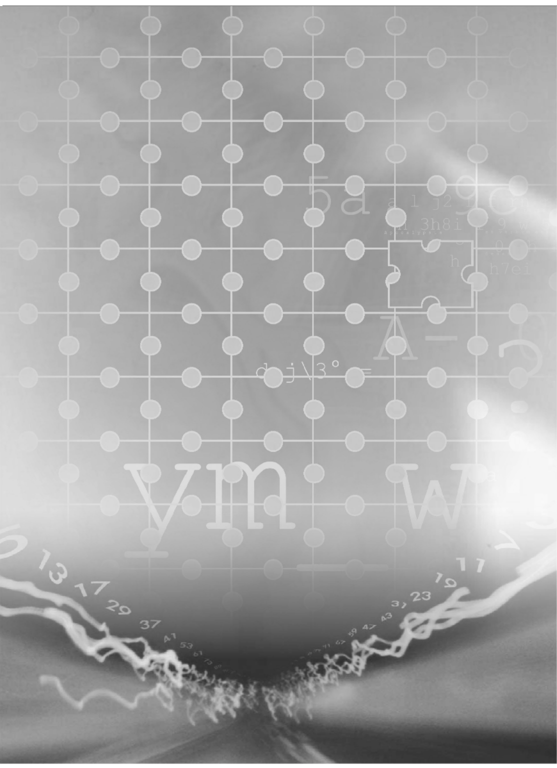


MATHEMATICS

Formula Book for Engineering Entrance Examinations

Best Wishes for

Your Success in Competitive Examinations ahead !!!

SHORT FORMULA (GYAN SUTRA)

MATHEMATICS

STRAIGHT LINE

1. Distance Formula: $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

2. Section Formula : $x = \dfrac{mx_2 \pm nx_1}{m \pm n}$; $y = \dfrac{my_2 \pm ny_1}{m \pm n}$.

3. Centroid, Incentre & Excentre:

Centroid G $\left(\dfrac{x_1 + x_2 + x_3}{3}, \dfrac{y_1 + y_2 + y_3}{3}\right)$, Incentre I $\left(\dfrac{ax_1 + bx_2 + cx_3}{a+b+c}, \dfrac{ay_1 + by_2 + cy_3}{a+b+c}\right)$

Excentre $I_1\left(\dfrac{-ax_1 + bx_2 + cx_3}{-a+b+c}, \dfrac{-ay_1 + by_2 + cy_3}{-a+b+c}\right)$

4. Area of a Triangle:

$$\Delta ABC = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

5. Slope Formula:

(i) Line Joining two points $(x_1\ y_1)$ & $(x_2\ y_2)$, $m = \dfrac{y_1 - y_2}{x_1 - x_2}$

6. Condition of collinearity of three points: $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

7. Angle between two straight lines : $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$.

8. Two Lines : $ax + by + c = 0$ and $a'x + b'y + c' = 0$ two lines

1. parallel if $\dfrac{a}{a'} = \dfrac{b}{b'} \neq \dfrac{c}{c'}$.

2. Distance between two parallel lines = $\left|\dfrac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$.

3 Perpendicular : If $aa' + bb' = 0$.

9. A point and line:

1. Distance between point and line = $\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}}\right|$.

2. Reflection of a point about a line: $\frac{x - x_1}{a} = \frac{y - y_1}{b} = -2\frac{ax_1 + by_1 + c}{a^2 + b^2}$

3. Foot of the perpendicular from a point on the line is $\frac{x-x_1}{a} = \frac{y-y_1}{b} = -\frac{ax_1+by_1+c}{a^2+b^2}$

10. Bisectors of the angles between two lines: $\frac{ax + by + c}{\sqrt{a^2+b^2}} = \pm \frac{a'x + b'y + c'}{\sqrt{a'^2+b'^2}}$

11. Condition of Concurrency :

of three straight lines $a_ix + b_iy + c_i = 0$, $i = 1,2,3$ is $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$.

12. A Pair of straight lines through origin: $ax^2 + 2hxy + by^2 = 0$

If θ is the acute angle between the pair of straight lines, then $\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a + b}\right|$.

CIRCLE

1. Intercepts made by Circle $x^2 + y^2 + 2gx + 2fy + c = 0$ **on the Axes:**

(a) $2\sqrt{g^2 - c}$ on x -axis (b) $2\sqrt{f^2 - c}$ on y - aixs

2. Parametric Equations of a Circle: $x = h + r\cos\theta$; $y = k + r\sin\theta$

3. Tangent :

(a) Slope form : $y = mx \pm a\sqrt{1+m^2}$

(b) Point form : $xx_1 + yy_1 = a^2$ or T = o

(c) Parametric form : $x\cos\alpha + y\sin\alpha = a$.

4. Pair of Tangents from a Point: $SS_1 = T^2$.

5. Length of a Tangent : Length of tangent is $\sqrt{S_1}$

6. Director Circle: $x^2 + y^2 = 2a^2$ for $x^2 + y^2 = a^2$

7. Chord of Contact: T = 0

1. Length of chord of contact = $\frac{2LR}{\sqrt{R^2+L^2}}$

2. Area of the triangle formed by the pair of the tangents & its chord of contact = $\frac{RL^3}{R^2+L^2}$

3. Tangent of the angle between the pair of tangents from $(x_1, y_1) = \left(\frac{2RL}{L^2 - R^2}\right)$

4. Equation of the circle circumscribing the triangle PT_1T_2 is : $(x - x_1)(x + g) + (y - y_1)(y + f) = 0$.

8. Condition of orthogonality of Two Circles: $2g_1g_2 + 2f_1f_2 = c_1 + c_2$.

9. Radical Axis : $S_1 - S_2 = 0$ i.e. $2(g_1 - g_2)x + 2(f_1 - f_2)y + (c_1 - c_2) = 0$.

10. Family of Circles: $S_1 + KS_2 = 0$, $S + KL = 0$.

PARABOLA

1. Equation of standard parabola :

$y^2 = 4ax$, Vertex is (0, 0), focus is (a, 0), Directrix is $x + a = 0$ and Axis is $y = 0$

Length of the latus rectum = 4a, ends of the latus rectum are L(a, 2a) & L' (a, – 2a).

2. Parametric Representation: $x = at^2$ & $y = 2at$

3. Tangents to the Parabola $y^2 = 4ax$:

1. Slope form $y = mx + \frac{a}{m}$ $(m \neq 0)$ 2. Parametric form $ty = x + at^2$

3. Point form T = 0

4. Normals to the parabola $y^2 = 4ax$:

$y - y_1 = -\frac{y_1}{2a}(x - x_1)$ at (x_1, y_1) ; $y = mx - 2am - am^3$ at $(am^2, -2am)$; $y + tx = 2at + at^3$ at $(at^2, 2at)$.

ELLIPSE

1. Standard Equation : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a > b$ & $b^2 = a^2(1 - e^2)$.

Eccentricity: $e = \sqrt{1 - \frac{b^2}{a^2}}$, $(0 < e < 1)$, **Directrices :** $x = \pm \frac{a}{e}$.

Focii : $S \equiv (\pm ae, 0)$. Length of, major axes = 2a and minor axes = 2b

Vertices : $A' \equiv (-a, 0)$ & $A \equiv (a, 0)$.

Latus Rectum : = $\frac{2b^2}{a} = 2a(1 - e^2)$

2. Auxiliary Circle : $x^2 + y^2 = a^2$

3. Parametric Representation : $x = a\cos\theta$ & $y = b\sin\theta$

4. Position of a Point w.r.t. an Ellipse:

The point $P(x_1, y_1)$ lies outside, inside or on the ellipse according as ; $\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - 1 > <$ or $= 0$.

5. Line and an Ellipse: The line $y = mx + c$ meets the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in two points real, coincident or imaginary according as c^2 is $< =$ or $> a^2m^2 + b^2$.

6. Tangents: Slope form: $y = mx \pm \sqrt{a^2m^2 + b^2}$, Point form : $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$,

Parametric form: $\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$

7. **Normals:** $\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2$, $ax.\sec\theta - by\,\text{cosec}\,\theta = (a^2 - b^2)$, $y = mx - \frac{(a^2 - b^2)m}{\sqrt{a^2 + b^2m^2}}$.

8. **Director Circle:** $x^2 + y^2 = a^2 + b^2$

HYPERBOLA

1. **Standard Equation:** Standard equation of the hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, where $b^2 = a^2(e^2 - 1)$.

Focii : $S \equiv (\pm ae, 0)$ **Directrices :** $x = \pm \frac{a}{e}$

Vertices : $A \equiv (\pm a, 0)$

Latus Rectum (ℓ) : $\ell = \frac{2b^2}{a} = 2a(e^2 - 1)$.

2. **Conjugate Hyperbola :** $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ & $-\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are conjugate hyperbolas of each.

3. **Auxiliary Circle :** $x^2 + y^2 = a^2$.

4. **Parametric Representation :** $x = a\sec\theta$ & $y = b\tan\theta$

5. **Position of A Point 'P' w.r.t. A Hyperbola :**

$S_1 \equiv \frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} - 1$ >, = or < 0 according as the point (x_1, y_1) lies inside, on or outside the curve.

6. **Tangents :**

(i) **Slope Form :** $y = mx \pm \sqrt{a^2m^2 - b^2}$

(ii) **Point Form :** at the point (x_1, y_1) is $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$.

(iii) **Parametric Form :** $\frac{x\sec\theta}{a} - \frac{y\tan\theta}{b} = 1$.

7. **Normals :**

(a) at the point $P(x_1, y_1)$ is $\frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2 + b^2 = a^2e^2$.

(b) at the point $P(a\sec\theta, b\tan\theta)$ is $\frac{ax}{\sec\theta} + \frac{by}{\tan\theta} = a^2 + b^2 = a^2e^2$.

(c) Equation of normals in terms of its slope 'm' are $y = mx \pm \frac{(a^2 + b^2)m}{\sqrt{a^2 - b^2m^2}}$.

8. **Asymptotes :** $\frac{x}{a} + \frac{y}{b} = 0$ and $\frac{x}{a} - \frac{y}{b} = 0$. Pair of asymptotes : $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$.

9. Rectangular Or Equilateral Hyperbola : $xy = c^2$, eccentricity is $\sqrt{2}$.

Vertices : ($\pm c \cdot \pm c$) ; Focii : $\left(\pm\sqrt{2}c, \pm\sqrt{2}c\right)$. Directrices : $x + y = \pm\sqrt{2}\,c$

Latus Rectum (l) : $\ell = 2\sqrt{2}\,c$ = T.A. = C.A.

Parametric equation $x = ct$, $y = c/t$, $t \in R - \{0\}$

Equation of the tangent at P (x_1, y_1) is $\frac{x}{x_1} + \frac{y}{y_1} = 2$ & at P (t) is $\frac{x}{t} + t\,y = 2c$.

Equation of the normal at P (t) is $x t^3 - y t = c(t^4 - 1)$.

Chord with a given middle point as (h, k) is $kx + hy = 2hk$.

LIMIT OF FUNCTION

1. Limit of a function f(x) is said to exist as $x \to a$ when,

$$\underset{h\to 0^+}{\text{Limit}}\ f(a-h) = \underset{h\to 0^+}{\text{Limit}}\ f(a+h) = \text{some finite value } M.$$

(Left hand limit) (Right hand limit)

2. Indeterminant Forms:

$$\frac{0}{0}, \frac{\infty}{\infty}, 0 \times \infty, \infty - \infty, \infty^0, 0^0, \text{ and } 1^\infty.$$

3. Standard Limits:

$$\underset{x\to 0}{\text{Limit}}\ \frac{\sin x}{x} = \underset{x\to 0}{\text{Limit}}\ \frac{\tan x}{x} = \underset{x\to 0}{\text{Limit}}\ \frac{\tan^{-1}x}{x} = \underset{x\to 0}{\text{Limit}}\ \frac{\sin^{-1}x}{x} = \underset{x\to 0}{\text{Limit}}\ \frac{e^x - 1}{x} = \underset{x\to 0}{\text{Limit}}\ \frac{\ell n(1+x)}{x} = 1$$

$$\underset{x\to 0}{\text{Limit}}\ (1+x)^{1/x} = \underset{x\to \infty}{\text{Limit}}\ \left(1+\frac{1}{x}\right)^x = e,\quad \underset{x\to 0}{\text{Limit}}\ \frac{a^x - 1}{x} = \log_e a,\ a > 0,\quad \underset{x\to a}{\text{Limit}}\ \frac{x^n - a^n}{x - a} = na^{n-1}.$$

4. Limits Using Expansion

(i) $a^x = 1 + \frac{x\ln a}{1!} + \frac{x^2 \ln^2 a}{2!} + \frac{x^3 \ln^3 a}{3!} + \ldots\ldots\ldots a > 0$

(ii) $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots$

(iii) $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\ldots\ldots \text{for } -1 < x \le 1$

(iv) $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots\ldots$

(v) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots\ldots$

(vi) $\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \ldots\ldots$

(vii) $\tan^{-1}x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots.$

(viii) $\sin^{-1}x = x + \frac{1^2}{3!}x^3 + \frac{1^2.3^2}{5!}x^5 + \frac{1^2.3^2.5^2}{7!}x^7 + \ldots\ldots$

(x) for $|x| < 1$, $n \in R$ $(1+x)^n = 1 + nx + \frac{n(n-1)}{1.2}x^2 + \frac{n(n-1)(n-2)}{1.2.3}x^3 + \ldots\ldots\ldots\ldots\infty$

5. Limits of form 1^∞, 0^0, ∞^0

Also for $(1)^\infty$ type of problems we can use following rules.

$\lim_{x\to 0} (1 + x)^{1/x} = e$, $\lim_{x\to a} [f(x)]^{g(x)}$, where $f(x) \to 1$; $g(x) \to \infty$ as $x \to a = \lim_{x\to a} = e^{\lim_{x\to a}[f(x)-1]\,g(x)}$

6. Sandwich Theorem or Squeeze Play Theorem:

If $f(x) \le g(x) \le h(x)\ \forall\ x$ & $\text{Limit}_{x\to a} f(x) = \ell = \text{Limit}_{x\to a} h(x)$ then $\text{Limit}_{x\to a} g(x) = \ell$.

METHOD OF DIFFERENTIATION

1. Differentiation of some elementary functions

1. $\frac{d}{dx}(x^n) = nx^{n-1}$ **2.** $\frac{d}{dx}(a^x) = a^x \ell n\, a$ **3.** $\frac{d}{dx}(\ell n\,|x|) = \frac{1}{x}$ **4.** $\frac{d}{dx}(\log_a x) = \frac{1}{x\,\ell n\, a}$

5. $\frac{d}{dx}(\sin x) = \cos x$ **6.** $\frac{d}{dx}(\cos x) = -\sin x$ **7.** $\frac{d}{dx}(\sec x) = \sec x\ \tan x$

8. $\frac{d}{dx}(\text{cosec } x) = -\text{cosec } x \cot x$ **9.** $\frac{d}{dx}(\tan x) = \sec^2 x$ **10.** $\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$

2. Basic Theorems

1. $\frac{d}{dx}(f \pm g) = f'(x) \pm g'(x)$ **2.** $\frac{d}{dx}(k\, f(x)) = k \frac{d}{dx} f(x)$ **3.** $\frac{d}{dx}(f(x)\,.\,g(x)) = f(x)\, g'(x) + g(x)\, f'(x)$

4. $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\, f'(x) - f(x) g'(x)}{g^2(x)}$ **5.** $\frac{d}{dx}(f(g(x))) = f'(g(x))\, g'(x)$

Derivative Of Inverse Trigonometric Functions.

$\frac{d\sin^{-1} x}{dx} = \frac{1}{\sqrt{1-x^2}}$, $\frac{d\cos^{-1} x}{dx} = -\frac{1}{\sqrt{1-x^2}}$, for $-1 < x < 1$.

$\frac{d\tan^{-1} x}{dx} = \frac{1}{1+x^2}$, $\frac{d\cot^{-1} x}{dx} = -\frac{1}{1+x^2}$ $(x \in R)$

$\frac{d\sec^{-1} x}{dx} = \frac{1}{|x|\sqrt{x^2-1}}$, $\frac{d\,\text{cosec}^{-1} x}{dx} = -\frac{1}{|x|\sqrt{x^2-1}}$, for $x \in (-\infty, -1) \cup (1, \infty)$

3. Differentiation using substitution

Following substitutions are normally used to sumplify these expression.

(i) $\sqrt{x^2 + a^2}$ by substituting $x = a \tan\theta$, where $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

(ii) $\sqrt{a^2 - x^2}$ by substituting $x = a \sin\theta$, where $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

(iii) $\sqrt{x^2 - a^2}$ by substituting $x = a \sec\theta$, where $\theta \in [0, \pi]$, $\theta \neq \frac{\pi}{2}$

(iv) $\sqrt{\frac{x+a}{a-x}}$ by substituting $x = a\cos\theta$, where $\theta \in (0, \pi]$.

4. Parametric Differentiation

If $y = f(\theta)$ & $x = g(\theta)$ where θ is a parameter, then $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$.

5. Derivative of one function with respect to another

Let $y = f(x)$; $z = g(x)$ then $\frac{dy}{dz} = \frac{dy/dx}{dz/dx} = \frac{f'(x)}{g'(x)}$.

6. If $F(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ l(x) & m(x) & n(x) \\ u(x) & v(x) & w(x) \end{vmatrix}$, where f, g, h, l, m, n, u, v, w are differentiable functions of x then

$$F'(x) = \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l(x) & m(x) & n(x) \\ u(x) & v(x) & w(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ l'(x) & m'(x) & n'(x) \\ u(x) & v(x) & w(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ l(x) & m(x) & n(x) \\ u'(x) & v'(x) & w'(x) \end{vmatrix}$$

APPLICATION OF DERIVATIVES

1. Equation of tangent and normal

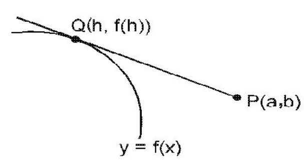
Tangent at (x_1, y_1) is given by $(y - y_1) = f'(x_1)(x - x_1)$; when, $f'(x_1)$ is real.

And normal at (x_1, y_1) is $(y - y_1) = -\frac{1}{f'(x_1)}(x - x_1)$, when $f'(x_1)$ is nonzero real.

2. Tangent from an external point

Given a point P(a, b) which does not lie on the curve y = f(x), then the equation of possible tangents to the curve y = f(x), passing through (a, b) can be found by solving for the point of contact Q.

$$f'(h) = \frac{f(h) - b}{h - a}$$



Figure

And equation of tangent is $y - b = \frac{f(h) - b}{h - a}(x - a)$

3. Length of tangent, normal, subtangent, subnormal

(i) $PT = |k|\sqrt{1 + \frac{1}{m^2}}$ = Length of Tangent

(ii) $PN = |k|\sqrt{1 + m^2}$ = Length of Normal

(iii) $TM = \left|\frac{k}{m}\right|$ = Length of subtangent

(iv) $MN = |km|$ = Length of subnormal.

4. Angle between the curves

Angle between two intersecting curves is defined as the acute angle between their tangents (or normals) at the point of intersection of two curves (as shown in figure).

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

5. Shortest distance between two curves

Shortest distance between two non-intersecting differentiable curves is always along their common normal. (Wherever defined)

6. Rolle's Theorem :

If a function f defined on [a, b] is

(i) continuous on [a, b] (ii) derivable on (a, b) and

(iii) f(a) = f(b),

then there exists at least one real number c between a and b ($a < c < b$) such that $f'(c) = 0$

7. Lagrange's Mean Value Theorem (LMVT) :

If a function f defined on [a, b] is

(i) continuous on [a, b] and (ii) derivable on (a, b)

then there exists at least one real numbers between a and b ($a < c < b$) such that $\frac{f(b) - f(a)}{b - a} = f'(c)$

8. Useful Formulae of Mensuration to Remember :

1. Volume of a cuboid = ℓbh.

2. Surface area of cuboid = $2(\ell b + bh + h\ell)$.

3. Volume of cube = a^3

4. Surface area of cube = $6a^2$

5. Volume of a cone = $\frac{1}{3}\pi r^2 h$.

6. Curved surface area of cone = $\pi r\ell$ (ℓ = slant height)

7. Curved surface area of a cylinder = $2\pi rh$.

8. Total surface area of a cylinder = $2\pi rh + 2\pi r^2$.

9. Volume of a sphere = $\frac{4}{3}\pi r^3$.

10. Surface area of a sphere = $4\pi r^2$.

11. Area of a circular sector = $\frac{1}{2} r^2 \theta$, when θ is in radians.

12. Volume of a prism = (area of the base) × (height).

13. Lateral surface area of a prism = (perimeter of the base) × (height).

14. Total surface area of a prism = (lateral surface area) + 2 (area of the base)
(Note that lateral surfaces of a prism are all rectangle).

15. Volume of a pyramid = $\frac{1}{3}$ (area of the base) × (height).

16. Curved surface area of a pyramid = $\frac{1}{2}$ (perimeter of the base) × (slant height).
(Note that slant surfaces of a pyramid are triangles).

INDEFINITE INTEGRATION

1. If f & g are functions of x such that g′(x) = f(x) then,

$\int f(x)\,dx = g(x) + c \Leftrightarrow \frac{d}{dx}\{g(x)+c\} = f(x)$, where c is called the **constant of integration**.

2. Standard Formula:

(i) $\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c,\ n \neq -1$

(ii) $\int \frac{dx}{ax+b} = \frac{1}{a}\,\ell n\,(ax+b) + c$

(iii) $\int e^{ax+b}\,dx = \frac{1}{a}\,e^{ax+b} + c$

(iv) $\int a^{px+q}\,dx = \frac{1}{p}\,\frac{a^{px+q}}{\ell n a} + c;\ a > 0$

(v) $\int \sin(ax+b)\,dx = -\frac{1}{a}\cos(ax+b) + c$

(vi) $\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b) + c$

(vii) $\int \tan(ax+b)\,dx = \frac{1}{a}\,\ell n \sec(ax+b) + c$

(viii) $\int \cot(ax+b)\,dx = \frac{1}{a}\,\ell n \sin(ax+b) + c$

(ix) $\int \sec^2(ax+b)\,dx = \frac{1}{a}\tan(ax+b) + c$

(x) $\int \operatorname{cosec}^2(ax+b)\,dx = -\frac{1}{a}\cot(ax+b) + c$

(xiii) $\int \sec x\,dx = \ell n\,(\sec x + \tan x) + c$ **OR** $\ell n \tan\left(\frac{\pi}{4} + \frac{x}{2}\right) + c$

(xiv) $\int \operatorname{cosec} x\,dx = \ell n\,(\operatorname{cosec}x - \cot x) + c$ **OR** $\ell n \tan\frac{x}{2} + c$ **OR** $-\ell n\,(\operatorname{cosec}x + \cot x) + c$

(xv) $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + c$

(xvi) $\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

(xvii) $\int \frac{dx}{|x|\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\frac{x}{a} + c$

(xviii) $\int \frac{dx}{\sqrt{x^2 + a^2}} = \ell n\left[x + \sqrt{x^2 + a^2}\right] + c$

(xix) $\int \frac{dx}{\sqrt{x^2 - a^2}} = \ell n\left[x + \sqrt{x^2 - a^2}\right] + c$

(xx) $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\,\ell n\left|\frac{a+x}{a-x}\right| + c$

(xxi) $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\,\ell n\left|\frac{x-a}{x+a}\right| + c$

(xxii) $\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c$

(xxiii) $\int \sqrt{x^2+a^2}\, dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\ \ell n\left(\frac{x+\sqrt{x^2+a^2}}{a}\right) + c$

(xxiv) $\int \sqrt{x^2-a^2}\, dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\ \ell n\left(\frac{x+\sqrt{x^2-a^2}}{a}\right) + c$

3. Integration by Subsitutions

If we subsitute f(x) = t, then f ′(x) dx = dt

4. Integration by Part :

$$\int (f(x)\, g(x))\, dx = f(x) \int (g(x))dx - \int\left(\frac{d}{dx}(f(x)) \int (g(x))\, dx\right)dx$$

5. Integration of type $\int \frac{dx}{ax^2+bx+c}, \int \frac{dx}{\sqrt{ax^2+bx+c}}, \int \sqrt{ax^2+bx+c}\ dx$

Make the substitution $x + \frac{b}{2a} = t$

6. Integration of type

$$\int \frac{px+q}{ax^2+bx+c}\ dx,\ \int \frac{px+q}{\sqrt{ax^2+bx+c}}\ dx,\ \int (px+q)\sqrt{ax^2+bx+c}\ dx$$

Make the substitution $x + \frac{b}{2a} = t$, then split the integral as some of two integrals one containing the linear term and the other containing constant term.

7. Integration of trigonometric functions

(i) $\int \frac{dx}{a + b\sin^2 x}$ **OR** $\int \frac{dx}{a + b\cos^2 x}$ **OR** $\int \frac{dx}{a\sin^2 x + b\sin x\cos x + c\cos^2 x}$ **put tan x = t.**

(ii) $\int \frac{dx}{a + b\sin x}$ **OR** $\int \frac{dx}{a + b\cos x}$ **OR** $\int \frac{dx}{a + b\sin x + c\cos x}$ **put** $\tan\frac{x}{2} = t$

(iii) $\int \frac{a.\cos x+b.\sin x+c}{\ell.\cos x+m.\sin x+n}$ **dx. Express Nr** $\equiv$ **A(Dr) + B** $\frac{d}{dx}$ **(Dr) + c & proceed.**

8. Integration of type

$\int \frac{x^2 \pm 1}{x^4 + Kx^2 + 1}$ **dx where K is any constant.**

Divide Nr & Dr by x² & put $x \mp \frac{1}{x} = t$.

9. Integration of type