric field is given by $\tau = pE\sin\theta$
$p = ql = 0.01 \times 0.4 \times 10^{-3} = 4 \times 10^{-6}$ cm
$\tau = 4 \times 10^{-6} \times 10 \times 10^{-5} \times \sin 30^{\circ} = 2 \times 10^{-10}$ N m

13. **Option (3) is correct.**
Dipole is made of equal and opposite charges, so net charge of surface containing dipole is zero.

So, $\phi = \dfrac{q}{\varepsilon_0} = \dfrac{0}{\varepsilon_0} = 0$

Hence both statements are correct and R is the correct explanation of Assertion.

14. **Option (2) is correct.**
As per Gauss law, $\phi = \dfrac{q_{in}}{\varepsilon_0} = \dfrac{CV}{\varepsilon_0}$

15. **Option (2) is correct.**
Energy stored in capacitor, $E_1 = \dfrac{1}{2}CV^2$
When charged capacitor is connected to uncharged capacitor, flow of charges will take place until both capacitors are at same potential.
New potential

$$V' = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{CV + 0}{2C} = \frac{V}{2}$$

$$C' = C + C = 2C$$

Energy stored in combination, $E_2 = \dfrac{1}{2}C'V'^2$

$$\Rightarrow \quad E_2 = \frac{1}{2} \times 2C \times \left(\frac{V}{2}\right)^2 = \frac{1}{4}CV^2$$

Now $\dfrac{E_2}{E_1} = \dfrac{\frac{1}{4}CV^2}{\frac{1}{2}CV^2} = \dfrac{1}{2}$

16. **None of the given Options (BONUS) is correct.**
Here mass of two charges in dipole is not same, hence we need to find its location.
Centre of mass will be at $\dfrac{l}{3}$ from positive charge and $\dfrac{2l}{3}$ from negative charge.

M.I. about axis, $I = 2m\left(\frac{l}{3}\right)^2 + m\left(\frac{2l}{3}\right)^2$

$\Rightarrow$ $I = 2\frac{ml^2}{3}$ Now, $pE = I\omega^2$

$$\omega = \sqrt{\frac{pE}{I}} = \sqrt{\frac{qlE}{\frac{2ml^2}{3}}} = \sqrt{\frac{3qE}{2ml}}$$

None of the option matches with the obtained value.

17. Option (4) is correct.

For a charged spherical shell, the potential inside the shell is given by, $V = \frac{KQ}{R}$, which is constant inside the shell.

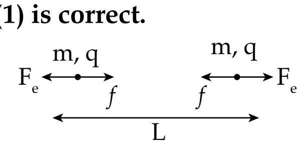
And potential due to charged spherical shell at a point outside the shell is given by,

$$V = \frac{KQ}{r} \text{ i.e. } V \propto \frac{1}{r}$$

It means potential decreases with increase in distance outside the given shell.

Therefore graph (4) represent the correct relation of distance and potential.

18. Option (1) is correct.



Since each mass is in equilibrium, the electrostatic repulsion force is balanced by the force of frictions

$$F_e = f$$

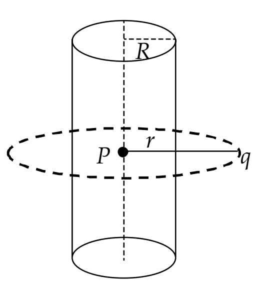
$$\frac{kq^2}{L^2} = \mu mg$$

$$L^2 = \frac{kq^2}{\mu mg} = \frac{9\times10^9\times4\times10^{-14}}{0\cdot25\times0\cdot01\times10}$$

$$L^2 = \frac{36}{0\cdot25\times0\cdot01}\times10^{-6}$$

$$L = \frac{6\times10^{-3}}{0\cdot5\times0\cdot1} = \frac{6\times10^{-1}}{5} = 0\cdot12\text{m} = 12\text{cm}$$

19. Option (1) is correct.



For q to move in a circular path $qE = \frac{mv^2}{r}$

Where r is the radius of the circular path

Applying Gauss's law, $E(2\pi rl) = \frac{q_{encl}}{\varepsilon_0} = \frac{\rho\pi R^2 l}{\varepsilon_0}$

$$E = \frac{\rho R^2}{2\varepsilon_0 r}$$

Hence $\frac{q\rho R^2}{2\varepsilon_0 r} = \frac{mv^2}{r}$

$$K\cdot E = \frac{1}{2}mv^2 = \frac{q\rho R^2}{4\varepsilon_0}$$

20. Option (3) is correct.

For an infinite sheet of charge,

$$E = \frac{\sigma}{2\varepsilon_0} = \text{constant} \therefore\ E_1 = E_2 = \frac{\sigma}{2\varepsilon_0}$$

21. Option (2) is correct.

Surface charge density of a spherical conductor is given by,

$$\sigma = \frac{q}{4\pi r^2}$$

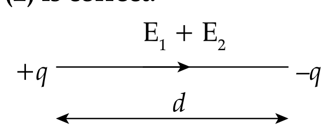
When all smaller drops combine, the radius of the bigger drop is given by $\frac{4}{3}\pi R^3 = 64\left(\frac{4}{3}\pi r^3\right)$

$$R^3 = 64r^3$$

$$R = 4r$$

$$\sigma' = \frac{64q}{4\pi(4r)^2} = 4\sigma \text{ Hence, } \frac{\sigma'}{\sigma} = 4:1$$

22. Option (2) is correct.



Net electric field at the midpoint $E = E_1 + E_2$

Here $E_1 = E_2 = \frac{kq}{(d/2)^2} = \frac{4kq}{d^2}$

$$E = \frac{8kq}{d^2}$$

$$d^2 = \frac{8kq}{E} = \frac{8\times9\times10^9\times8\times10^{-6}}{6.4\times10^4} = \frac{72\times8}{64}$$

$$d^2 = 9$$

$\therefore$ $d = 3$ m

23. Option (1) is correct.

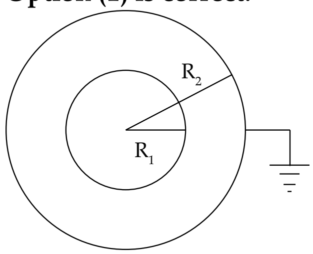
For a condenser $V \propto q$

$$V = \frac{q}{C}$$

At $q_1 = 0$, $V_1 = V_0$

At $q_2 = 5$ C, $V_2 = \dfrac{5}{2\times10^{-6}} = 2.5 \times 10^6$ V

24. Option (1) is correct.



$$C = 4\pi \in_0 \frac{R_1R_2}{R_2 - R_1} = n\cdot 4\pi \in_0 R_1$$

$$\frac{R_2}{R_2 - R_1} = n$$

$$R_2 = nR_2 - nR_1$$

$$R_2(n-1) = nR_1$$

$$\frac{R_2}{R_1} = \frac{n}{n-1}$$

25. Option (2) is correct

Charges will flow until both conductors have same potential. Hence, $V_1 = V_2$

$$\frac{kQ_1}{r_1} = \frac{kQ_2}{r_2} \Rightarrow \frac{Q_1}{Q_2} = \frac{5}{10} = \frac{1}{2}$$

Electric field intensity on the surface of a spherical conductor is given by $E = \dfrac{kQ}{r^2}$

$$\Rightarrow \frac{E_1}{E_2} = \frac{kQ_1}{kQ_2}\times\frac{r_2^2}{r_1^2} \Rightarrow \frac{E_1}{E_2} = \frac{1}{2}\times\frac{10^2}{5^2} = \frac{100}{50}$$

$$\Rightarrow E_1 : E_2 = 2:1$$

26. Option (3) is correct.

Case 1 : Switch is closed and without dielectric

$$E_1 = \frac{1}{2}C_{eq}V^2 = \frac{1}{2}\times 2C\times V^2 = CV^2$$

Case 2 : Switch is open and dielectric is inserted , the new capacitance for A is 5C and it is still connected to the battery. Whereas the new capacitance for B is also 5C but it is now disconnected from the battery.

So, charge on B remains same as in case 1.

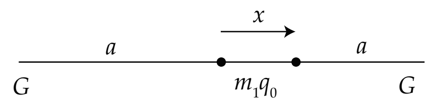
$$E_2 = \frac{1}{2}C_{eq}V^2 + \frac{1}{2}\frac{Q^2}{C_{eq}}$$

$$E_2 = \frac{1}{2}\times 5C\times V^2 + \frac{1}{2}\times\frac{(CV)^2}{5C}$$

$$E_2 = \frac{1}{2}5CV^2 + \frac{1}{2}\frac{C^2V^2}{5C} = \frac{13}{5}CV^2$$

$$\frac{E_1}{E_2} = \frac{CV^2}{13/5CV^2} = 5:13$$

27. Option (1) is correct.



Let q be displaced towards right by x ($x << a$)

Restoring force

$$F = -KQq_0\left[\frac{1}{(a-x)^2} - \frac{1}{(a+x)^2}\right] = mA$$

Where A is the acceleration,

$$A = -\frac{KQq_0}{m}\left[\frac{4ax}{(a^2-x^2)^2}\right]$$

$$A = \frac{-KQq_0}{m}\frac{4ax}{a^4} \quad as\ x << a\ (a^2 - x^2) \approx a^2$$

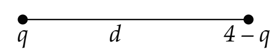
$$A = -\frac{4KQq_0x}{ma^3} = -\omega^2x$$

$$\omega = \sqrt{\frac{Qq_0}{\pi\varepsilon_0 ma^3}} = \frac{2\pi}{T}$$

$$T = 2\pi\sqrt{\frac{\pi\varepsilon_0 ma^3}{Qq_0}} = \sqrt{\frac{4\pi^3\varepsilon_0 ma^3}{Qq_0}}$$

28. Option (2) is correct.

Let the two parts be q and (4 – q) separated by a given distance



$$F = \frac{kq(4-q)}{d^2}$$

For F to be maximum $\dfrac{dF}{dq} = 4 - 2q = 0$

$$q = 2\ \mu C$$

$$4 - q = 2\ \mu C$$

29. Option (2) is correct

Let's split the motion of particle in two directions.

In x direction

$u_x = 2$ ms^{-1} $a_x = 0$ $s_x = 1$

From 1st equation of motion, $v_x = u_x + a_xt$

$v_x = 2 + 0 \times t = 2$ ms^{-1}

From 2nd equation of motion, $s_x = u_xt + \frac{1}{2}a_xt^2$

$$1 = 2\times t + \frac{1}{2}\times 0\times t^2$$

$$t = \frac{1}{2}\ or\ 0.5\,s$$

Force on a particle placed in an electric field, F = $qE = ma$

$$a_y = \frac{eE}{m} = \frac{e\times\left(\frac{8m}{e}\right)}{m} = 8\ ms^{-2}$$

Similarly, $v_y = u_y + a_y t = 0 + 8 \times 0.5 = 4\ \text{ms}^{-1}$

$$\tan\theta = \frac{v_y}{v_x} = \frac{4}{2} = 2$$

$$\theta = \tan^{-1} 2$$

30. Option (1) is correct

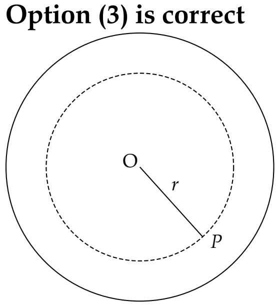
Original capacitance of capacitor, $C_0 = \dfrac{\varepsilon_0 A}{d}$

When dielectric is inserted

$$C = \frac{\varepsilon_0 A}{\dfrac{d}{4} + \dfrac{3d}{4}K} = \frac{4\varepsilon_0 A}{d(1+\dfrac{3}{K})}$$

$$C = \frac{4C_0 K}{3+K}$$

31. Option (3) is correct



Given, Charge density,

$$\rho(r) = \begin{cases} \rho_0\left(\dfrac{3}{4} - \dfrac{r}{R}\right) & \text{for } r \le R \\ \text{zero} & \text{for } r > R \end{cases}$$

Let us take a differential shell of thickness dr, at distance r from the centre

$dV = 4\pi r^2 . dr$

$$\phi = E.ds = \frac{q_{in}}{\varepsilon_0} = E \times 4\pi r^2 = \frac{q_{in}}{\varepsilon_0} \qquad ...(i)$$

$q_{in} = \rho\ dV = \rho_0\left(\dfrac{3}{4} - \dfrac{r}{R}\right) \times 4\pi r^2 dr$ Put in Eq. (i)

$$E.4\pi r^2 = \frac{\int_0^r \rho_o\left(\dfrac{3}{4} - \dfrac{r}{R}\right) 4\pi r^2 dr}{\varepsilon_o}$$

$$E.4\pi r^2 = \rho_o \frac{4\pi}{\varepsilon_0}\left(\frac{3r^3}{4\times 3} - \frac{r^4}{4R}\right)$$

$$E.r^2 = \rho_o \frac{r^3}{4\varepsilon_0}\left\{1 - \frac{r}{R}\right\} \Rightarrow E = \rho_o \frac{r}{4\varepsilon_o}\left[1 - \frac{r}{R}\right]$$

32. Option (3) is correct.

For the reduction of the problem, we can replace –Q charge at origin by +Q and –2Q.

Now, the electric field due to +Q charge of the corners at the centre of the cube will be zero.

Hence, the net electric field at the centre due to –2Q charge of the origin will work.

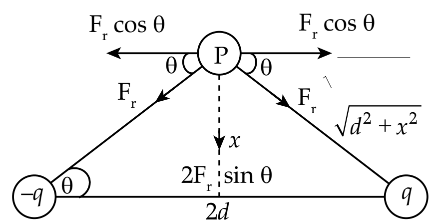
$$\vec{E} = \frac{kq\vec{r}}{r^3} = \frac{1(-2Q)\dfrac{a}{2}(\hat{x}+\hat{y}+\hat{z})}{4\pi\varepsilon_0\left(\dfrac{a}{2}\sqrt{3}\right)^3}$$

$$\vec{E} = \frac{-2Q(\hat{x}+\hat{y}+\hat{z})}{3\sqrt{3}\pi a^2 \varepsilon_0}$$

33. Option (1) is correct.

We can depict the given situation in the following diagram:

Now, taking the component of forces.



The restoring force will be

$$= 2F_r \sin\theta$$

$$= \frac{2kq^2}{(d^2+x^2)} \times \frac{x}{\sqrt{d^2+x^2}} = \frac{2kq^2 x}{(d^2+x^2)^{\frac{3}{2}}}$$

As $x << d$

So, $x^2 \simeq 0$

$$kx = \frac{2kq^2 x}{d^3}$$

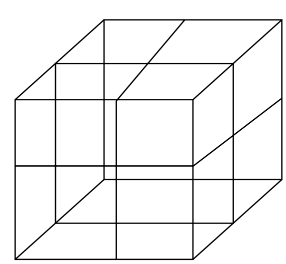
$$k = \frac{2kq^2}{d^3} = 2\times\frac{1}{4\pi\varepsilon_0}\frac{q^2}{d^3}$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{q^2}{2\pi m\varepsilon_0 d^3}}$$

34. Option (3) is correct.

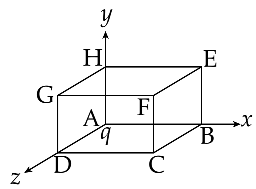
Let this large cube formed with 8 small cube.

And charge q is placed at one corner of smaller cube.



Total flux through the cube $\phi = \dfrac{q}{\varepsilon_0}$

So, total flux through 1 cube $= \dfrac{q}{8\varepsilon_0}$



As the three adjacent surface are perpendicular, So, flux through ABEH, ADGH and ABCD will be zero.

So, ϕ (EFGH) $= \left(\dfrac{q}{8\varepsilon_0}\right)\times\dfrac{1}{3}$

ϕ (EFCB) $= \dfrac{q}{8\varepsilon_0}\times\dfrac{1}{3}$

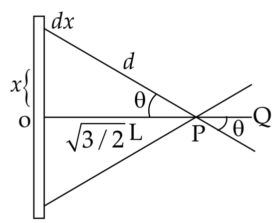
Hence, the total flux through the shaded portion

$$\phi = \frac{\phi(\text{EFGH})}{2}+\frac{\phi(\text{EFCB})}{2}=\frac{1}{2}\left(\frac{q}{24\varepsilon_0}+\frac{q}{24\varepsilon_0}\right)$$

$$= \frac{q}{24\varepsilon_0}$$

35. Option (1) is correct.

Let the length of wire be L, charge on it be Q



So, $\rho = \dfrac{Q}{L}$

$dQ = \rho.dx$

$= \dfrac{Q}{L}.dx$

$dE_1 = \dfrac{dQ}{4\pi\varepsilon_0.d^2}$

Now, taking vertical and horizontal component of electric field,

$dE = 2dE_1 \cos\theta$

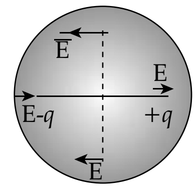
$$\int dE = \int 2\frac{dQ}{4\pi\varepsilon_0}d^2\,\frac{\frac{\sqrt{3}}{2}L}{\sqrt{d}}$$

$$= \frac{2\times\sqrt{3}}{2}k\int_{-L/2}^{+L/2}\frac{Q}{L}\frac{dx}{\left(\sqrt{\frac{3}{4}L^2+x^2}\right)^{\frac{3}{2}}}$$

$$E_{net} = \frac{Q}{2\sqrt{3}\pi\varepsilon_0 L^2}$$

36. Option (1) is correct.

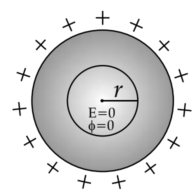
Given,



Electric flux $(\phi) = \dfrac{Q_{enclosed}}{\varepsilon_0} = \dfrac{q-q}{\varepsilon_0} = 0$

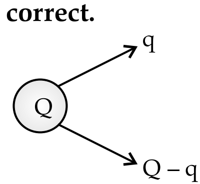
So, we can conclude that electric flux in sphere will be zero but electric field will not be zero.

Hence, statement (I) is the correct answer.



So, we can conclude that statement II is incorrect.

37. Option (3) is correct.



$$F = \frac{kq(Q-q)}{r^2}$$

$\dfrac{dF}{dq} = 0$ for maxima and minima

$$\frac{k}{r^2}[-2q+Q] = 0$$

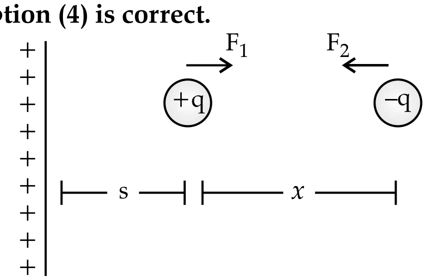
Now, $\left(\dfrac{df^2}{dq^2}\right) < 0$

$Q = 2q$

Hence, for maximum force

$Q = 2q$

38. Option (4) is correct.



$$F_1 = \frac{2k\lambda}{s}q$$

$$F_2 = \frac{2k\lambda}{s+x}q$$

$$F_{net} = F_1 - F_2$$

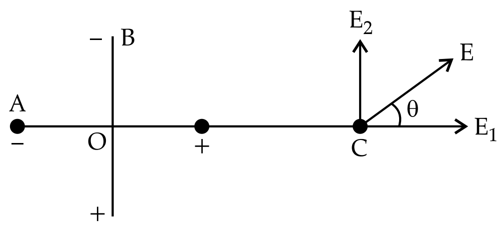
$$= 2k\lambda q\left[\frac{1}{s} - \frac{1}{s+x}\right]$$

$$= 2k\lambda q\left[\frac{(s+x)-s}{s(s+x)}\right]$$

$$4 = \frac{2\times 9\times 10^9 \times 3\times 10^{-6}\times q\times 2\times 10^{-3}}{10\times 12\times 10^{-6}}$$

$q = 4.44\ \mu C$

39. Option (2) is correct.



$\theta = 37°$, $E_1 = E\cos 37°$, $E_2 = E\sin 37°$

$$(E_1)_A = \frac{2kP_1}{r^3}$$

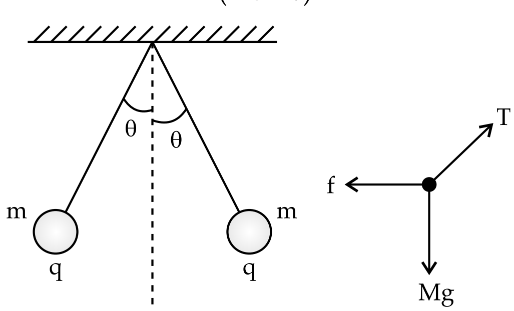
$$(E_2)_B = \frac{kP_2}{r^3}$$

$$\frac{2P_1}{P_2} = \frac{\frac{4}{5}}{\frac{3}{5}} = \frac{4}{3}$$

$$\frac{P_1}{P_2} = \frac{2}{3}$$

40. Option (3) is correct.

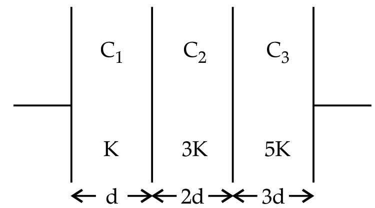
$$f = \frac{kq^2}{(2l\sin\theta)^2}$$



41. Option (2) is correct.

Capacitors are in series

$$\frac{1}{\mathbf{C}} = \frac{1}{\mathbf{C_1}} + \frac{1}{\mathbf{C_2}} + \frac{1}{\mathbf{C_3}}$$



$$C_1 = \frac{kA\varepsilon_0}{d}, C_2 = \frac{3kA\varepsilon_0}{2d}, C_3 = \frac{5kA\varepsilon_0}{3d}$$

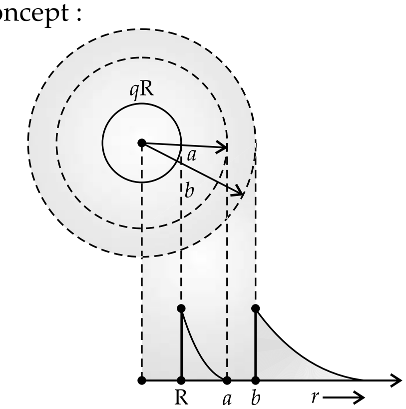
$$\frac{1}{C} = \frac{1}{\frac{kA\varepsilon_0}{d}} + \frac{1}{\frac{3kA\varepsilon_0}{2d}} + \frac{1}{\frac{5kA\varepsilon_0}{3d}}$$

$$\frac{1}{C} = \frac{d}{kA\varepsilon_0} + \frac{2d}{3kA\varepsilon_0} + \frac{3d}{5kA\varepsilon_0}$$

$$C = \frac{15k\varepsilon_0 A}{34d}$$

42. Option (2) is correct.

Concept :



As we know there is no charge inside the conductor.

When we consider innermost sphere

For region or $0 < r < R$, $E = 0$

$$R < r < a,\ E \propto \frac{1}{r^2}$$

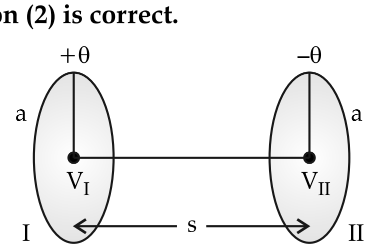
The shell is hollow. So, no charge is there inside the shell.

$$a < r < b \Rightarrow E = 0$$

For region, $r > b$,

$$E \propto \frac{1}{r^2}$$

43. Option (2) is correct.



Potential at centre of Ring I,

$$V_I = \frac{kQ}{a} - \frac{kQ}{\sqrt{a^2 + S^2}}$$

Potential at centre of Ring II,

$$V_{II} = \frac{-kQ}{a} - \frac{kQ}{\sqrt{a^2 + S^2}}$$

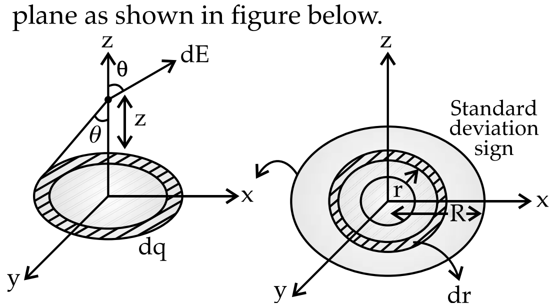
$$\Delta V = V_I - V_{II}$$

$$= \left(\frac{kQ}{a} - \frac{kQ}{\sqrt{a^2+S^2}}\right) - \left(\frac{-kQ}{a} - \frac{kQ}{\sqrt{a^2+S^2}}\right)$$

$$= \frac{Q}{2\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{\sqrt{S^2+a^2}}\right)$$

44. Option (2) is correct.

Concept : Consider a differential ring in $x - y$ plane as shown in figure below.



Only component along z-axis will exist rest will cancel out.

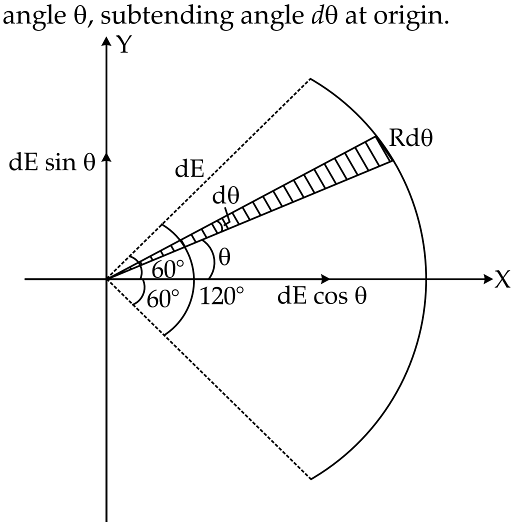
$$dE_z = \frac{k\,dq\,z}{(r^2+z^2)^{3/2}} = \frac{K.\sigma\,2\pi r dr}{(z^2+r^2)^{3/2}}$$

Where $k = \frac{1}{4\pi\varepsilon_0}$

$$E_z = \int_0^R dE_z = \frac{\sigma}{2\varepsilon_0}\left[\frac{1-z}{\sqrt{R^2+z^2}}\right]$$

45. Option (2) is correct.

Concept : Consider a differential element at angle θ, subtending angle $d\theta$ at origin.



let linear charge density be λ

$$\lambda = \frac{-Q}{(R\theta)} = \frac{-Q}{R\frac{2\pi}{3}} \Rightarrow \lambda = \frac{-3Q}{2\pi R}$$

The net field will exist along axis of x, only y-component will cancell each other for the given two halves of ring.

$$E = \int dE\cos\theta$$

$$= \int \frac{1}{4\pi\varepsilon_0}\frac{dQ}{R^2}\cos\theta$$

$$dq = \lambda dx = \lambda R d\theta$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{1}{R^2}\times\lambda\int_{-\pi/3}^{\pi/3} R d\theta\cos\theta$$

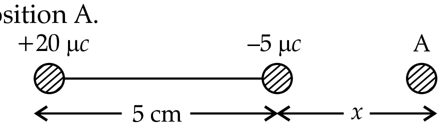
$$= \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{R}\int_0^{\pi/3}\cos\theta d\theta$$

$$E = \frac{2}{4\pi\varepsilon_0}\times\left(\frac{-3Q}{2\pi R}\right).\frac{1}{R}\times(\sin\theta)_0^{\pi/3}\ (-\hat{i})$$

$$E = \frac{3\sqrt{3}Q}{8\pi^2\varepsilon_0 R^2}\hat{i}$$

46. Option (4) is correct.

Let we place third charge q(Positive in nature) at position A.



The net force at position A is zero.

$$F_1 - F_2 = 0$$

$$\Rightarrow \quad F_1 = F_2$$

$$\frac{k\times 20\times q}{(5+x)^2} = k\frac{5q}{x^2}$$

$$4x^2 = (5+x)^2$$

$$\Rightarrow \quad 4x^2 = 25 + x^2 + 10x$$

$$3x^2 - 10x - 25 = 0$$

$$(x-5)(3x+5) = 0$$

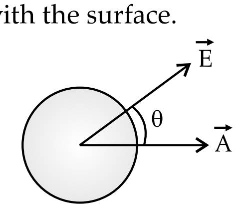
$$x = 5\text{ cm}$$

$$x = -\frac{5}{3}\text{ cm Not possible}$$

So, the charge should be placed at 5 cm right of –5 μc.

47. Option (1) is correct.

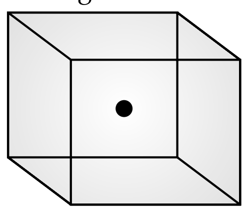
Using Gauss law ϕ_E= F. $ds\cos\theta$, where ϕ_E is flux enclosed with the surface.



(1) The electric lines of forces entering in to Gaussian surface provide negative flux.

Correct – entering flux is negative

(2) A charge q is placed at centre of the cube. The flux through all faces will be same.

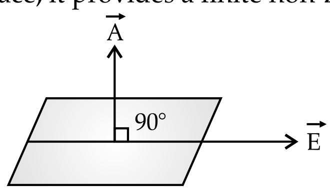


Correct $\quad \phi_E = \dfrac{q}{6A\varepsilon_0}$

(3) In an uniform electric field net flux through a closed Gaussion surface containing no net charge is zero.

Correct $\quad \phi = \int E.ds = \dfrac{q}{\varepsilon_0} = \dfrac{0}{\varepsilon_0} = 0$

(4) When electric field is parallel to a Gaussion surface, it provides a finite non-zero flux.

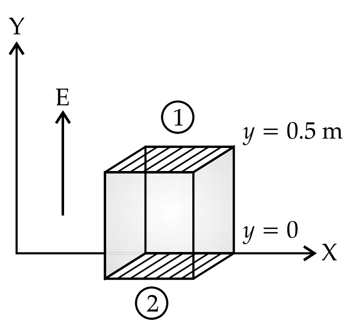


$$\phi = E.ds \cos\phi$$
$$\theta = 90°$$
$$\phi = 0$$

So incorrect.

48. Option (4) is correct.

Given, $E = 150y^2\,\hat{j}$



Here, the electric field is directed in y direction only. So the flux will be linked only with top and bottom surface, as shaded with number (1) and (2), For rest surfaces $\phi = E.ds \cos 90 = 0$

Now flux linked with surface (2)

$$\phi_1 = \vec{E}.\vec{A}$$
$$= (150y^2)|\hat{j}||\hat{j}|\cos 0$$

Put $y = 0.5 \quad \Rightarrow \phi_1 = 150 \times 0.5^2 \times (0.5 \times 0.5)$

Flux linked with surface (1)

$$\phi_2 = \vec{E}.\vec{A}$$
$$= (150y^2)|\hat{j}||\hat{j}|\cos 180$$
$$= -150 \times y^2 \times 0.25$$

Put $y = 0 \qquad \phi_2 = 0$

Net flux $= \phi_1 - \phi_2 = \phi_1 = \dfrac{150 \times 0.25}{4} = \dfrac{150}{4} \times 0.25$

$$\phi = \frac{150}{16}$$

Charge linked, $q = \varepsilon_0\phi = 8.85 \times 10^{-12} \times \dfrac{150}{16}$

$$= 8.3 \times 10^{-11} C$$

49. Option (2) is correct.

Given : Charge density on infinite plane sheet-1 is σ_+, charge density on infinite plane sheet-2 is σ_-, $|\sigma_+| > |\sigma_-|$, the two sheets intersect at right angles.

To find : The best representation of electric field lines in the region of space between the two sheets.

The electric field lines coming from an infinite plane sheet of charge will be straight lines and the electric field will be uniform in space. Graph-2 depicts these two qualities correctly.

50. Option (1) is correct.

Given : Three charges are located along the x-axis as, $+Q$ at $x = 0$, q at $x = \dfrac{d}{2}$ and $+Q$ at $x = d$, the net force experienced by the charge at $x = 0$ is 0.

To find : The value of q.

The net force on the charge at $x = 0$:

$$\frac{kqQ}{\left(\frac{d}{2}\right)^2} + \frac{kQ^2}{d^2} = 0$$

$$4q + Q = 0$$

$$q = -\frac{Q}{4}$$

51. Option (3) is correct.

$$E = \frac{-P}{4\pi\varepsilon_0 r^3}$$

$$= \frac{5 \times 10^{-6} \times 8 \times 10^{-3} \times 9 \times 10^9}{(20)^3 \times 10^{-6}}$$

$$= -4.5 \times 10^4 \text{ N/C}$$

52. Option (3) is correct.

$$W = Q.\Delta V$$

$\Rightarrow \qquad 2 = 20 \times \Delta V$

$$\Delta V = 0.1 \text{ Volt}$$

Topic-2
Capacitors

JEE (Main) Previous Year Questions

Multiple Choice Questions

1. Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: Two metallic spheres are charged to the same potential. One of them is hollow and another is solid, and both have the same radii. Solid sphere will have lower charge than the hollow one.

Reason R: Capacitance of metallic spheres depend on the radii of spheres.

In the light of the above statements, choose the correct answer from the options given below.

(1) Both A and R are true and R is the correct explanation of A
(2) A is true but R is false
(3) A is false but R is true
(4) Both A and R are true but R is not the correct explanation of A

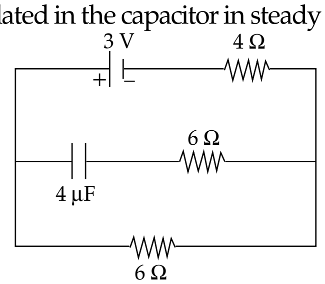
[JEE (Main) – 1st Feb. 2023 - Shift-2]

2. A parallel plate capacitor has plate area 40 cm^2 and plates separation 2 mm. The space between the plates is filled with a dielectric medium of a thickness 1 mm and dielectric constant 5. The capacitance of the system is:

(1) $24\varepsilon_0 F$ **(2)** $\frac{10}{3}\varepsilon_0 F$ **(3)** $\frac{3}{10}\varepsilon_0 F$ **(4)** $10\varepsilon_0 F$

[JEE (Main) – 25th Jan. 2023 - Shift-1]

3. In the network shown below, the charge accumulated in the capacitor in steady state will be:



(1) 4.8 μC **(2)** 12 μC **(3)** 7.2 μC **(4)** 10.3 μC

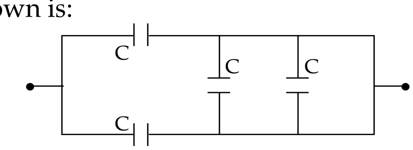
[JEE (Main) – 13th April 2023 - Shift-1]

4. The distance between two plates of a capacitor is d and its capacitance is C_1, when air is the medium between the plates. If a metal sheet of thickness $\frac{2d}{3}$ and of the same area as plate is introduced between the plates, the capacitance of the capacitor becomes C_2. The ratio $\frac{C_2}{C_1}$ is:

(1) 4 : 1 **(2)** 3 : 1 **(3)** 2 : 1 **(4)** 1 : 1

[JEE (Main) – 10th April 2023 - Shift-2]

5. The equivalent capacitance of the combination shown is:



(1) 4C **(2)** $\frac{5}{3}C$ **(3)** $\frac{C}{2}$ **(4)** 2C

[JEE (Main) – 10th April 2023 - Shift-1]

6. A parallel plate capacitor is formed by two plates each of area 30π cm^2 separated by 1 mm. A material of dielectric strength 3.6×10^7 Vm^{-1} is filled between the plates. If the maximum charge that can be stored on the capacitor without causing any dielectric breakdown is 7×10^{-6} C, the value of dielectric constant of the material is :

$$\left[\text{Use } \frac{1}{4\pi\varepsilon_0} = 9\times10^9 \text{ Nm}^2\text{C}^{-2}\right]$$

(1) 1.66 **(2)** 1.75 **(3)** 2.25 **(4)** 2.33

[JEE (Main) – 24th June 2022 - Shift-1]

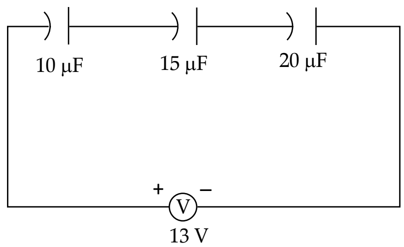
7. Two metallic plates form a parallel plate capacitor. The distance between the plates is 'd'. A metal sheet of thickness $\frac{d}{2}$ and of area equal to area of each plate is introduced between the plates.

What will be the ratio of the new capacitance to the original capacitance of the capacitor?

(1) 2 : 1 **(2)** 1 : 2 **(3)** 1 : 4 **(4)** 4 : 1

[JEE (Main) – 25th June 2022 - Shift-2]

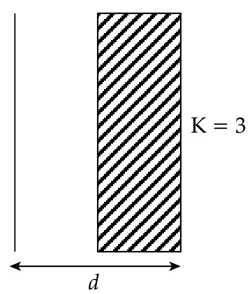
8. The charge on capacitor of capacitance 15 μF in the figure given below is:



(1) 60 μC **(2)** 130 μC **(3)** 260 μC **(4)** 585 μC

[JEE (Main) – 26th June 2022 - Shift-2]

9. A parallel plate capacitor with plate area A and plate separation d = 2 m has a capacitance of 4 μF. The new capacitance of the system if half of the space between them is filled with a dielectric material of dielectric constant K = 3 (as shown in the figure) will be:



(1) 2 μF **(2)** 32 μF **(3)** 6 μF **(4)** 8 μF

[JEE (Main) – 26th June 2022 - Shift-2]

10. A force of 10 N acts on a charged particle placed between two plates of a charged capacitor. If one plate of capacitor is removed, then the force acting on that particle will be.

(1) 5 N **(2)** 10 N **(3)** 20 N **(4)** Zero

[JEE (Main) – 27th June 2022 - Shift-1]

11. The total charge on the system of capacitors C_1 = 1 μF, C_2 = 2μF, C_3 = 4μF and C_4 = 3 μF Connected in parallel is :
(Assume a battery of 20 V is connected to the combination)

(1) 200 μC **(2)** 200 C **(3)** 10 μC **(4)** 10 C

[JEE (Main) – 26th July 2022 - Shift-1]

12. An alpha particle and a proton are accelerated from rest through the same potential difference. The ratio of linear momenta acquired by above two particles will be:

(1) $\sqrt{2}$: 1 **(2)** $2\sqrt{2}$: 1 **(3)** $4\sqrt{2}$: 1 **(4)** 8 : 1

[JEE (Main) – 29th July 2022 - Shift-2]

13. An electron with kinetic energy k_1 enters between parallel plates of a capacitor at an angle 'α' with the plates. It leaves the plates at angle 'β' with kinetic energy k_2. Then the ratio of kinetic energies $k_1 : k_2$ will be:

(1) $\frac{\sin^2\beta}{\cos^2\alpha}$ **(2)** $\frac{\cos^2\beta}{\cos^2\alpha}$ **(3)** $\frac{\cos\beta}{\sin\alpha}$ **(4)** $\frac{\cos\beta}{\cos\alpha}$

[JEE (Main) – 25th Feb. 2021 - Shift-2]

14. Two equal capacitors are first connected in series and then in parallel. The ratio of the equivalent capacities in the two cases will be :

(1) 2 : 1 **(2)** 1 : 4 **(3)** 4 : 1 **(4)** 1 : 2

[JEE (Main) – 24th Feb 2021 - Shift-1]

15. Consider the combination of 2 capacitors C_1 and C_2, with $C_2 > C_1$, when connected in parallel, the equivalent capacitance is $\frac{15}{4}$ times the equivalent capacitance of the same connected in series. Calculate the ratio of capacitors, $\frac{C_2}{C_1}$.

(1) $\frac{15}{11}$ **(2)** $\frac{29}{15}$ **(3)** $\frac{15}{4}$ **(4)** $\frac{111}{80}$

[JEE (Main) – 26th Feb 2021 - Shift-1]

16. For changing the capacitance of a given parallel plate capacitor, a dielectric material of dielectric constant K is used, which has the same area as the plates of the capacitor. The thickness of the dielectric slab is $\frac{3}{4}d$, where 'd' is the separation between the plate of parallel plate capacitor.

The new capacitance (C′) in terms of original capacitance (C_0) is given by the following relation :

(1) $C' = \frac{4K}{K+3}C_0$ **(2)** $C' = \frac{4}{3+K}C_0$

(3) $C' = \frac{3+K}{4K}C_0$ **(4)** $C' = \frac{4+K}{3}C_0$

[JEE (Main) – 16th March 2021 - Shift-1]

17. A parallel plate capacitor with plate area 'A' and distance of separation 'd' is filled with a dielectric. What is the capacity of the capacitor when permittivity of the dielectric varies as :

$\varepsilon(x) = \varepsilon_0 + Kx$, for $\left(0 < x \le \frac{d}{2}\right)$

$\varepsilon(x) = \varepsilon_0 + K(d-x)$, for $\left(\frac{d}{2} \le x \le d\right)$

(1) 0 **(2)** $\frac{KA}{2\ln\left(\frac{2\varepsilon_0 + Kd}{2\varepsilon_0}\right)}$

(3) $\left(\varepsilon_0 + \frac{Kd}{2}\right)^{2/KA}$ **(4)** $\frac{KA}{2}\ln\left(\frac{2\varepsilon_0}{2\varepsilon_0 - Kd}\right)$

[JEE (Main) – 25th July 2021 - Shift-1]

18. If q_f is the free charge on the capacitor plates and q_b is the bound charge on the dielectric slab of dielectric constant K placed between the capacitor plates, then bound charge q_b can be expressed as:

(1) $q_b = q_f\left(1-\frac{1}{K}\right)$ **(2)** $q_b = q_f\left(1-\frac{1}{\sqrt{K}}\right)$

(3) $q_b = q_f\left(1+\frac{1}{\sqrt{K}}\right)$ **(4)** $q_b = q_f\left(1+\frac{1}{K}\right)$

[JEE (Main) – 25th July 2021 - Shift-2]

19. An electron and proton are separated by a large distance. The electron starts approaching the proton with energy 3 eV. The proton captures the electrons and forms a hydrogen atom in second excited state. The resulting photon is incident on a photosensitive metal of threshold wavelength 4000 Å. What is the maximum kinetic energy of the emitted photoelectron?

(1) No photoelectron would be emitted
(2) 3.3 eV
(3) 1.41 eV **(4)** 7.61 eV

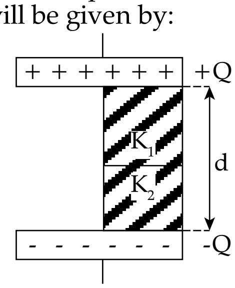
[JEE (Main) – 25th July 2021 - Shift-1]

20. The material filled between the plates of a parallel plate capacitor has resistivity 200 Ωm The value of capacitance of the capacitor is 2 pF. If a potential difference of 40 V is applied across the plates of the capacitor, then the value of leakage current flowing out of the capacitor is: (given the value of relative permitivity of material is 50)

(1) 9.0 μA **(2)** 0.9 μA **(3)** 9.0 μA **(4)** 0.9 mA

[JEE (Main) – 26th Aug 2021 - Shift-1]

21. A parallel-plate capacitor with plate area A has separation d between the plates. Two dielectric slabs of dielectric constant K_1 and K_2 of same area A/2 and thickness d/2 are inserted in the space between the plates. The capacitance of the capacitor will be given by:

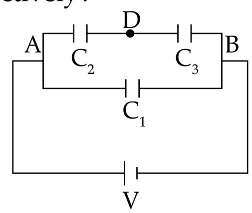


(1) $\frac{\varepsilon_0 A}{d}\left(\frac{1}{2}+\frac{K_1K_2}{K_1+K_2}\right)$ **(2)** $\frac{\varepsilon_0 A}{d}\left(\frac{1}{2}+\frac{2(K_1+K_2)}{K_1K_2}\right)$

(3) $\frac{\varepsilon_0 A}{d}\left(\frac{1}{2}+\frac{K_1+K_2}{K_1K_2}\right)$ **(4)** $\frac{\varepsilon_0 A}{d}\left(\frac{1}{2}+\frac{K_1K_2}{2(K_1+K_2)}\right)$

[JEE (Main) – 26th Aug 2021 - Shift-2]

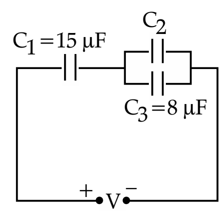
22. Three capacitors C_1 = 2 μF, C_2 = 6 μF, and C_3 = 12 μF are connected as shown in figure. Find the ratio of the charges on capacitor C_1, C_2 and C_3 respectively:



(1) 2:1:1 **(2)** 2:3:3 **(3)** 3:4:4 **(4)** 1:2:2

[JEE (Main) – 27th Aug 2021 - Shift-2]

23. In the circuit shown in the figure, the total charge is 750 μC and the voltage across capacitor C_2 is 20 V. Then the charge on capacitor C_2 is :



(1) 650 μC **(2)** 450 μC **(3)** 590 μC **(4)** 160 μC

[JEE (Main) – 3rd Sep. 2020 - Shift-1]

24. A parallel plate capacitor has 1 mF capacitance. One of its two plates is given +2 μC charge and the other plate, +4 μC charge. The potential difference developed across the capacitor is :

(1) 3 V **(2)** 1 V **(3)** 5 V **(4)** 2 V

[JEE (Main) – 8th April 2019 - Shift-2]

ANSWER – KEY

1. (3)	2. (2)	3. (3)	4. (2)
5. (4)	6. (4)	7. (1)	8. (1)
9. (3)	10. (1)	11. (1)	12. (2)
13. (2)	14. (2)	15. Bonus	16. (1)
17. (2)	18. (1)	19. (3)	20. (4)
21. (1)	22. (4)	23. (3)	24. (2)

ANSWERS WITH EXPLANATIONS

1. Option (3) is correct.

Charge on a conductor resides only on its outer surface, so both conductor will have same charge as both have same radii.

Now, capacitance of spherical conductor,

$$C = 4\pi\varepsilon_0 r \Rightarrow C \propto r$$

2. Option (2) is correct.

For a partially filled dielectric of thickness t

$$C = \frac{\varepsilon_0 A}{d - t\left(1-\frac{1}{K}\right)}$$

where, t = thickness

K = dielectric constant

Putting the given values, we get

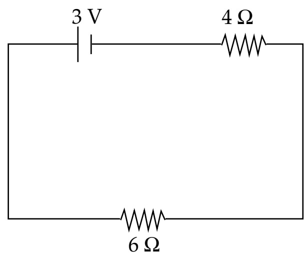
$$C = \frac{\varepsilon_0\left(40\times10^{-4}\right)}{\left[2-1\left(1-\frac{1}{5}\right)\right]\times10^{-3}} = \frac{4\varepsilon_0}{1+\frac{1}{5}}$$

$$= \frac{10\varepsilon_0}{3}(\text{F})$$

3. **Option (3) is correct.**

In the steady state, capacitor acts as an open circuit.

The equivalent circuit becomes:



$$i = \frac{3}{10} = 0.3 \text{ A}$$

$$V_C = 6i = 1.8 \text{ V}$$

$$q = CV_C$$

$$= 4 \times 1.8 = 7.2\,\mu\text{C}$$

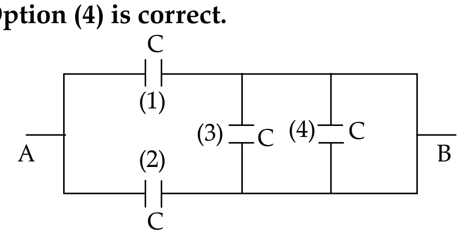
4. **Option (2) is correct.**

$$C_1 = \frac{\varepsilon_0 A}{d} \Rightarrow C_2 = \frac{\varepsilon_0 A}{d-t}$$

where, t = thickness of the metal sheet

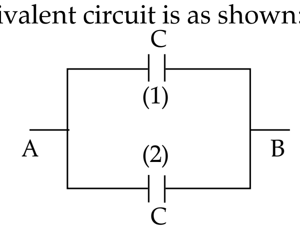
Given, $t = \frac{2d}{3}, C_2 = 3\frac{\varepsilon_0 A}{d} = 3C_1 \Rightarrow \frac{C_2}{C_1} = 3:1$

5. **Option (4) is correct.**



Capacitors numbered (3) and (4) are short circuit as their both plates are connected to terminal B which makes the potential difference across both to be zero.

The equivalent circuit is as shown:



Hence $C_{AB} = 2C$ as capacitors numbered (1) and (2) are connected in parallel.

6. **Option (4) is correct.**

Since the capacitor is filled completely with a dielectric, the electric field inside is given by $\text{E} = \frac{\text{E}_0}{\text{K}}$ where, E_0 is the electric field in vacuum and is given by $\text{E}_0 = \frac{q}{\text{A}\varepsilon_0}$

$$\therefore \quad E = \frac{q}{\text{K A}\varepsilon_0} \Rightarrow \text{K} = \frac{q}{\text{E A}\varepsilon_0}$$

$$= \frac{7\times10^{-6}}{3\cdot6\times10^{7}\times30\pi\times10^{-4}\times8\cdot85\times10^{-12}}$$

$$= \frac{700}{3\cdot6\times3\lambda\times8\cdot85} = 2\cdot33$$

7. **Option (1) is correct.**

Original capacitance $\text{C}_0 = \frac{\varepsilon_0 \text{A}}{d}$

New capacitance $\text{C} = \frac{\varepsilon_0 \text{A}}{d-\frac{d}{2}} = 2\frac{\varepsilon_0 \text{A}}{d} = 2\text{C}_0$

$$\Rightarrow \qquad \frac{\text{C}}{\text{C}_0} = 2$$

Note that when a metal plate of thickness t is inserted between the plates $\text{C} = \frac{\varepsilon_0 t}{d-t}$

8. **Option (1) is correct.**

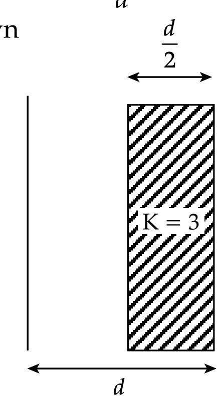
$$\text{C}_{eq} = \frac{1}{\frac{1}{10}+\frac{1}{15}+\frac{1}{20}} = \frac{60}{6+4+3} = \frac{60}{13}\mu\text{F}$$

$$\text{Q} = \text{C}_{eq}\text{V} = \frac{60}{13}\times13 = 60\,\mu\text{C} \text{ (on all capacitors)}$$

9. **Option (3) is correct.**

Let initial capacitor, $\text{C}_0 = \frac{\varepsilon_0 \text{A}}{d}$

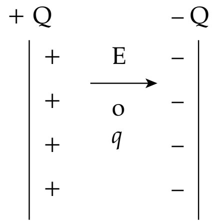
For the case shown



$$\text{C} = \frac{\varepsilon_0 \text{A}}{d-\frac{d}{2}\left(1-\frac{1}{3}\right)} = \frac{\varepsilon_0 \text{A}}{\frac{d}{2}+\frac{d}{6}} = \frac{6\varepsilon_0 \text{A}}{4d} = \frac{3}{2}\text{C}_0$$

$$\text{C} = \frac{3}{2}\times4 = 6\,\mu\text{F}$$

10. Option (1) is correct.



Electric field between a parallel plate capacitor,

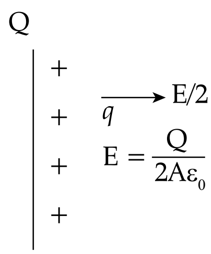
$$E = \frac{Q}{A\varepsilon_0}$$

Force on the charge, $F = q.E$

$$q \times \frac{Q}{A\varepsilon_0} = 10 \text{ N}$$

Now, when one plate is removed then electric field gets halved in region. Hence, force on the charge particle will be,

$$F' = q\frac{Q}{2A\varepsilon_0} = \frac{10}{2} = 5 \text{ N}$$



11. Option (1) is correct.

Equivalent capacitance when capacitors are connected in parallel,

$C = C_1 + C_2 + C_3 + C_4$

$C = 1 + 2 + 4 + 3 = 10\ \mu F$

Now, $q = CV = 10\ \mu F \times 20\ V = 200\ \mu C$

12. Option (2) is correct

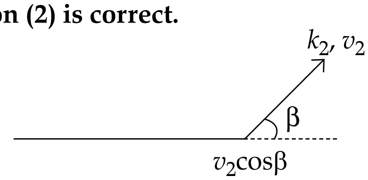
An α particle and a proton are accelerated from rest though the same potential difference, so momentum of each can be given

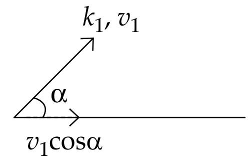
$$P = \sqrt{2mE} = \sqrt{2mqV}$$

$$\frac{P_\alpha}{P_p} = \sqrt{\frac{m_\alpha q_\alpha}{m_p q_p}} = \sqrt{\frac{4}{1} \times \frac{2}{1}}$$

$$\frac{P_\alpha}{P_p} = \frac{2\sqrt{2}}{1}$$

13. Option (2) is correct.





Here, α, β are the angles made with capacitor plates by electrons.

Since the electric field inside a parallel plate capacitor is perpendicular to the plates, the parallel component of velocity won't change.

So, we can write it as $V_1 \cos\alpha = V_2 \cos\beta$ ...(i)

Rearranging equation (i) we get:

$$\frac{V_1}{V_2} = \frac{\cos\beta}{\cos\alpha} \quad \text{...(ii)}$$

Since, kinetic energy $(k) = \left(\frac{1}{2}\right)mv^2$

The ratio of kinetic energy can be written as:

$$\frac{k_1}{k_2} = \frac{\left(\frac{1}{2}\right)mv_1^2}{\left(\frac{1}{2}\right)mv_2^2}$$

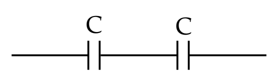
$$\frac{k_1}{k_2} = \left(\frac{v_1}{v_2}\right)^2 \quad \text{...(iii)}$$

Using equations (ii) and (iii) we get:

$$\frac{k_1}{k_2} = \left(\frac{\cos\beta}{\cos\alpha}\right)^2$$

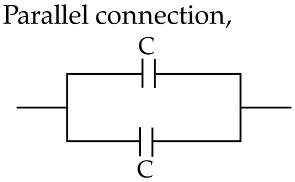
14. Option (2) is correct.

Case I : Series connection,



$$\frac{1}{C_1} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C} \Rightarrow C_1 = \frac{C}{2}$$

Case II : Parallel connection,



$$C_2 = C + C = 2C$$

Hence, the required ratio, $\frac{C_1}{C_2} = \frac{1}{4}$

15. (Bonus)

Given, $C_2 > C_1$

Case I connected in series

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

Case II connected in parallel

$$C_{eq} = C_1 + C_2$$

So, $$\frac{15}{4}\frac{C_1C_2}{C_1+C_2} = C_1 + C_2$$

$$\Rightarrow \quad \frac{15}{4}C_1C_2 = (C_1+C_2)^2$$

$$\Rightarrow \quad 15C_1C_2 = 4C_1^2 + 4C_2^2 + 8C_1C_2$$

$$\Rightarrow \quad 4C_1^2 + 4C_2^2 - 7C_1C_2 = 0$$

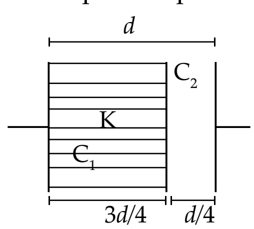
$$\Rightarrow \quad 4\left(\frac{C_1}{C_2}\right)^2 + 4 - \frac{7C_1}{C_2} = 0$$

$$b^2 - 4ac < 0 \Rightarrow \sqrt{49-64} < 0$$

Hence, root of the equation will be imaginary.
All the given options are inccorect.

16. Option (1) is correct.

The expression of a parallel place capacitor



Here, $$C_0 = \frac{A\varepsilon_0}{d}$$

So, $$C_1 = \frac{K A\varepsilon_0}{\frac{3d}{4}} \text{........at } d \rightarrow \frac{3d}{4}$$

And $$C_2 = \frac{A\varepsilon_0}{\frac{d}{4}} \text{.......... at } d \rightarrow \frac{d}{4}$$

Now, equivalent capacitance

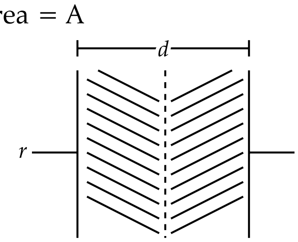
$$C' = \frac{C_1C_2}{C_1+C_2}$$

$$\Rightarrow \frac{\left(4K\frac{A\varepsilon_0}{3d}\right)\left(4\frac{A\varepsilon_0}{d}\right)}{4K\frac{A\varepsilon_0}{3d} + 4\frac{A\varepsilon_0}{d}}$$

$$\Rightarrow \frac{\frac{4KA^2\varepsilon_0^2}{3d^2}\times 4}{\frac{4A\varepsilon_0}{d}\left[\frac{K}{3}+1\right]} \Rightarrow \frac{4K}{(3+K)}C_0$$

17. Option (2) is correct.

Plate area = A (given)



Capacitance of a parallel plate capacitor

$$C = \frac{A\varepsilon_0}{d} \qquad \text{[without dielectric]}$$

$$C = \frac{KA\varepsilon_0}{d}$$

[with dielectric constant 'K']

$$K = \varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

$$C = \frac{KA\varepsilon_0}{d} = \frac{\varepsilon A}{d}$$

[when dielectric is inserted]

$$C_1 = \frac{(\varepsilon_0 + Kx)A}{dx} \text{ for } \left[0 < x \le \frac{d}{2}\right]$$

$$C_2 = \frac{\varepsilon_0 + K(d-x)A}{dx} \text{ for } \left[\frac{d}{2} < x \le d\right]$$

As the capacitors are in series

$$\frac{1}{C_{\text{equi.}}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\int_0^{d/2}\frac{dx}{A(\varepsilon_0 + Kx)} + \int_{d/2}^{d}\frac{dx}{A[\varepsilon_0 + K(d-x)]}$$

$$\Rightarrow \frac{1}{A}\left[\log_e\frac{(\varepsilon_0+Kx)}{K}\right]_0^{d/2} + \frac{1}{A}\left[\log_e\frac{[\varepsilon_0 + K(d-x)]}{-K}\right]_{d/2}^{d}$$

$$\Rightarrow \frac{1}{AK}\left[\log_e\left(\varepsilon_0 + \frac{Kd}{2}\right) - \log_e\varepsilon_0\right]$$

$$-\frac{1}{AK}\left[\log_e\varepsilon_0 - \log_e\left(\varepsilon_0 + \frac{Kd}{2}\right)\right]$$

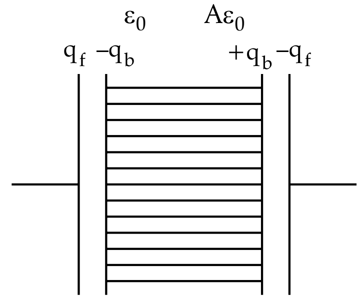
On simplifying

$$C_{\text{equi.}} = \frac{KA}{2\log_e\left[\frac{2\varepsilon_0 + Kd}{2\varepsilon_0}\right]}$$

18. Option (1) is correct.

Electric field due to capacitor is

$$\frac{\sigma_f}{\varepsilon_0} = \frac{q_f}{A\varepsilon_0}$$



Electric field due to dielectric slab is

$$\frac{\sigma_f}{K\varepsilon_0} = \frac{q_f}{KA\varepsilon_0}$$

$$\text{Net electric field} = \frac{\sigma_b}{\varepsilon_0} = \frac{q_b}{A\varepsilon_0}$$

$$\frac{q_b}{A\varepsilon_0} = \frac{q_f}{A\varepsilon_0} - \frac{q_f}{KA\varepsilon_0} = \frac{q_f}{A\varepsilon_0}\left(1-\frac{1}{K}\right)$$

$$q_b = q_f\left(1-\frac{1}{K}\right)$$

19. Option (3) is correct.

Using photoelectric effect equation

$$KE_{maxi} = \frac{hc}{\lambda} - \phi \Rightarrow E = \frac{-(13.6)}{(3)^2}$$

$$= -1.51\ eV$$

$$(KE)_{maxi} = (3 + 1.51)eV - \phi$$

$$= (4.51)eV - \left[\frac{12400}{4000}\right]eV = 1.41eV$$

20. Option (4) is correct.

Current leakage is small current that flows through capacitor when voltage is applied.

Given, $\rho = 200\ \Omega m, C = 2 \times 10^{-12}$ F, V = 40 V, $k = 50$

Leakage current as function of time

$$i(f) = i_0\, e^{\frac{-t}{\tau}} \qquad ...(i)$$

Here, $i_0 = \frac{q_0}{\rho\, k\varepsilon_0}$ for maximum current.

Put in equation (i)

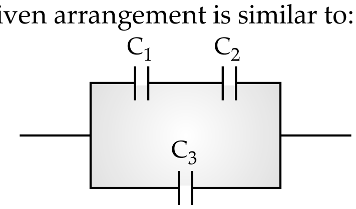
$$i = \frac{CV_0}{\rho k\varepsilon_0} e^{-t/k\varepsilon_0\rho}$$

$$i_{leakage} = \frac{CV_0}{\rho\, k\varepsilon_0} \text{ as } t\to\infty = \frac{2\times10^{-12}\times40}{200\times50\times8.85\times10^{-12}}$$

$$= 903\ \mu A = 0.9\ mA$$

21. Option (1) is correct.

The given arrangement is similar to:

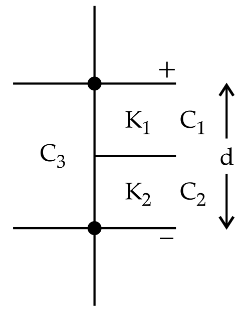


Where, C_1 has plate separation of $d/2$. Area A/2 and dielectric constant K_1

$$C_1 = \frac{K_1\varepsilon_0 A/2}{d/2} = \frac{K_1\varepsilon_0 A}{d}$$

C_2 also with same dimensions as C_1

$$C_2 = \frac{K_2\varepsilon_0 A/2}{d/2} = \frac{K_2\varepsilon_0 A}{d}$$



Both C_1 and C_2 are in series,

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\Rightarrow \qquad C' = \frac{C_1C_2}{C_1+C_2}$$

$$C' = \frac{\varepsilon_0 A}{d}\ \frac{K_1K_2}{K_1+K_2}$$

Now C_3 has dimensions d, Area A/2

$$K_3 = 1$$

$$C_3 = \frac{\varepsilon_0 A/2}{d} = \frac{\varepsilon_0 A}{2d}$$

C_3 is in parallel with C′

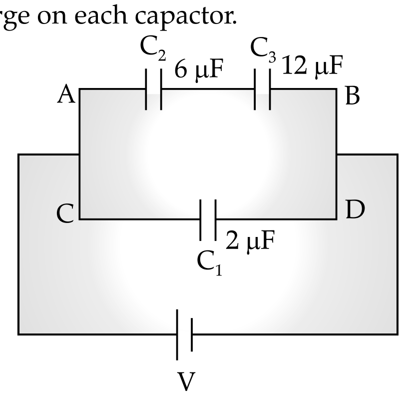
$$C_{net} = C_3 + C_3$$

$$= \frac{\varepsilon_0 A}{d}\frac{K_1K_2}{K_1+K_2} + \frac{\varepsilon_0 A}{2d}$$

$$c_{net} = \frac{\varepsilon_0 A}{d}\left(\frac{1}{2} + \frac{k_1k_2}{k_1+k_2}\right)$$

22. Option (4) is correct.

By Finding voltage across arms and the equivalent capactane for each arm we can get charge on each capactor.



AB and CD arms are parallel to voltage across each will be 'V' volts.

charge on $C_1 \Rightarrow q_1 = C_1V = 2\mu C$

Now For AB,

$$\frac{1}{C_{eq}} = \frac{1}{6} + \frac{1}{12} \Rightarrow C_{eq} = \frac{12\times6}{12\times6}$$

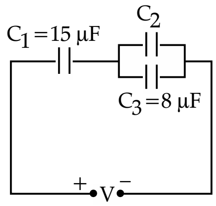
$$= 4\ \mu F$$

Both C_2 and C_3 are in series so charges will be same $q = C_{eq} \times V$, So $q_2 = 4$ V, $q_3 = 4$ V

$$q_1 : q_2 : q_3 = 2V : 4\,V : 4\,V = 1 : 2 : 2$$

23. Option (3) is correct.

Given : A capacitive circuit with three capacitors, total charge in the circuit is Q = 750 μC, voltage drop across C_2 is $V_2 = 20$ V.



To find : Q_2, charge on C_2.

Voltage drop across C_3 :

$$V_3 = 20\text{ V}$$

Charge on C_3 :

$$Q_3 = C_3V_3 = 8\times10^{-6}\times20 = 160\ \mu C$$

Charge on C_2 :

$$Q_2 = Q - Q_3 = 750 - 160 = 590\ \mu C$$

24. Option (2) is correct.

Given : Capacitance of a parallel plate capacitor is C = 1 μF, charge on plate 1 of the capacitor is $Q_1 = +\ 2$ μC, charge on plate 2 of the capacitor is $Q_2 = +\ 4$ μC

To find : V, the potential difference developed across the capacitor.

Net value of charge on the plates of capacitor after steady state is reached :

$$Q_{net} = \frac{Q_2 - Q_1}{2} = \frac{(4-2)\times10^{-6}}{2} = 1\ \mu C \quad ...(i)$$

The potential difference developed across the capacitor :

$$V = \frac{Q_{net}}{C} = \frac{1\times10^{6}}{1\times10^{6}} = 1\text{ V}$$

Subjective Questions (Chapter Based)

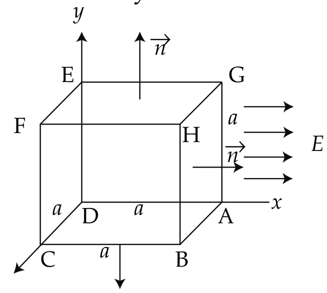
1. A cubical volume is bounded by the surfaces $x = 0, x = a, y = 0, y = a, z = 0, z = a$. The electric field in the region is given by $\vec{E} = E_0x\hat{i}$. Where $E_0 = 4\times10^4$ N C^{-1} m^{-1}. If a = 2 cm, the charge contained in the cubical volume is $Q\times10^{-14}$ C. The value of Q is ________. (Take $\varepsilon_0 = 9\times10^{-12}$ C^2 N^{-1} m^{-2})

[JEE (Main) – 1st Feb. 2023 - Shift-2]

Sol. Correct answer is [288]

$$E = E_0x\hat{i}$$

Electric field is in only x direction



$\therefore$ $\phi_{EFGH} = \phi_{ABCD} = \phi_{ADEG} = \phi_{BCFH} = 0$

$\phi_{ABHG} = E_0a\times a^2 = E_0a^3$

$\phi_{CDEF} = 0$, as $x = 0, E = 0$

$\phi_{total} = E_0a^3 = 4\times10^4\times8\times10^{-6}$

$\frac{q_{in}}{\varepsilon_0} = 32\times10^{-2}$

$q_{in} = 32\varepsilon_0\times10^{-2}$

$= 32\times9\times10^{-12}\times10^{-2}$

$= 288\times10^{-14}$

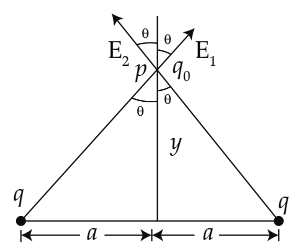
$Q = 288$

2. Two equal positive point charges are separated by a distance $2a$. The distance of a point from the centre of the line joining two charges on the equatorial line (perpendicular bisector) at which force experienced by a test charge q_0 becomes maximum is $\frac{a}{\sqrt{x}}$. The value of x is ________.

[JEE (Main) – 1st Feb. 2023 - Shift-1]

Sol. Correct answer is [2]

For force on q_0 to be maximum, electric field at Point P should be maximum.



At P_1, $E_1 = \frac{kq}{a^2+y^2} \Rightarrow E_2 = \frac{kq}{a^2+y^2}$

$$E_p = E_1\cos\theta + E_2\cos\theta = \frac{2kqy}{(a^2+y^2)^{3/2}}$$

For E_p to be maximum

$$\frac{d}{dy}E_p = 0 \Rightarrow \frac{d}{dy}\frac{2kqy}{(a^2+y^2)^{3/2}} = 0$$

$$y = \pm\frac{a}{\sqrt{2}}$$

On comparision $x = 2$

3. Two parallel plate capacitors C_1 and C_2 each having capacitance of 10 μF are individually charged by a 100 V D.C. source. Capacitor C_1 is kept connected to the source and a dielectric slab is inserted between its plates. Capacitor C_2 is disconnected from the source and then a dielectric slab is inserted in it. Afterwards the capacitor C_1 is also disconnected from the source and the two capacitors are finally connected in parallel combination. The common potential of the combination will be ____V. (Assuming Dielectric constant = 10)

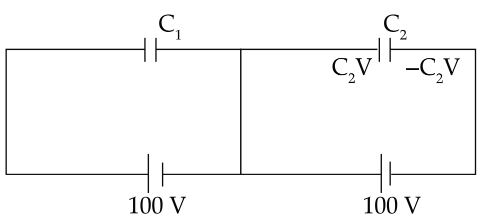
[JEE (Main) – 31st Jan. 2023 - Shift-2]

Sol. Correct answer is [55]

Given $C_1 = C_2 = 10\ \mu\text{F}$

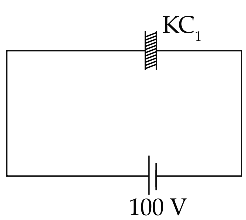
$V = 100\text{ V}$ & $K = 10$

Let's take two individual parallel plate capacitors

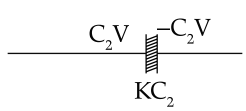


Charge on each plate of (ii) capacitor is $+C_2V$ and $-C_2V$ respectively

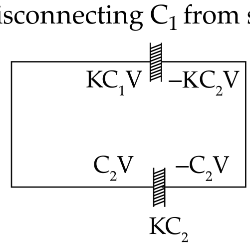
Afterwards, dielectric slab is inserted in C_1.



Afterwards C_2 is disconnected from the source and then dielectic slab is inserted.



Finally both the capacitors are connected in parallel by disconnecting C_1 from source.



Applying charge conservation, $Q_1 = Q_2$

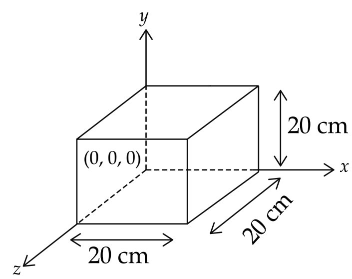
$$KC_1V + C_2V = (KC_1V + KC_2)V_{\text{common}}$$

$$V_{\text{common}} = \frac{(K+1)CV}{2KC} = \frac{K+1}{2K}.V$$

$$\Rightarrow \quad V_{\text{common}} = \frac{10+1}{2\times 10}\times 100$$

$$\Rightarrow \quad V_{\text{common}} = 55\text{ V}$$

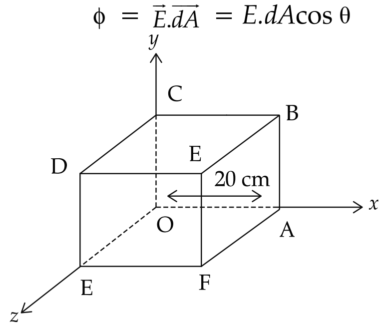
4. Expression for an electric field is given by $\vec{E} = 4000\ x^2\hat{i}\ \text{Vm}^{-1}$. The electric flux through the cube of side 20 cm when placed in electric field (as shown in the figure) is _____ V cm.



[JEE (Main) – 31st Jan. 2023 - Shift-1]

Sol. Correct answer is [640]

$$\phi = \vec{E}.\overrightarrow{dA} = E.dA\cos\theta$$



Given,

$$\vec{E} = 4000x^2\ \hat{i}\ \text{V m}^{-1}$$

Here the electric field is directed towards x-axis

So, the flux along y and z-axis would be zero, as E and A are perpendicular

$\phi_{DCBE} = \phi_{DEGF} = \phi_{COAB} = \phi_{GOFA} = 0$

$\phi_{GODC} = 0$ as $\vec{E}$ depends on x.

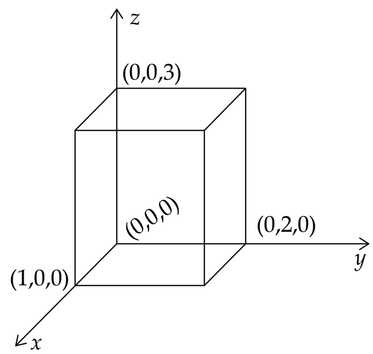
And $x = 0$ for surface *GODC*

Now for surface *AFEB*

$$\phi_{AFEB} = \vec{E}.\overrightarrow{dA}$$
$$= 4000x^2 \times (0.2)^2$$
$$= 4000 \times (0.2)^2 \times (0.2)^2$$
$$= 6.4\text{ V m} = 640\text{ Vcm}$$

5. As shown in figure, a cuboid lies in a region with electric field $= 2x^2\hat{i} - 4y\hat{j} + 6\hat{k}\ \text{N C}^{-1}$. The magnitude of charge within the cuboid is $n\varepsilon_0$C. The value of n is _______.

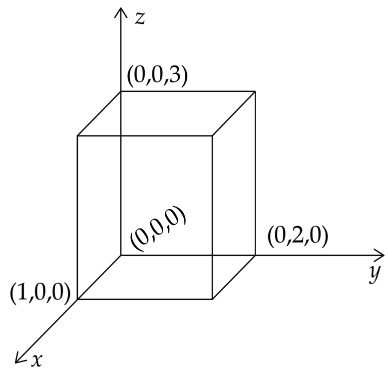
(if dimension of cuboid is $1 \times 2 \times 3\ \text{m}^3$).



[JEE (Main) – 30th Jan. 2023 - Shift-2]

Sol. Correct answer is [12]

Given: $\vec{E} = 2x^2\hat{i} - 4y\hat{j} + 6g\hat{k}$ NC^{-1}



Flux through planes parallel to yz

$= 2 \times (1)^2 \times 2 \times 3$

$= 12\ N\ m^2 C^{-1}$

Flux through planes parallel to xz

$= -4 \times 2 \times (1 \times 3)$

$= -24\ N\ m^2 C^{-1}$

Flux through planes parallel to $xy = 0$

Hence, net flux $(\phi) = 12 - 24 + 0$

$$\frac{q_{enc}}{\varepsilon_0} = -12$$

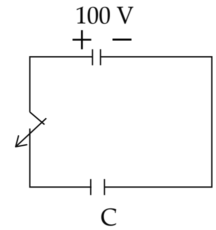
or, $q_{enc} = -12\varepsilon_0 C$

$|q_{enc}| = 12\varepsilon_0 C = n\varepsilon_0 C$

Hence, $n = 12$

6. A capacitor of capacitance 900 μF is charged by a 100 V battery. The capacitor is disconnected from the battery and connected to another uncharged identical capacitor such that one plate of uncharged capacitor connected to positive plate and another plate of uncharged capacitor connected to negative plate of the charged capacitor. The loss of energy in this process is measured as $x \times 10^{-2}$ J. The value of x is ____. **[JEE (Main) – 30th Jan. 2023 - Shift-1]**

Sol. Correct answer is [225]



When equilibrium is attained i.e. both capacitor acquire the same potential,

loss of energy $= \frac{1}{2}\frac{C_1C_2}{C_1+C_2} \therefore (V_1 - V_2)^2$

Here $C_1 = C_2 = C = 900\ \mu F$

$V_1 = 100$ V

$V_2 = 0$

Loss $= \frac{1}{2}\frac{C}{2}V_1^2 = \frac{900}{4} \times 10^4 \times 10^{-6}$ J

$= 2.25$ J

$= 225 \times 10^{-2}$ J $= x \times 10^{-2}$ J (given)

On comparing, $x = 225$

7. For a charged spherical ball, electrostatic potential inside the ball varies with r as $V = 2ar^2 + b$. Here, a and b are constant and r is the distance from the center. The volume charge density inside the ball is $-\lambda a\varepsilon$. The value of λ is ________.

(where, ε = permittivity of the medium)

[JEE (Main) – 29th Jan. 2023 - Shift-2]

Sol. Correct answer is [12]

Given, $V = 2ar^2 + b$

By $E = \frac{-dV}{dr}$

$E = -4qr$

Also, inside a solid charged sphere

$$E = \frac{\rho r}{3\varepsilon_0}$$

where, ρ = volume charge density

$$\frac{\rho r}{3\varepsilon_0} = -4ar$$

$\rho = -12a\varepsilon_0$

$\lambda = 12$

8. A point charge $q_1 = 4q_0$ is placed at origin. Another point charge $q_2 = -q_0$ is placed at = 12 cm. Charge of proton is q_0. The proton is placed on x-axis so that the electrostatic force on the proton is zero. In this situation, the position of the proton from the origin is ____________ cm.

[JEE (Main) – 29th Jan. 2023 - Shift-1]

Sol. Correct answer is [24]

For net force zero on the proton, the net electric field must be zero.

Hence, $\frac{q_0}{x^2} = \frac{4q_0}{(x+12)^2}$

$\Rightarrow \quad x + 12 = 2x$

$\Rightarrow \quad x = 12$

So, the distance from origin = $x + 12$

$= 12 + 12 = 24$ cm

9. A capacitor has capacitance 5 μF when its parallel plates are separated by air medium of thickness d. A slab of material of dielectric constant 1.5 having area equal to that of plates but thickness $\frac{d}{2}$ is inserted between the plates. Capacitance of the capacitor in the presence of slab will be ______ μ **[JEE (Main) – 25th Jan. 2023 - Shift-2]**

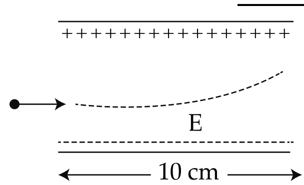
Sol. Correct answer is [6]

For a partially filled dielectric slab of thickness t and dielectric constant K, capacitance is given by

$$C = \frac{C_0}{1 - \frac{t}{d}\left(1 - \frac{1}{K}\right)} = \frac{5}{1 - \frac{d/2}{d}\left(1 - \frac{1}{1.5}\right)}$$

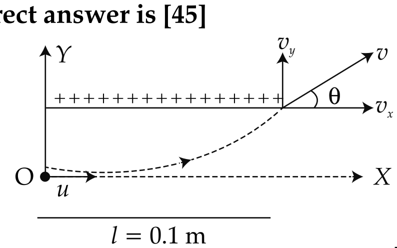
$$= \frac{5}{1 - \frac{1}{2}\left(\frac{1}{3}\right)} = \frac{30}{5} = 6\ \mu\text{F}$$

10. A uniform electric field of 10 N/C is created between two parallel charged plates (as shown in figure). An electron enters the field symmetrically between the plates with a kinetic energy 0.5 eV. The length of each pate is 10 cm. The angle (θ) of deviation of the path of electron as it comes out of the field is ______ (in degree).



[JEE (Main) – 25th Jan. 2023 - Shift-1]

Sol. Correct answer is [45]



Let u be the initial speed

$$\text{KE} = 0.5 \text{ eV} = 0.5 \times 1.6 \times 10^{-19} = \frac{1}{2}mu^2$$

$$u = \sqrt{\frac{1.6 \times 10^{-19}}{m}}$$

Time taken before enticing the field, $t = \frac{l}{u}$

$v_x = u_x$ as $a_x = 0$

$v \cos\theta = u$ ....(i)

$v_y = u_y + a_y t$

since $u_y = 0$

$v_y = v\sin\theta = \frac{eE}{m}t$

$\therefore \quad v\sin\theta = \frac{eE}{m}\left(\frac{l}{u}\right)$ ....(ii)

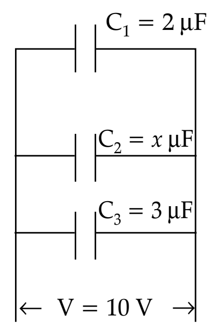
From (i) and (ii) on diving, we get

$$\tan\theta = \frac{eE}{m}\frac{l}{u^2} = \frac{eE}{m}\frac{l}{(e/m)}$$

$= El = 10 \times 01 = 1$

$\theta = 45°$

11. In the given figure the total charge stored in the combination of capacitors is 100 μ C. The value of 'x' is ______.



[JEE (Main) – 15th April 2023 - Shift-1]

Sol. Correct answer is [5]

Since all three capacitors are connected in parallel,

$q_1 = C_1V = 20$ μC

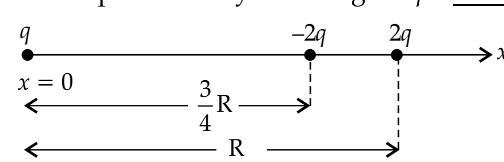
$q_2 = C_2V = 10x$ μC

$q_3 = C_3V = 30$ μC

$q = q_1 + q_2 + q_3 = 50 + 10x = 100$

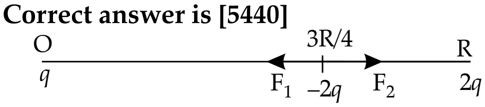
$x = 5$ μF

12. Three point charges q, $-2q$ and $2q$ are placed on x-axis at a distance $x = 0$, $x = \frac{3}{4}R$ and $x = R$ respectively from origin as shown. If $q = 2 \times 10^{-6}$ C and $R = 2$ cm, the magnitude of net force experienced by the charge $-2q$ is ______ N.



[JEE (Main) – 13th April 2023 - Shift-2]

Sol. Correct answer is [5440]



$$F_{net} = F_2 - F_1 = \frac{k4q^2}{\left(\frac{R}{4}\right)^2} - \frac{k2q^2}{\left(\frac{3R}{4}\right)^2}$$

$$= \frac{16kq^2}{R^2}\left(4 - \frac{2}{9}\right) = \frac{16kq^2}{R^2} \times \frac{34}{9} = \frac{544kq^2}{9R^2}$$

Given, $q = 2 \times 10^{-6}$ C

$R = 2 \times 10^{-2}$ m

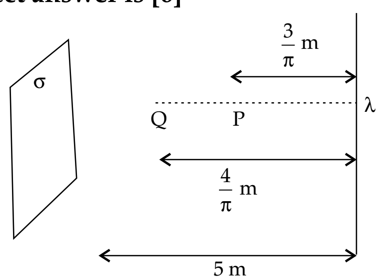
$$F_{net} = \frac{544}{9} \times \frac{9 \times 10^{9} \times 4 \times 10^{-12}}{4 \times 10^{-4}}$$

$$= 5440 \text{ N}$$

13. A thin infinite sheet charge and an infinite line charge of respective charge densities $+\sigma$ and $+\lambda$ are placed parallel at 5 m distance from each other. Points 'P' and 'Q' are at $\frac{3}{\pi}$ m and $\frac{4}{\pi}$ m perpendicular distances from line charge towards sheet charge, respectively. 'E_P' and 'E_Q' are the magnitudes of resultant electric field intensities at point 'P' and 'Q' respectively. If $\frac{E_P}{E_Q} = \frac{4}{a}$ for $2|\sigma| = |\lambda|$, then the value of a is ___________.

[JEE (Main) – 13th April 2023 - Shift-1]

Sol. Correct answer is [6]



$$E_P = \frac{\sigma}{2\varepsilon_0} - \frac{\lambda}{2\pi\varepsilon_0\left(\frac{3}{\pi}\right)} = \frac{\sigma}{2\varepsilon_0} - \frac{\lambda}{6\varepsilon_0}$$

Given $|\lambda| = 2|\sigma|$

$$E_P = \frac{\sigma}{2\varepsilon_0} - \frac{2\sigma}{6\varepsilon_0} = \frac{\sigma}{6\varepsilon_0}$$

$$E_Q = \frac{\sigma}{2\varepsilon_0} - \frac{\lambda}{2\pi\varepsilon_0\left(\frac{4}{\pi}\right)} = \frac{\sigma}{2\varepsilon_0} - \frac{\lambda}{8\varepsilon_0}$$

Putting $|\lambda| = 2|\sigma|$

$$E_Q = \frac{\sigma}{2\varepsilon_0} - \frac{2\sigma}{8\varepsilon_0} = \frac{\sigma}{4\varepsilon_0}$$

$$\frac{E_P}{E_Q} = \frac{\sigma}{6\varepsilon_0} \times \frac{4\varepsilon_0}{\sigma} = \frac{2}{3} = \frac{4}{a}$$

On comparing, $a = 6$

14. 64 identical drops each charged upto potential of 10 mV are combined to form a bigger drop. The potential of the bigger drop will be _____ mV. **[JEE (Main) – 12th April 2023 - Shift-1]**

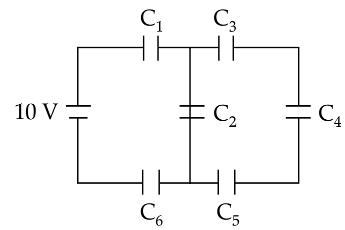
Sol. Correct answer is [0.16]

$$V' = Vn^{2/3}$$

$$\Rightarrow V' = 10 \times 10^{-3} \times 64^{2/3} = 0.16 \text{ volt}$$

15. In the given circuit:

$C_1 = 2$ μF, $C_2 = 0.2$ μF, $C_3 = 2$ μF, $C_4 = 4$ μF, $C_5 = 2$ μF, $C_6 = 2$ μF. The charge stored on capacitor C_4 is _________ μC.



[JEE (Main) – 11th April 2023 - Shift-2]

Sol. Correct answer is [4]

$$\frac{1}{C'} = \frac{1}{C_3} + \frac{1}{C_4} + \frac{1}{C_5} = \frac{1}{2} + \frac{1}{4} + \frac{1}{2}$$

$$C' = \frac{5}{4} \mu F$$

$$C'' = C' + C_2 = \frac{5}{4} + 0.2 = 1.45 \mu\text{F}$$

$$\frac{1}{C_{eq}} = \frac{1}{C''} + \frac{1}{C_1} + \frac{1}{C_6} = \frac{1}{C''} + \frac{1}{2} + \frac{1}{2}$$

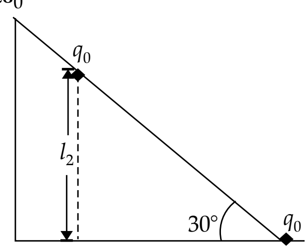
$\Rightarrow$ $C_{eq} = 0.5$ μF

Now, $Q = C_{eq} \times V = 0.5 \times 10 = 5$ μC

$$Q' = \frac{5 \mu\text{C} \times 0.8}{0.8 + 0.2} = 4 \mu\text{C}$$

16. As shown in the figure, a configuration of two equal point charges ($q_0 = +2$ μ C) is placed on an inclined plane. Mass of each point charge is 20 g. Assume that there is no friction between charge and plane. For the system of two point charges to be in equilibrium (at rest) the height, $h = x \times 10^{-3}$ m. The value of x is _________.

(Take $\frac{1}{4\pi\varepsilon_0}$ 9×10^{9} N m^2 C^{-2}, $g = 10$ m s^{-2})

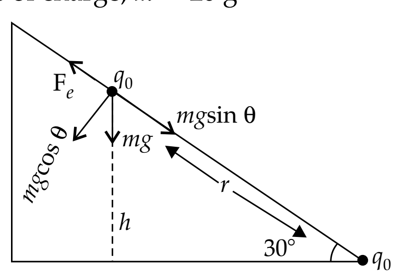


[JEE (Main) – 11th April 2023 - Shift-1]

Sol. Correct answer is [300]

Given, $q_1 = q_2 = 2$ μC

Mass of charge, $m = 20$ g



At equilibrium, $F_e = mg\sin\theta$

$$\Rightarrow \quad \frac{kq_1q_2}{r^2} = mg\sin 30$$

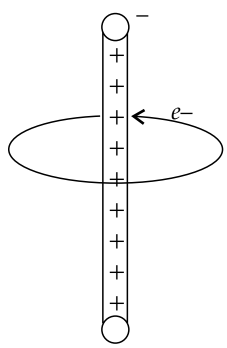
$$\Rightarrow \quad \frac{kq_1q_2}{\left(\frac{h}{\sin 30}\right)^2} = mg\sin 30$$

$$\Rightarrow \quad \frac{(9\times10^9\times2\times10^{-6}\times2\times10^{-6})}{4h^2} = 20\times10^{-3}\times10\times\frac{1}{2}$$

$$\Rightarrow \quad h = 300\times10^{-3}\text{ m}$$

On comparing with $h = x\times10^{-3}$, we get $x = 300$

17. An electron revolves around an infinite cylindrical wire having uniform linear charge density 2×10^{-8} C m^{-1} in circular path under the influence of attractive electrostatic field as shown in the figure. The velocity of electron with which it is revolving is ________ $\times 10^6$ m s^{-1}. Given mass of electron $= 9\times10^{-31}$ kg



[JEE (Main) – 10th April 2023 - Shift-2]

Sol. Correct answer is [8]

For circular motion, centripetal force is provided by attractive central electrostatic force

$$\frac{\lambda e}{2\pi\varepsilon_0 r} = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{2ke\lambda}{m}}$$

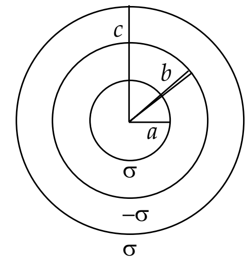
$$= \sqrt{\frac{2\times10^{-8}\times1.6\times10^{-19}\times2\times9\times10^9}{9\times10^{-31}}}$$

$$= \sqrt{\frac{2\times16\times18}{9}\times10^{12}} = 8\times10^6$$

18. Three concentric spherical metallic shells X, Y and Z of radius a, b and c respectively [$a < b < c$] have surface charge densities σ, $-\sigma$ and σ, respectively. The shells X and Z are at same potential. If the radii of X & Y are 2 cm and 3 cm, respectively. The radius of shell Z is ________ cm.

[JEE (Main) – 10th April 2023 - Shift-1]

Sol. Correct answer is [5]



Let $\quad q_a = \sigma(4\pi a^2)$
$q_b = -\sigma(4\pi b^2)$
$q_c = \sigma(4\pi c^2)$

$$V_a = k\left(\frac{q_a}{a}+\frac{q_b}{b}+\frac{q_c}{c}\right) = \frac{\sigma}{\varepsilon_0}(a-b+c)$$

$$V_b = k\left(\frac{q_a}{b}+\frac{q_b}{b}+\frac{q_c}{c}\right) = \frac{\sigma}{\varepsilon_0}\left(\frac{a^2}{b}-b+c\right)$$

$$V_c = k\left(\frac{q_a}{c}+\frac{q_b}{c}+\frac{q_c}{c}\right) = \frac{\sigma}{\varepsilon_0}\left(\frac{a^2}{c}-\frac{b^2}{c}+c\right)$$

Given: $\quad V_a = V_c$

$$\frac{\sigma}{\varepsilon_0}(a-b+c) = \frac{\sigma}{\varepsilon_0}\left(\frac{a^2-b^2}{c}+c\right)$$

$a = 2$ cm and $b = 3$ cm

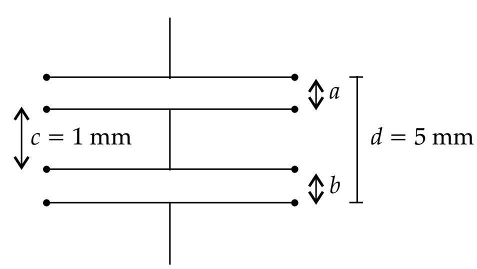
$$2-3+c = \frac{4-9}{c}+c$$

$$-1+c = \frac{-5}{c}+c$$

$$1 = \frac{5}{c}$$

$c = 5$ cm

19. As shown in the figure, two parallel plate capacitors having equal plate area of 200 cm^2 are joined in such a way that $a \neq b$. The equivalent capacitance of the combination is $x\varepsilon_0$F. The value of x is ________.



[JEE (Main) – 6th April 2023 - Shift-2]

Sol. Correct answer is [5]

Here, $\quad b = 5 - 1 - a = (4 - a)$ mm

Capacitance, $\quad C = \dfrac{\varepsilon_0 A}{d}$

In series, $\quad \dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$

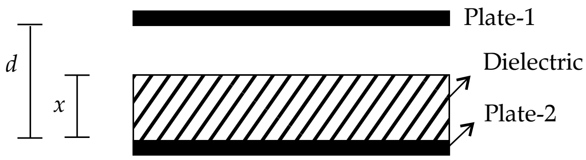
$$\Rightarrow \quad \frac{1}{C} = \frac{a}{\varepsilon_0 A} + \frac{4-a}{\varepsilon_0 A}$$

$\Rightarrow \quad C = \frac{\varepsilon_0 A}{4} = \frac{200 \times 10^{-4}}{4 \times 10^{-3}} \varepsilon_0$

$\Rightarrow \quad C = 5\varepsilon_0$

On comparing with $C = x\varepsilon_0$, we get, $x = 5$

20. A parallel plate capacitor with plate area A and plate separation d is filed with a dielectric material of dielectric constant K = 4. The thickness of the dielectric material is x, where $x < d$.



Let C_1 and C_2 be the capacitance of the system for $x = \frac{1}{3}d$ and $x = \frac{2d}{3}$, respectively. If $C_1 = 2\mu F$ the value of C_2 is ________ μF.

[JEE (Main) – 6th April 2023 - Shift-1]

Sol. Correct answer is [3]

Given, $\quad C_1 = 2\mu F$ and $K = 4$

$\Rightarrow \quad C_1 = \frac{\varepsilon_0 A}{\left(\frac{\frac{d}{3}}{K} + \frac{2d}{3}\right)} = \frac{\varepsilon_0 A}{\left(\frac{d}{3 \times 4} + \frac{2d}{3}\right)}$

$\Rightarrow \quad 2\mu F = \frac{\varepsilon_0 A \times 12}{9d} = \frac{4}{3}\frac{\varepsilon_0 A}{d}$

$\Rightarrow \quad 2\mu F = \frac{4}{3}\frac{\varepsilon_0 A}{d} \quad \left(\text{for } x = \frac{1}{3}d\right)$

$\Rightarrow \quad \frac{\varepsilon_0 A}{d} = \frac{3}{2}\mu F$

Now, $\quad C_2 = \frac{\varepsilon_0 A}{\left(\frac{\frac{2d}{3}}{K} + \frac{d}{3}\right)} \quad \left(\text{for } x = \frac{2d}{3}\right)$

$\Rightarrow \quad C_2 = \frac{\varepsilon_0 A}{\left(\frac{2d}{12} + \frac{d}{3}\right)}$

$\Rightarrow \quad C_2 = \frac{\varepsilon_0 A}{\left(\frac{2d}{12} + \frac{d}{3}\right)}$

$\Rightarrow \quad C_2 = \frac{12\varepsilon_0 A}{2d + 4d} = \frac{2\varepsilon_0 A}{d} = \frac{3}{2} \times 2$

$= 3\ \mu F$

21. Two parallel plate capacitors of capacity C and 3C are connected in parallel combination and charged to a potential difference 18 V. The battery is then disconnected and the space between the plates of the capacitor of capacity C is completely filled with a material of dielectric constant 9. The final potential difference across the combination of capacitors will be ________ V.

[JEE (Main) – 25th July 2022 - Shift-2]

Sol. Correct answer is [6]

When C and 3C are charged by 18 V, let q_1 (on C)=18 C and q_2 (on 3C) = 54 C

When a dielectric ($k = 9$) is inserted in C, its capacity becomes C′ = 9C

Let the new values of charges acquired by them be $q_1' = 9CV'$ and $q_2' = 3CV'$ where V′ is the common potential across each now.

As charge is conserved in this system

$q_1 + q_2 = q_1' + q_2'$

$18C + 54C = (9C + 3C)V'$

$V' = \frac{72}{12} = 6V$

22. Three–point charges of magnitudes 5μC, 0.16μC and 0.3μC are located at the vertices A, B, C of a right–angled triangle whose sides are AB = 3 cm, BC = $3\sqrt{2}$cm and CA = 3 cm and point A is the right–angle corner. Charge at point A experiences ________N of electrostatic force due to the other two charges.

[JEE (Main) – 26th July 2022 - Shift-2]

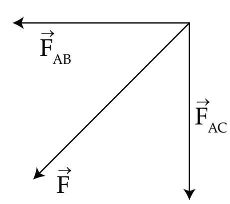
Sol. Correct answer is [17]

Net Force on charge at A, $\vec{F} = \vec{F}_{AB} + \vec{F}_{BC}$

$$F_{AB} = \frac{kQ_A Q_B}{r_{AB^2}} = k \times \frac{5 \times 10^{-6} \times 0.3 \times 10^{-6}}{9 \times 10^{-4}} = 15\ \text{N}$$

$$F_{AC} = \frac{kQ_B Q_C}{r_{BC^2}} = k \times \frac{5 \times 10^{-6} \times 0.16 \times 10^{-6}}{9 \times 10^{-4}} = 8\text{N}$$

$$|F| = \sqrt{15^2 + 8^2} = \sqrt{289} = 17\ \text{N}$$



23. The electric dipoles of dipole moments 1.2×10^{-30} cm and 2.4×10^{-30} cm are placed in two different uniform electric fields of strengths 5×10^4 NC^{-1} and 15×10^4 NC^{-1} respectively. The ratio of maximum torque experienced by the electric dipoles will be $\frac{1}{x}$. The value of x is ________.

[JEE (Main) – 28th July 2022 - Shift-1]

Sol. Correct answer is [6]

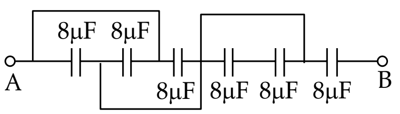
Torque on an electric dipole placed in uniform electric field is given by, $\tau = pE \cos\theta$ $\tau_{max} = pE$ $[\cos\theta = 1]$

$$\frac{\tau_2}{\tau_1} = \frac{p_2E_2}{p_1E_1}$$

$$\frac{\tau_2}{\tau_1} = \frac{1.2\times10^{-30}\times5\times10^4}{2.4\times10^{-30}\times15\times10^4} = \frac{1}{6}$$

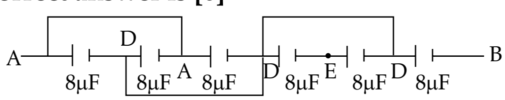
On comparing above equation with given $\frac{1}{x}$, we get $x = 6$

24. The equivalent capacitance between points A and B in below shown figure will be _______ μF.

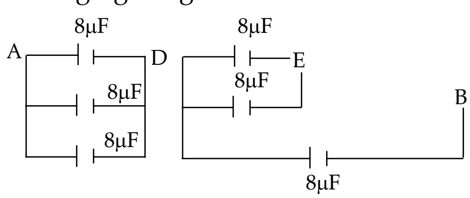


[JEE (Main) – 25th June 2022 - Shift-1]

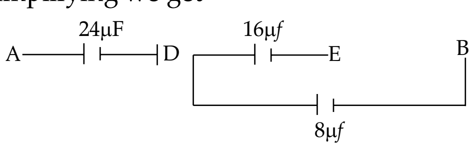
Sol. Correct answer is [6]



Rearranging the given circuit



Simplifying we get



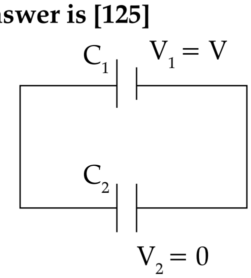
Since, terminal E is not connected to any part of the given circuit, 24 μf and 8 μf are seen to be in series across A and B

$$C_{AB} = \frac{24\times8}{32} = 6\,\mu F$$

25. A capacitor of capacitance 50 pF is charged by 100 V source. It is then connected to another uncharged identical capacitor. Electrostatic energy loss in the process is ___________ nJ.

[JEE (Main) – 27th June 2022 - Shift-1]

Sol. Correct answer is [125]



The loss of energy when two capacitors are connected at different voltages is given by

$$\Delta E = \frac{1}{2}\times\frac{C_1C_2}{(C_1+C_2)}(V_1 - V_2)^2$$

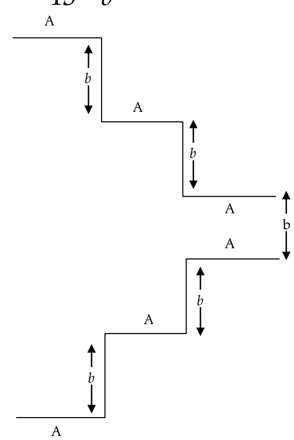
Here, $V_2 = 0, V_1 = 100$ V

$C_1 = 50$ pF, $C_2 = 50$ pF

$$\Delta E = \frac{1}{2}\times\frac{50\times50\times10^{-24}}{2\times50\times10^{-12}}\times(100)^2$$

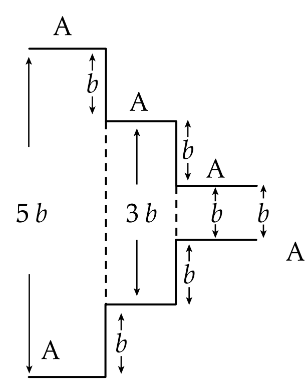
$\Delta E = 125$ nJ

26. A parallel plate capacitor is made up of stair like structure with a plate area A of each stair and that is connected with a wire of length b, as shown in the figure. The capacitance of the arrangement is $\frac{x}{15}\frac{\varepsilon_0 A}{b}$, The value of x is ______.



[JEE (Main) – 27th June 2022 - Shift-2]

Sol. Correct answer is [23]



This arrangement behaves as combination of parallel capacitors of 3 capacitors.

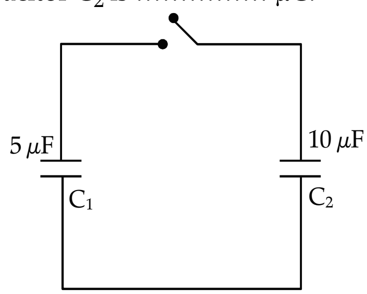
$$C_{eq} = C_1 + C_2 + C_3$$

$$= \varepsilon_0 A\left(\frac{1}{5b}+\frac{1}{3b}+\frac{1}{b}\right) = \frac{23}{15}\frac{\varepsilon_0 A}{b}$$

$$C_{eq} = \frac{23}{15}\times\frac{\varepsilon_0 A}{b}$$

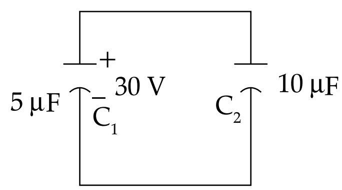
On comparing, we get, $x = 23$

27. A capacitor C_1 of capacitance 5 μF is charged to a potential of 30 V using a battery. The battery is then removed and the charged capacitor is connected to an uncharged capacitor C_2 of capacitance 10 μF as shown in figure. When the switch is closed charge flows between the capacitors. At equilibrium, the charge on the capacitor C_2 is …………… μC.



[JEE (Main) – 28th June 2022 - Shift-2]

Sol. Correct answer is [100]



At equilibrium both capacitors acquire a common potential

$$V_0 = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{5(30) + 10(0)}{5 + 10}$$

$= 10$ V

$Q_2 = C_2V_0 = 10 \times 10^{-6} \times 10 = 100\ \mu C$

28. The volume charge density of a sphere of radius 6 m is 2 $\mu C/cm^{-3}$. The number of lines of force per unit surface area coming out from the surface of the sphere is ….. $\times 10^{10}$ NC^{-1}

[Given: Permittivity of vacuum $\in_0 = 8.85 \times 10^{-12}\ C^2 N^{-1} - m^{-2}$)

[JEE (Main) – 25th July 2022 - Shift-1]

Sol. Correct answer is [45]

Electric field at the surface $= \frac{eR}{3\in_0} = E$

Electric flux $\phi_E = EA$

Number of electric lines (ϕ_E) per unit area

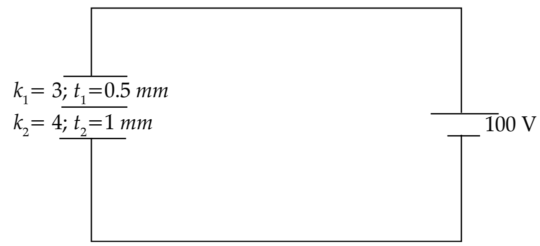
$$= \frac{\rho R}{3\in_0}$$

$$= \frac{2\times10^{-6}\times10^{+6}\times6}{3\times8.85\times10^{-12}}$$

$$= \frac{4}{8.85}\times10^{12}$$

$$= 45\times10^{10}$$

29. A composite parallel plate capacitor is made up of two different dielectric materials with different thickness (t_1 and t_2) as shown in figure. The two different dielectric materials are separated by a conducting foil F. The voltage of the conducting foil is __________V.



[JEE (Main) – 26th July 2022 - Shift-1]

Sol. Correct answer is [60]

$$C_1 = \frac{\varepsilon_0 k_1 A}{t} = \frac{3\varepsilon_0 A}{0.5} = 6\varepsilon_0 A$$

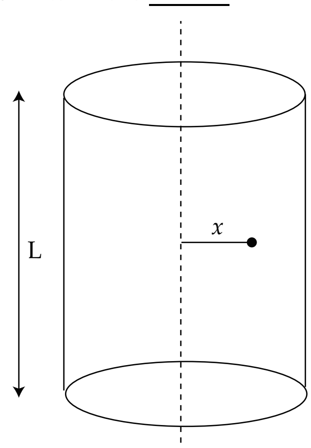
$$C_2 = \frac{\varepsilon_0 k_2 A}{t} = \frac{4\varepsilon_0 A}{1} = 4\varepsilon_0 A$$

Equivalent capacitance, $C = C_1 + C_2 = 10\varepsilon_0 A$

$$V_2 = \frac{C_1}{C}\times V$$

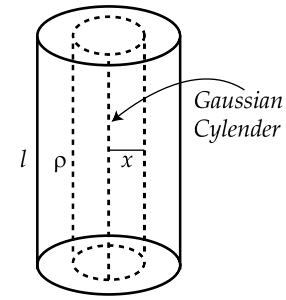
$$V_2 = \frac{6\varepsilon_0 A}{10\varepsilon_0 A}\times 100 = 60\,V$$

30. A long cylindrical volume contains a uniformly distributed charge of density $\rho\ Cm^{-3}$. The electric field inside the cylindrical volume at a distance $x = \frac{2\varepsilon_0}{\rho}$ from its axis is ______ Vm^{-1}.



[JEE (Main) – 27th July 2022 - Shift-1]

Sol. Correct answer is [1]



Applying gauss's theorem $= \vec{E}\vec{ds} = \frac{q_{encl}}{\varepsilon_0}$

$$E(2\pi x l) = \frac{\rho.\pi x^2 l}{\varepsilon_0}$$

$$E = \frac{\rho x}{2\varepsilon_0}$$

$$E\left(x = \frac{2\varepsilon_0}{\rho}\right) = \frac{\rho}{2\varepsilon_0} \times \frac{2\varepsilon_0}{\rho} = 1 \text{V/m}$$

31. *A* parallel plate capacitor with width 4 cm, length 8 cm and separation between the plates of 4 mm is connected to a battery of 20 V. *A* dielectric slab of dielectric constant 5 having length 1 cm, width 4 cm and thickness 4 mm is inserted between the plates of parallel plate capacitor. The electrostatic energy of this system will be ______ε_0J.

[JEE (Main) – 27th July 2022 - Shift-2]

Sol. Correct answer is [240]

Let A_1 = Area covered by vacuum (Air)

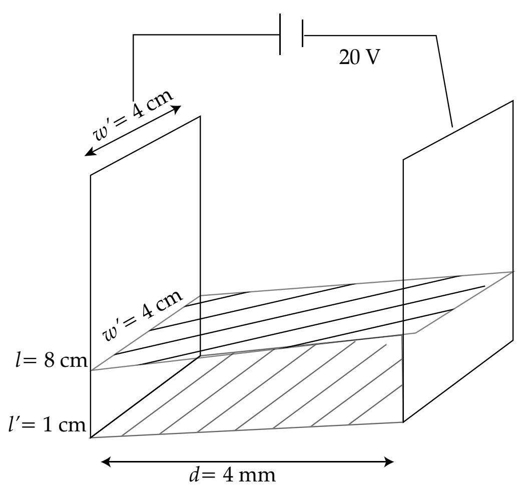
$= 7 \times 4 \times 10^{-4}$

$= 28 \times 10^{-4} m^2$

A_2 = Area covered by dielectric

$= 1 \times 4 \times 10^{-4}$

$= 4 \times 10^{-4} m^2$



Energy stored $= \frac{1}{2} C_{eq} V^2$

$C_{eq} = C_1 + C_2$

$= \frac{\varepsilon_0 A_1}{d} + \frac{K\varepsilon_0 A_2}{d}$

$= \frac{\varepsilon_0}{d}(A_1 + KA_2)$

$= \frac{\varepsilon_0}{4 \times 10^{-3}}$ (28 + 5 x 4) x 10^{-4}

$= \frac{\varepsilon_0}{40}$ x 48

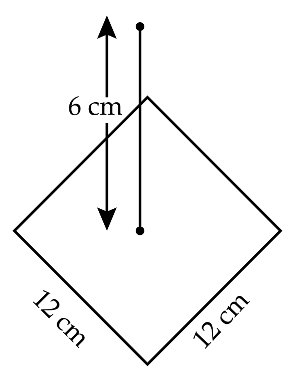
$C_{eq} = \frac{6\varepsilon_0}{5}$

$\therefore$ Energy $= \frac{1}{2} (\frac{6}{5}\varepsilon_0) (20)^2$

$= \frac{3}{5} \varepsilon_0 \times 400$

$= 240\ \varepsilon_0$ J

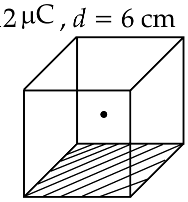
32. A point charge of +12 μC is at a distance 6 cm vertically above the centre of a square of side 12 cm as shown in figure. The magnitude of the electric flux through the square will be ______ $\times 10^3$ Nm^2/C.



[JEE (Main) – 24th Feb 2021 - Shift-2]

Sol. Correct answer is [226]

Given, Q = 12 μC , d = 6 cm



Electric flux through the cube $\phi = \frac{Q}{\varepsilon_0}$

Hence, electric flux through the one face of the cube

$$\phi_1 = \frac{Q}{6\varepsilon_0} = \frac{12}{6 \times 8.8 \times 10^{-12}} \frac{Nm^2}{C}$$

$$= 226 \times 10^3 \frac{Nm^2}{C}$$

33. The electric field in a region is given by $\vec{E} = \left(\frac{3}{5}E_0\hat{i} + \frac{4}{5}E_0\hat{j}\right)\frac{N}{C}$. The ratio of flux of reported field through the rectangular surface of area 0.2 m^2 (parallel to *y-z* plane) to that of the surface of area 0.3 m^2 (parallel to *x-z* plane) is *a* : *b*, where *a* = ...

[Here $\hat{i}.\hat{j}$ and $\hat{k}$ are unit vectors along *x*, *y* and *z*-axes respectively].

[JEE (Main) – 25th Feb 2021 - Shift-1]

Sol. Correct answer is [1]

Given,

$$\vec{E} = \left(\frac{3}{5}E_0\hat{i} + \frac{4}{5}E_0\hat{j}\right)\frac{M}{C}$$

Surface area $\vec{A}_x = 0.2\ m^2(\hat{i})$

$$\vec{A}_y = 0.3\ m^2\,\hat{j}$$

$$\phi_a = \vec{E}\cdot\vec{A}_x$$

$$= \left(\frac{3}{5}E_0\hat{i} + \frac{4}{5}E_0\hat{j}\right)(0.2\hat{i})$$

$$= \frac{0.6}{5}E_0$$

$$\phi_b = \vec{E}\cdot\vec{A}_y = \left(\frac{3}{5}E_0\hat{i} + \frac{4}{5}E_0\hat{j}\right)(0.3\ m^2\hat{j})$$

$$= \frac{1.2}{5}E_0$$

$$a/b = \phi a/\phi b$$

$$\frac{a}{b} = \frac{\frac{0.6}{5}E^0}{\frac{1.2}{5}E^0} = \frac{0.6}{1.2}$$

$$\frac{a}{b} = \frac{1}{2}$$

34. 512 identical drops of mercury are charged to a potential of 2 V each. The drops are joined to form a single drop. The potential of this drop is .... V. **[JEE (Main) – 25th Feb 2021 - Shift-1]**

Sol. Correct answer is [128]

Total no of drops (n) = 512

$$V = 2\ V$$

Let Charge on drops = q

Let radius on drops = r

$$V = \frac{kq}{r}$$

$$\Rightarrow \quad 2 = \frac{kq}{r}$$

⇒ Let the radius of bigger drops = R

$$\Rightarrow \quad \frac{4}{3}\pi R^3 = 512 \times \frac{4}{3}\pi r^3$$

$$\Rightarrow \quad R^3 = 512\,r^3$$

$$\Rightarrow \quad R^3 = 8^3 r^3$$

$$\Rightarrow \quad R = 8r$$

$$V = \frac{k(512)q}{R}$$

$$\Rightarrow \quad V = \frac{512}{8} \times \frac{kq}{r}$$

$$\Rightarrow \quad V = \frac{512}{8} \times 2$$

$$V = 128\ V$$

35. Two identical conducting spheres with negligible volume have 2.1 nC and –0.1 nC charges, respectively. They are brought into contact and then separated by a distance of 0.5 m. The electrostatic force acting between the spheres is ________ $\times 10^{-9}$ N.

[Given : $\frac{1}{4\pi\varepsilon_0} = \frac{1}{9 \times 10^9}$ SI unit]

[JEE (Main) – 25th Feb 2021 - Shift-2]

Sol. Correct answer is [36]

Initially, charges on each sphere

$$Q_1 = 2.1\ nC$$

$$Q_2 = -0.1\ nC$$

After, touching each other

$$Q = \frac{Q_1 + Q_2}{2} = \frac{2.1 - 0.1}{2} = 1\ nC$$

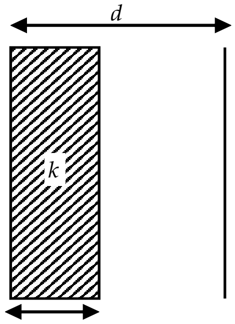
$$F = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{0.5^2}$$

$$\Rightarrow \quad F = 9 \times 10^9 \times \frac{(1nC)^2}{\frac{1}{4}}$$

$$= 36 \times 10^{-9}\ N$$

36. In a parallel plate capacitor set up, the plate area of capacitor is 2 m^2 and the plates are separated by 1 m. If the space between the plates are filled with a dielectric material of thickness 0.5 m and area 2 m^2 (see fig) the capacitance of the set – up will be ______ ε_0.

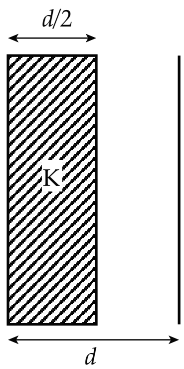
(Dielectric constant of the material = 3.2)
(Round off to the nearest integer)



[JEE (Main) – 16th March 2021 - Shift-2]

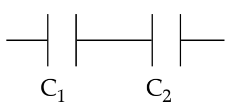
Sol. Correct answer is [3]

$A = 2\ m^2 \qquad d = 1\ m$



$K = 3.2$

$C_1 = \dfrac{KA\varepsilon_o}{\frac{d}{2}}, \quad C_2 = \dfrac{A\varepsilon_o}{\frac{d}{2}}$



$$\frac{1}{C_{equi}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C_{eq} = \frac{C_1\ C_2}{C_1 + C_2} = \frac{\frac{2A\varepsilon_o}{d}\left(\frac{K2A\varepsilon_o}{d}\right)}{\frac{2A\varepsilon_o}{d}[K+1]}$$

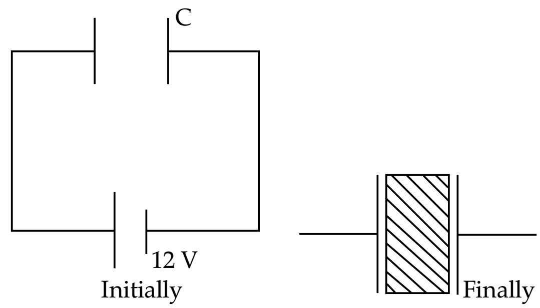
$$C_{eq} = \frac{3.2\times 2\times 2\varepsilon_o}{[3.2+1]} \Rightarrow \frac{4\times 3.2\varepsilon_o}{4.2} \Rightarrow 3.04\ \varepsilon_o$$

37. A parallel plate capacitor whose capacitance C is 14 pF is charged by a battery to a potential difference V = 12 V between its plates. The charging battery is now disconnected and a porcelin plate with K = 7 is inserted between the plates, then the plate would oscillate back and forth between the plates, with a constant mechanical energy of ____ pJ (Assume no friction).

[JEE (Main) – 17th March 2021 - Shift-1]

Sol. Correct answer is [864]

Given, C = 14 pF



Initially energy 'U_i' $U_i = \dfrac{Q^2}{2C}$ ...(i)

Finally energy 'U_f' $U_f = \dfrac{Q^2}{2KC}$ ...(ii)

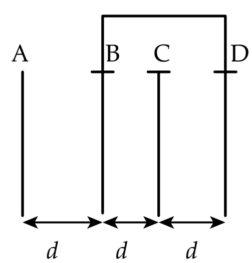
Oscillating energy = $U_i - U_f$

$$= \frac{Q^2}{2C} - \frac{Q^2}{2kC} = \frac{Q^2}{2C}\left[1 - \frac{1}{K}\right]$$

$$= \frac{C^2V^2}{2C}\left[1 - \frac{1}{7}\right]$$

$$= \frac{6}{7}\left[\frac{14}{2}\times 144\right] = 864\ \text{pJ}$$

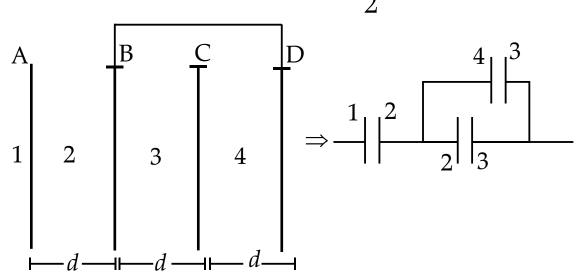
38. Four identical rectangular plates with length, l = 2 cm and breadth, b = 3/2 cm are arranged as shown in figure. The equivalent capacitance between A and C is $\dfrac{x\varepsilon_0}{d}$. The value of x is .... (Round off to the nearest integer)



[JEE (Main) – 17th March 2021 - Shift-1]

Sol. Correct answer is [2]

Area of each plate $= 2\times\dfrac{3}{2} = 3\ cm^2$



Capacitance of a parallel plate

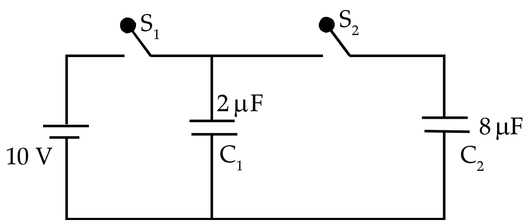
Capacitor 'C' $= \dfrac{A\varepsilon_0}{d}$

$$C_{equi.} = \frac{2C \times C}{2C + C} = \frac{2C^2}{3C} = \frac{2}{3}C$$

$$\Rightarrow \quad = \frac{2}{3}\left[\frac{A\varepsilon_0}{d}\right] = \frac{2}{3}\left[\frac{3\varepsilon_0}{d}\right]$$

$$x = 2$$

39. A 2 μF capacitor C_1 is first charged to a potential difference of 10 V using a battery. Then the battery is removed and the capacitor is connected to an uncharged capacitor C_2 of 8 μF. The charge in C_2 on equilibrium condition is ______ μC.
(Round off to the nearest integer)



[JEE (Main) – 17th March 2021 - Shift-2]

Sol. Correct answer is [16]

$$\text{Common Potential (V)} = \frac{\text{total charge}}{\text{total capacitance}}$$

$$V = \frac{10 \times 2\mu f}{C_1 + C_2} = \frac{20\,\mu C}{10\,\mu f} = 2 \text{ volt}$$

Charge on C_2 after equilibrium $= 8\mu F \times 2$ volt

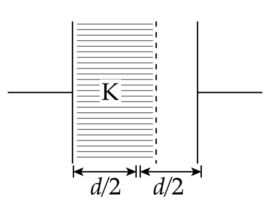
$\Rightarrow \quad 16\ \mu$ C

40. A parallel plate capacitor has plate area 100 m^2 and plate separation of 10 m. The space between the plates is filled up to a thickness 5 m with a material of dielectric constant of 10. The resultant capacitance of the system is 'x' pF.
The value of $\varepsilon_0 = 8.85 \times 10^{-12}$ F.m^{-1}
The value of 'x' to the nearest integer is _______.
[JEE (Main) – 18th March 2021 - Shift-1]

Sol. Correct answer is [161]

Given, Area (A) $= 100$ m^2
Plate separation $= 10$ m
Thickness of a dielectric material (t) $= 5$ m
Dielectric constant (k) $= 10$

$$C_1 = \frac{KA\varepsilon_0}{\frac{d}{2}}, C_2 = \frac{A\varepsilon_0}{\frac{d}{2}}$$

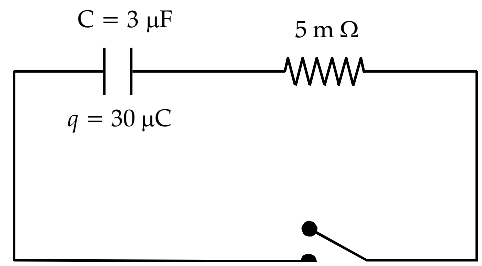


$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{K\left(\frac{2A\varepsilon_0}{d}\right)\left(\frac{2A\varepsilon_0}{d}\right)}{\frac{2A\varepsilon_0}{d} + \frac{2A\varepsilon_0}{d}K}$$

$$= \frac{\frac{2KA\varepsilon_0}{d}}{(K+1)} = \frac{2 \times 10 \times 100 \times 8.85 \times 10^{-12}}{5(11)}$$

$$= 160.90 \times 10^{-12} = 161\,\text{pF}$$

41.



The circuit shown in the figure consists of a charged capacitor of capacity 3 μF and a charge of 30 μC. At time $t = 0$, when the key is closed, the value of current flowing through the 5 μΩ resistor is 'x' μA.

The value of 'x' to the nearest integer is _____.**[JEE (Main) – 18th March 2021 - Shift-1]**

Sol. Correct answer is [2]

Using the expression

$$I = I_0 e^{\frac{-t}{RC}}$$

$$I_0 = \frac{Q_0}{t} = \frac{Q_0}{R_C}$$

$$\text{At} \quad t = 0, \ I = \frac{Q}{RC} = \frac{30 \times 10^{-6}}{5 \times 10^{-3} \times 3 \times 10^{-6}}$$

$$I = 2\ \mu A.$$

42. An infinite number of point charges, each carrying 1 μC charge, are placed along the y–axis at $y = 1$ m, 2 m, 4 m, 8 m _______.

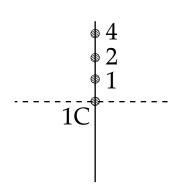
The total force on a 1C point charge, placed at the origin, is $x \times 10^3$ N.

The value of x, to the nearest integer, is _______.

$$\left[\text{Take } \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \frac{Nm^2}{C^2}\right]$$

[JEE (Main) – 18th March 2021 - Shift-2]

Sol. Correct answer is [12]



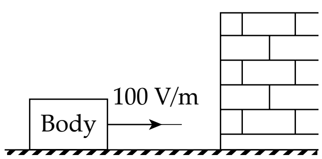
Using Coulomb's law Total force on 1C point change due 1 μc charge

$$F_{total} = \frac{kq_1q_2}{r_1^2} + \frac{kq_1q_3}{r_2^2} + \frac{kq_1q_4}{r_3^2}$$

$$= k \times 10^{-6}\left[1 + \frac{1}{(2)^2} + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{8}\right)^2 ...\right]$$

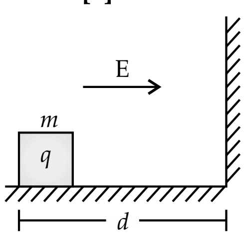
$$= 9\times10^9\times10^{-6}\left[\frac{1}{1-\frac{1}{4}}\right] = 12\times10^3 \text{ N}$$

43. A body having specific charge 8 μC/g is resting on a frictionless plane at a distance 10 cm from the wall (as shown in the figure). It starts moving towards the wall when a uniform electric field of 100 V/m is applied horizontally toward the wall. If the collision of the body with the wall is perfectly elastic, then the time period of the motion will be _________ *s*.



[JEE (Main) – 20th July 2021 - Shift-1]

Sol. Correct answer is [1]



Electric force, $F = qE$

$$ma = qE$$

$$a = \frac{qE}{m}$$

Use second equation of motion

$$d = ut + \frac{1}{2}at^2 \quad [\because u = 0 \text{ m/s}]$$

$$d = \frac{1}{2}at^2 = \frac{1}{2}\left[\frac{qE}{m}\right]t^2$$

$$t = \sqrt{\frac{2d}{\frac{qE}{m}}} = \sqrt{\frac{2\times0.1}{8\times10^{-3}(100)}}$$

$$= \sqrt{\frac{2000}{8000}} = \frac{1}{2}$$

It will take same time to return back its initial position.

$$\text{Time period (T)} = 2\left(\frac{1}{2}\right) = 1 \text{ s.}$$

44. The total charge enclosed in an incremental volume of 2×10^{-9} m^3 located at the origin is _____ nC, if electric flux density of its field is found as $D = e^{-x}\sin y\hat{i} - e^{-x}\cos y\hat{j} + 2z\hat{k}$ C/m^2

[JEE (Main) – 22nd July 2021 - Shift-2]

Sol. Correct answer is [4]

Electric flux density $(\vec{D})$

$$\vec{D} = \frac{\text{charge}}{\text{area}}\hat{r}$$

$$= \frac{\theta}{4\pi r^2}\hat{r}$$

$$= \varepsilon_0\left(\frac{\theta}{4\pi\varepsilon_0 r^2}\hat{r}\right)$$

$$\vec{E} = \frac{\vec{D}}{\varepsilon_0}$$

$$= \left[ex\sin y\,\hat{i} - ex\cos y\,\hat{j} + 2z\hat{k}\right]\frac{C}{m^2}$$

By Gauss's Law

$$\frac{\rho}{\varepsilon_0} = \left(\frac{\partial}{\partial_x}\hat{i} + \frac{\partial}{\partial_y}\hat{j} + \frac{\partial}{\partial_z}\hat{k}\right)\vec{E}$$

$$\Rightarrow \frac{\partial}{\partial_x}(ex\sin y) + \frac{\partial}{\partial_y}(ex\cos y) + \frac{\partial}{\partial_z}(2z)$$

$$\rho = -ex\sin y + ex\sin y + 2$$

At $x = 0$ $\rho = 2$ c/m^3

Charge $= \rho \times$ volume

$= 2 \times 2 \times 10^{-9}$

$= 4$ μC

45. If the maximum value of accelerating potential provided by a radio frequency oscillator is 12 kV. The number of revolution made by a proton in a cyclotron to achieve one sixth of the speed of light is ___________.

[$m_p = 1.67 \times 10^{-27}$kg, $e = 1.6 \times 10^{-19}$ C, speed of light $= 3 \times 10^8$ m/s]

[JEE (Main) – 22nd July 2021 - Shift-2]

Sol. Correct answer is [543.4]

Applied accelerating potential

$= 12 \times 10^3$ V

In one rotation energy gained by proton

$= (2q_p.v)$

In n rotations $= n \times 2q_p v$

Which is equal to kinetic eneryg of proton

$$n \times q_p v = \frac{1}{2}m_p v^2 = \frac{1}{2}m_p \times \left(\frac{c}{6}\right)^2$$

$$n[2 \times 1.6 \times 10^{-19} \times 12 \times 10^3] = \frac{1}{2}\times 1.6\times 10^{-27}\left(\frac{3\times 10^8}{6}\right)^2$$

$$n(384 \times 10^{-16}) = 0.02087 \times 10^{11}$$

$$n = 543.4$$

46. A parallel plate capacitor of capacitance 200 μF is connected to a battery of 200 V. A dielectric slab of dielectric constant 2 is now inserted into the space between plates of capacitor while the battery remain connected. The change in the electrostatic energy in the capacitor will be ______ J. **[JEE (Main) – 31st Aug 2021 - Shift-2]**

Sol. Correct answer is [4]

Energy of capacitor when no dielectric plate is inserted

$$U_1 = \frac{1}{2}CV^2$$

Now when dielectric slab of dielectric constant k is inserted between the plates of capacitor

$$\text{Energy } U_2 = \frac{1}{2}(KC)V^2$$

$$\Delta U = U_2 - U_1 = \frac{1}{2}V^2(KC - C)$$

$$= \frac{1}{2}CV^2(K-1)$$

$$= \frac{1}{2}\times 200\times 10^{-6}\times 200\times 200$$

$$\Delta U = 4 \text{ J}$$

47. Assume that an electric field $\vec{E} = 30x^2\hat{i}$ exists in space. Then the potential difference $V_A - V_O$, where V_O is the potential at the origin and V_A the potential at x = 2 m is :

Sol. Correct answer is [80]

Given :

$$\vec{E} = 30x^2\hat{i}. \quad \text{...(i)}$$

To find : $V_A - V_O$, V_A is potential at $x = 2$ m and V_O is potential at $x = 0$ m.

$$dV = -Edx;$$

$$\int_{V_O}^{V_A} dV = -\int_0^2 30x^2 dx$$

$$V_A - V_O = -30\frac{[x^3]_0^2}{3} = -80 \text{ J}$$

❐❐❐