

Solution

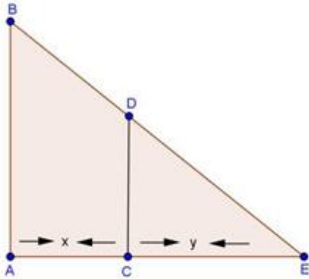
APPLICATION OF DERIVATIVES WS 1

Class 12 - Mathematics

Section A

1. 0.4

Explanation:



Let AB be the height of pole, Suppose at time t, the man CD is at a distance x meters from the lamp-post and y meters be the length of his shadow CE, then  $\frac{dx}{dt} = 1.1\text{m/sec}$

$$\frac{AB}{CD} = \frac{AE}{CE}$$

$$\frac{600}{160} = \frac{x+y}{y}$$

$$\frac{15}{4} = \frac{x+y}{y}$$

$$15y = 4x + 4y$$

$$11y = 4x$$

$$11 \frac{dy}{dt} = 4 \frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{4}{11}(1.1)$$

$$\frac{dy}{dt} = 0.4 \text{ m/sec}$$

Rate of increasing of shadow = 0.4 m/sec.

2. 37.68

Explanation:

Let height of the cone is x cm and radius of sphere is r cm.

here given,

$$x = 2r$$

$$h = x + r$$

$$h = 2r + r$$

$$h = 3r$$

v = Volume of cone + Volume of hemisphere

$$= \frac{1}{3}\pi r^2 x + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3}\pi r^2(2r) + \frac{2}{3}\pi r^3$$

$$v = \frac{2}{3}\pi r^3 + \frac{2}{3}\pi r^3$$

$$= \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}\pi \left(\frac{h}{3}\right)^3$$

$$v = \frac{4}{81}\pi h^3$$

$$\frac{dv}{dh} = \frac{4}{81}\pi \times 3h^2$$

$$\left(\frac{dv}{dh}\right)_{h=9} = \frac{4}{27}\pi(9)^2$$

$$\left(\frac{dv}{dh}\right)_{h=9} = 12\pi = 12 \times 3.14 = 37.68 \text{ cm}^2$$

Volume is increasing at rate of 37.68 cm<sup>2</sup> with respect to total height.

3. 66

Explanation:

By definition of marginal revenue we have,

$$\begin{aligned} \text{marginal revenue (MR)} &= \frac{dR}{dx} \\ &= \frac{d}{dx}(3x^2 + 36x + 5) = 6x + 36 \end{aligned}$$

When  $x = 5$ , then

$$\text{MR} = 6(5) + 36 = 30 + 36 = 66$$

Hence, the required marginal revenue is Rs. 66.

More amount of money spent for the welfare of the employees with the increase in marginal revenue show affinity and care for employees.

4. 1256

Explanation:

$$\text{Since, } V = \frac{4}{3}\pi r^3$$

$$\begin{aligned} \therefore \frac{dV}{dx} &= \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) \\ &= \frac{4}{3}\pi \cdot 3r^2 = 4\pi r^2 \end{aligned}$$

At  $x = 10$  cm

$$\Rightarrow \frac{dV}{dx} = 4\pi(10)^2 = 400\pi = 400 \times 3.14 = 1256$$

Therefore, the volume is increasing at the rate of 1256 cm<sup>3</sup>/sec.

5. 10

Explanation:

Let  $r$  be the radius and  $V$  be the volume of the sphere at any  $t$  then,

$$V = \frac{4}{3}\pi r^3$$

Differentiating both sides with respect to  $t$ ,

$$\begin{aligned} \Rightarrow \frac{dV}{dt} &= 4\pi r^2 \frac{dr}{dt} \\ \Rightarrow \frac{dr}{dt} &= \frac{1}{4\pi r^2} \frac{dV}{dt} \\ \Rightarrow \frac{dr}{dt} &= \frac{25}{4\pi(5)^2} \\ \Rightarrow \frac{dr}{dt} &= \frac{1}{4\pi} \text{ cm/sec} \end{aligned}$$

Now, let  $S$  be the surface area of the sphere area at any  $t$  then

$$S = 4\pi r^2$$

Differentiating both sides with respect to  $t$ ,

$$\begin{aligned} \Rightarrow \frac{dS}{dt} &= 8\pi r \frac{dr}{dt} \\ \Rightarrow \frac{dS}{dt} &= 8\pi(5) \times \frac{1}{4\pi} \\ \Rightarrow \frac{dS}{dt} &= 10 \text{ cm}^2/\text{sec} \end{aligned}$$

6. 188.4

Explanation:

Let  $r$  cm be the radius and  $A$  be the area of the circle at any time  $t$ .

Rate of increase of radius of circle = 3 cm/sec

$\Rightarrow \frac{dr}{dt}$  is positive and = 3 cm/sec

$$\frac{dr}{dt} = 3 \dots (i)$$

Now,  $A = \pi r^2$

$$\therefore \text{Rate of change of area of circle} = \frac{dA}{dt} = \pi 2r \frac{dr}{dt}$$

$$\Rightarrow \pi 2r(3) = 6\pi r \text{ cm}^2/\text{sec (from (i))}$$

putting  $r = 10$  cm (given),

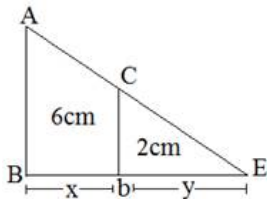
$$\frac{dA}{dt} = 6\pi(10) = 60\pi = 60 \times 3.14$$

Since  $\frac{dA}{dt}$  is positive, therefore surface area is increasing at the rate of  $188.4 \text{ cm}^2/\text{sec}$ .

7. 2.5

Explanation:

AB is lamp post DC is man



$$\frac{dx}{dt} = 5 \text{ km/h}, \frac{dy}{dt} = ?$$

$\triangle DEC \sim \triangle BEA$

$$\frac{2}{6} = \frac{y}{x+y}$$

$$x + y = 3y$$

$$x = 2y$$

$$\frac{dx}{dt} = 2 \frac{dy}{dt}$$

$$5 = 2 \frac{dy}{dt}$$

$$\frac{5}{2} \text{ km/h} = \frac{dy}{dt}$$

$$\frac{dy}{dt} = 2.5 \text{ km/h}$$

8. 251.2

Explanation:

Let A be the area of the circle of radius r.

Then,  $A = \pi r^2$

Therefore, the rate of change of area A with respect to time 't' is

$$\frac{dA}{dt} = \frac{d}{dt}(\pi r^2) = \frac{d}{dr}(\pi r^2) \cdot \frac{dr}{dt} = 2\pi r \frac{dr}{dt} \dots (\text{By Chain Rule})$$

Given that  $\frac{dr}{dt} = 4 \text{ cm/s}$

$$\text{Therefore, when } r = 10, \frac{dA}{dt} = 2\pi \times 10 \times 4 = 80\pi = 80 \times 3.14 = 251.2$$

Thus, the enclosed area is increasing at a rate of  $251.2 \text{ cm}^2/\text{s}$ , when  $r = 10$  cm.

9. 1368

Explanation:

$$\text{Given, } C(x) = 0.005x^3 - 0.02x^2 + 30x + 50$$

On differentiating both sides w.r.t. x, we get

$$\frac{dC}{dx} = 0.005(3x^2) - 0.02(2x) + 30$$

$$= 0.015x^2 - 0.04x + 30$$

On putting  $x = 300$ , we get

$$\frac{dC}{dx} = 0.015(300)^2 - 0.04(300) + 30$$

$$= 1350 - 12 + 30 = 1368$$

Therefore, the marginal cost of food for 300 students is Rs 1368.

By providing free mid-day meals to the students of primary classes, care and concern is shown towards their health and nutritional status.

10. 208

Explanation:

$$\text{Marginal Revenue (MR)} = \frac{dR}{dx}$$

$$= \frac{d}{dx} (13x^2 + 26x + 15)$$

$$= 26x + 26$$

$$\text{Now, when } x = 7, \text{ MR} = 26 \times 7 + 26 = 208$$

Therefore, the required marginal revenue is Rs 208

11. 0.5

Explanation:

Let AB be the height of light source, Suppose at time t, the man CD is at a distance of x meters from the lamp- post and y meters be the length of his shadow CE, then

$$\frac{dx}{dt} = 2 \text{ m/sec}$$

$$\frac{AB}{CD} = \frac{AE}{CE}$$

$$\frac{9}{1.8} = \frac{x+y}{y}$$

$$5y = x + y$$

$$4y = x$$

$$4 \frac{dy}{dt} = 2$$

$$4 \frac{dy}{dt} = 2$$

$$\frac{dy}{dt} = \frac{2}{4}$$

$$= \frac{1}{2}$$

$$\frac{dy}{dt} = 0.5 \text{ m/sec}$$

So, rate of increase of shadow is 0.5 m / sec

12. Since Marginal Revenue is the rate of change of total revenue with respect to the number of units sold, we have

$$\text{Marginal Revenue (MR)} = \frac{dR}{dx} = 6x + 36$$

$$\text{When } x = 5, \text{ MR} = 6(5) + 36 = 66$$

Hence, the required marginal revenue is ₹ 66.

13. Let x cm be the radius and y be the circumference of the circle at any time t.

Rate of increase of radius of circle = 0.7 cm/sec

$$\Rightarrow \frac{dx}{dt} \text{ is positive and } = 0.7 \text{ cm/sec}$$

$$\Rightarrow \frac{dx}{dt} = 0.7 \text{ cm / sec ... (i)}$$

$$\Rightarrow y = 2\pi x$$

$$\therefore \text{Rate of change of circumference of circle} = \frac{dy}{dt}$$

$$= 2\pi \frac{d}{dt} x = 2\pi(0.7) \text{ (from (i))}$$

$$= 1.4\pi \text{ cm/sec}$$

14. Let AB be the tower. Let at time t, the man be at a distance of x metres from the tower AB and let  $\theta$  be the angle of elevation at that time. Then,

$$\tan \theta = \frac{BC}{PC}$$

$$\Rightarrow \tan \theta = \frac{40}{x} \dots\dots\dots (i)$$

$$\Rightarrow x = 40 \cot \theta$$

$$\Rightarrow \frac{dx}{dt} = -40 \operatorname{cosec}^2 \theta \frac{d\theta}{dt}$$

We are given that  $\frac{dx}{dt} = 2\text{m/sec}$

$$\therefore 2 = -40\text{cosec}^2\theta \frac{d\theta}{dt}$$

$$\Rightarrow \frac{d\theta}{dt} = -\frac{1}{20\text{cosec}^2\theta} \dots\dots\dots(ii)$$

When  $x = 30$ ,  $\cot \theta = \frac{30}{40} = \frac{3}{4}$

$$\therefore \text{cosec}^2\theta = 1 + \cot^2\theta = 1 + \frac{9}{16} = \frac{25}{16}$$

Substituting  $\text{cosec}^2\theta = \frac{25}{16}$  in (ii), we get

$$\frac{d\theta}{dt} = -\frac{1}{20 \times \frac{25}{16}} = -\frac{4}{125} \text{ radians/sec}$$

Thus, the angle of elevation of the top of the tower is decreasing at the rate of  $\frac{4}{125}$  radians/sec.

15.  $MR = \frac{d}{dx}(R(x)) = 26x + 26$

$$MR]_{x=17} = 25 \times 17 + 26$$

$$= 425 + 26$$

$$= 451$$

16. Area of a circle of radius  $r$  is given by  $A = \pi r^2$

$$\text{Now, } A = \pi r^2 \Rightarrow \frac{dA}{dr} = 2\pi r$$

$$\Rightarrow \left[ \frac{dA}{dr} \right]_{r=5} = (2\pi \times 5)\text{cm} = 10\pi\text{cm}$$

Also,  $\delta r = (5.1 - 5) \text{ cm} = 0.1 \text{ cm}$

$$\therefore \delta A = \frac{dA}{dr} \cdot \delta r = (10\pi \times 0.1)\text{cm} = \pi\text{cm}$$

Hence, the increase in area of the given circle is  $\pi \text{ cm}^2$ .

17. We have,  $R(x) = 3x^2 + 40x + 10$

$$\Rightarrow MR = \frac{dR}{dx}$$

$$= \frac{d}{dx}(3x^2 + 40x + 10) = 6x + 40$$

$$\Rightarrow [MR]_{x=5} = (6 \times 5 + 40) = 70$$

Hence, the required marginal revenue is ₹70.

18.

Let  $AB$  be the height of light source, Suppose at time  $t$ , the man  $CD$  is at a distance of  $x$  meters from the lamp- post and  $y$  meters be the length of his shadow  $CE$ , then

$$\frac{dx}{dt} = 2\text{m/sec}$$

$$\frac{AB}{CD} = \frac{AE}{CE}$$

$$\frac{9}{1.8} = \frac{x+y}{y}$$

$$5y = x + y$$

$$4y = x$$

$$4 \frac{dy}{dt} = 2$$

$$4 \frac{dy}{dt} = 2$$

$$\frac{dy}{dt} = \frac{2}{4}$$

$$= \frac{1}{2}$$

$$\frac{dy}{dt} = 0.5\text{m/sec}$$

So, rate of increase of shadow is  $0.5 \text{ m / sec}$

$$19. \text{ Marginal Revenue (MR)} = \frac{dR}{dx}$$

$$= \frac{d}{dx} (13x^2 + 26x + 15)$$

$$= 26x + 26$$

Now, when  $x = 7$ ,  $MR = 26 \times 7 + 26 = 208$

Thus, the required marginal revenue is Rs 208.

$$20. \text{ Given, } C(x) = 0.005x^3 - 0.02x^2 + 30x + 50$$

On differentiating both sides w.r.t.  $x$ , we get

$$\frac{dC}{dx} = 0.005(3x^2) - 0.02(2x) + 30$$

$$= 0.015x^2 - 0.04x + 30$$

On putting  $x = 300$ , we get

$$\frac{dC}{dx} = 0.015(300)^2 - 0.04(300) + 30$$

$$= 1350 - 12 + 30 = 1368$$

Therefore, the marginal cost of food for 300 students is Rs 1368.

By providing free mid-day meals to the students of primary classes, care and concern is shown towards their health and nutritional status.

$$21. \text{ Since, } V = \frac{4}{3}\pi r^3$$

$$\therefore \frac{dV}{dx} = \frac{d}{dt} \left( \frac{4}{3}\pi r^3 \right)$$

$$= \frac{4}{3}\pi \cdot 3r^2 = 4\pi r^2$$

At  $x = 10$  cm

$$\Rightarrow \frac{dV}{dx} = 4\pi(10)^2 = 400\pi$$

Therefore, the volume is increasing at the rate of  $400\pi \text{ cm}^3/\text{sec}$ .

22. Let  $V$  be the volume of sphere

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dv}{dt} = 900 \text{ cm}^3/\text{s} \text{ (given)}$$

$$\text{Now, } \frac{dv}{dt} = \frac{4}{3} \cdot \pi \cdot 3r^2 \cdot \frac{dr}{dt}$$

$$\Rightarrow 900 = 4\pi r^2 \cdot \frac{dr}{dt}$$

$$\Rightarrow 900 = 4\pi \times (15)^2 \cdot \frac{dr}{dt} \text{ [r = 15 cm]}$$

$$\Rightarrow \frac{900}{4\pi \times 225} = \frac{dr}{dt}$$

$$\therefore \frac{1}{\pi} \text{ cm/s} = \frac{dr}{dt}$$

23. Let  $r$  be the radius,  $V$  be the volume and  $S$  be the surface area of sphere

$$\text{Then, we have } \frac{dV}{dt} = 8 \text{ cm}^3/\text{s}$$

To find  $\frac{dS}{dt}$ , when  $r = 12$  cm

$$\text{Since, } V = \frac{4}{3}\pi r^3$$

$$\therefore \frac{dV}{dt} = \frac{4}{3}\pi 3r^2 \frac{dr}{dt} \Rightarrow 8 = 4\pi \times r^2 \frac{dr}{dt}$$

$$\Rightarrow \frac{dr}{dt} = \frac{2}{\pi r^2} \text{ cm/s} \dots\dots\dots(i)$$

Now,  $S = 4\pi r^2$

$$\therefore \frac{dS}{dt} = \frac{d}{dt} (4\pi r^2) = 4\pi \times 2r \cdot \frac{dr}{dt}$$

$$= 8\pi r \times \frac{2}{\pi r^2} \text{ [From Eq(i)]}$$

$$= \frac{16}{r}$$

$$\Rightarrow \left( \frac{dS}{dt} \right)_{r=12} = \frac{16}{12} = \frac{4}{3} \text{ cm}^2/\text{s}$$

24. Let  $x$  cm be the edge and  $y$  be the volume of the variable cube at any time  $t$ .

Rate of increase of edge = 3 cm/sec

$$\Rightarrow \frac{dx}{dt} \text{ is positive and } = 3 \text{ cm/sec}$$

$$\Rightarrow \frac{dx}{dt} = 3 \text{ cm/sec} \dots (i)$$

$$\Rightarrow y = x^3$$

$$\therefore \text{Rate of change of volume of cube} = \frac{dy}{dt} = \frac{d}{dt} x^3$$

$$= 3x^2 \frac{dx}{dt} = 3x^2(3) \text{ [from (1)]}$$

$$= 9x^2 \text{ cm}^3/\text{sec}$$

Putting  $x = 10$  cm (given),

$$\frac{dy}{dt} = 9(10)^2 = 900 \text{ cm}^3/\text{sec}$$

Since  $\frac{dy}{dt}$  is positive, therefore volume of cube is increasing at the rate of 900  $\text{cm}^3/\text{sec}$ .

25. Let  $x$  cm be the radius of the air bubble at time  $t$ .

According to question,  $\frac{dx}{dt}$  is positive =  $\frac{1}{2}$  cm/sec .....(i)

$$\text{Volume of air bubble (z)} = \frac{4\pi}{3} x^3$$

$$\Rightarrow \frac{dz}{dt} = \frac{4\pi}{3} \frac{d}{dt} x^3$$

$$= \frac{4\pi}{3} \cdot 3x^2 \frac{dx}{dt}$$

$$= 4\pi x^2 \left( \frac{1}{2} \right)$$

$$\Rightarrow \frac{dz}{dt} = 2\pi x^2$$

$$= 2\pi(1)^2 = 2\pi$$

Therefore, required rate of increase of volume of air bubble is  $2\pi \text{ cm}^3/\text{sec}$

26. Since,  $x$  and  $y$  are the sides of two squares such that  $y = x - x^2$

$$\therefore \text{Area of the first square } (A_1) = x^2$$

$$\text{And area of the second square } (A_2) = y^2 = (x - x^2)^2$$

$$\therefore \frac{dA_2}{dt} = \frac{d}{dt} (x - x^2)^2 = 2(x - x^2) \left( \frac{dx}{dt} - 2x \cdot \frac{dx}{dt} \right)$$

$$= \frac{dx}{dt} (1 - 2x) 2(x - x^2)$$

$$\text{and } \frac{dA_1}{dt} = \frac{d}{dt} x^2 = 2x \cdot \frac{dx}{dt}$$

$$\therefore \frac{dA_2}{dA_1} = \frac{dA_2/dt}{dA_1/dt} = \frac{\frac{dx}{dt} \cdot (1 - 2x) (2x - 2x^2)}{2x \cdot \frac{dx}{dt}}$$

$$= \frac{(1 - 2x) 2x(1 - x)}{2x}$$

$$= (1 - 2x)(1 - x)$$

$$= 1 - x - 2x + 2x^2$$

$$= 2x^2 - 3x + 1$$

27. We have,  $C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$

The marginal cost,  $MC(x) = \frac{d}{dx} C(x)$

$$\text{Now, } \frac{d}{dx} (0.005x^3 - 0.02x^2 + 30x + 5000)$$

$$= 0.005 \times 3x^2 - 0.02 \times 2x + 30 + 0$$

$$= 0.015x^2 - 0.04x + 30$$

Marginal cost when 3 units are produced is

$$MC(3) = 0.015(9) - 0.04(3) + 30$$

$$= 0.135 - 0.12 + 30 = 30.015$$

28. Given: cost of producing:  $C(x) = 0.005x^3 - 0.02x^2 + 30x + 6000$

$$\Rightarrow MC = \frac{dC}{dx}$$

$$= \frac{d}{dx}(0.005x^3 - 0.02x^2 + 30x + 6000)$$

$$= [(0.005 \times 3x^2) - (0.02 \times 2x) + 30]$$

$$\Rightarrow [MC]_{x=4} = \{(0.005 \times 3 \times 4^2) - (0.02 \times 2 \times 4) + 30\}$$

$$= (0.24 - 0.16 + 30) = 30.08$$

Hence, the required marginal cost is ₹30.08

29. At any instant  $t$ , let the length of each edge of the cube be  $x$ ,  $V$  be its volume and  $S$  be its surface area. Then,

$$\frac{dV}{dt} = 7 \text{ cm}^3 / \text{sec} \dots \text{(given)} \dots \text{(i)}$$

$$\text{Now, } V = x^3 \Rightarrow \frac{dV}{dt} = \frac{dV}{dx} \cdot \frac{dx}{dt}$$

$$\Rightarrow 7 = \frac{d}{dx}(x^3) \cdot \frac{dx}{dt} \dots [\because V = x^3]$$

$$\Rightarrow 3x^2 \cdot \frac{dx}{dt} = 7$$

$$\Rightarrow \frac{dx}{dt} = \frac{7}{3x^2}$$

$$\therefore S = 6x^2 \Rightarrow \frac{dS}{dt} = \frac{dS}{dx} \cdot \frac{dx}{dt}$$

$$= \frac{d}{dx}(6x^2) \cdot \frac{7}{3x^2}$$

$$= \left(12x \times \frac{7}{3x^2}\right) = \frac{28}{x}$$

$$\Rightarrow \left[\frac{dS}{dt}\right]_{x=12} = \left(\frac{28}{12}\right) \text{ cm}^2 / \text{sec} = 2\frac{1}{3} \text{ cm}^2 / \text{sec}$$

Hence, the surface area of the cube is increasing at the rate of  $2\frac{1}{3} \text{ cm}^2 / \text{sec}$  at the instant when its edge is 12 cm.

30. Let  $x$  cm be the edge of the cube and  $y$  be the surface area of the cube at any time  $t$

Given: Rate of increase of volume of cube =  $8 \text{ cm}^3 / \text{sec}$

$$\frac{d}{dt}(x^3) = 8$$

$$\Rightarrow 3x^2 \frac{dx}{dt} = 8$$

$$\Rightarrow \frac{dx}{dt} = \frac{8}{3x^2} \dots \text{(i)}$$

$$y = 6x^2$$

$$\therefore \text{Rate of change of surface area of the cube} = \frac{dy}{dt} = 6 \frac{d}{dt}x^2$$

$$= 6 \left(2x \frac{dx}{dt}\right)$$

$$= 12x \left(\frac{8}{3x^2}\right) \text{ [from (1)]}$$

$$= 4 \left(\frac{8}{x}\right) = \frac{32}{x} \text{ cm}^2 / \text{sec}$$

Putting  $x = 12$  cm (given),

$$\frac{dy}{dt} = \frac{32}{12} = \frac{8}{3} \text{ cm}^2 / \text{sec}$$

Since  $\frac{dy}{dt}$  is positive, therefore surface area is increasing at the rate of  $\frac{8}{3} \text{ cm}^2/\text{sec}$

31. A soap bubble is in the form of a sphere. At an instant  $t$ , let its radius be  $r$  and surface area  $S$ . Then,

$$\frac{dr}{dt} = 0.02 \text{ cm/sec} \dots \text{(given)} \dots \text{(i)}$$

$$\text{Now, } S = 4\pi r^2$$

$$\Rightarrow \frac{dS}{dt} = 8\pi r \cdot \frac{dr}{dt} = (8 \times 3.14 \times r \times 0.02) \text{ cm}^2/\text{sec}$$

$$\Rightarrow \left[ \frac{dS}{dt} \right]_{r=5} = (8 \times 3.14 \times 5 \times 0.02) \text{ cm}^2/\text{sec} = 2.512 \text{ cm}^2/\text{sec}$$

Hence, the surface area of the bubble is increasing at the rate of  $2.512 \text{ cm}^2/\text{sec}$  at the instance when its radius is 5 cm.

32. Let  $A$  be the area of the circle of radius  $r$ .

$$\text{Then, } A = \pi r^2$$

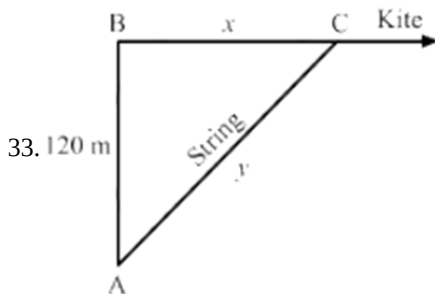
Therefore, the rate of change of area  $A$  with respect to time ' $t$ ' is

$$\frac{dA}{dt} = \frac{d}{dt}(\pi r^2) = \frac{d}{dr}(\pi r^2) \cdot \frac{dr}{dt} = 2\pi r \frac{dr}{dt} \dots \text{(By Chain Rule)}$$

$$\text{Given that } \frac{dr}{dt} = 4 \text{ cm/s}$$

$$\text{Therefore, when } r = 10, \frac{dA}{dt} = 2\pi \times 10 \times 4 = 80\pi$$

Thus, the enclosed area is increasing at a rate of  $80\pi \text{ cm}^2/\text{s}$ , when  $r = 10 \text{ cm}$ .



Here,

$$AB^2 + BC^2 = AC^2$$

$$\Rightarrow x^2 + (120)^2 = y^2$$

Differentiating both sides with respect to  $t$ ,

$$\Rightarrow 2x \frac{dx}{dt} = 2y \frac{dy}{dt}$$

$$\Rightarrow \frac{dy}{dt} = \frac{x}{y} \frac{dx}{dt}$$

$$\Rightarrow \frac{dy}{dt} = \frac{50}{130} \times 52 \left[ \because x = \sqrt{(130)^2 - (120)^2} = 50 \right]$$

$$\Rightarrow \frac{dy}{dt} = 20 \text{ m/sec}$$

34. At any instant  $t$ , let  $r$  be the radius,  $V$  the volume and  $S$  the surface area of the balloon. Then,

$$\frac{dV}{dt} = 20 \text{ cm}^3/\text{sec} \dots \text{(given)} \dots \text{(i)}$$

$$\text{Now, } V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt}$$

$$\Rightarrow 20 = \frac{d}{dr} \left( \frac{4}{3}\pi r^3 \right) \cdot \frac{dr}{dt}$$

$$\Rightarrow 20 = \frac{4}{3}\pi \times 3r^2 \times \frac{dr}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$$

$$\Rightarrow \frac{dr}{dt} = \frac{5}{\pi r^2} \dots \text{(ii)}$$

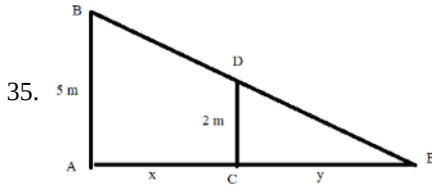
$$\therefore S = 4\pi r^2 \Rightarrow \frac{dS}{dt} = \frac{dS}{dr} \cdot \frac{dr}{dt}$$

$$= \frac{d}{dr} (4\pi r^2) \cdot \frac{5}{\pi r^2}$$

$$= \left( 8\pi r \times \frac{5}{\pi r^2} \right) = \frac{40}{r}$$

$$\Rightarrow \left[ \frac{dS}{dt} \right]_{r=8\text{cm}} = \left( \frac{40}{8} \right) \text{cm}^2 / \text{sec} = 5 \text{ cm}^2 / \text{sec}$$

Hence, the rate of change of surface area at the instant when  $r = 8 \text{ cm}$  is  $5 \text{ cm}^2 / \text{sec}$ .



Let  $AB$  be the lamp-post. Let at any time  $t$ , the man  $CD$  be at a distance  $x$  metres from the lamp-post and let the length of his shadow be  $y$  metres. Then,

$$\frac{dx}{dt} = 6 \text{ metres/minute}$$

Clearly, triangles  $ABE$  and  $CDE$  are similar.

$$\therefore \frac{AB}{CD} = \frac{AE}{CE}$$

$$\Rightarrow \frac{5}{2} = \frac{x+y}{y}$$

$$\Rightarrow 5y = 2x + 2y$$

$$\Rightarrow 3y = 2x$$

$$\Rightarrow 3 \frac{dy}{dt} = 2 \frac{dx}{dt}$$

$$\Rightarrow 3 \frac{dy}{dt} = 2(6)$$

$$\Rightarrow \frac{dy}{dt} = 4 \text{ Thus, the shadow increases at a rate of } 4 \text{ metres/minute.}$$

36. Let at any time  $t$ ,  $V$  be the volume of the water in the cone i.e., the volume of the water-cone  $VA'B'$ , and let  $l$  be the slant height. Then,

$$AO' = l \sin 60^\circ = \frac{\sqrt{3}l}{2} \text{ and } VO' = l \cos 60^\circ = \frac{l}{2}$$

$$\therefore V = \frac{1}{3} \pi \left( \frac{\sqrt{3}l}{2} \right)^2 \left( \frac{l}{2} \right) = \frac{\pi l^3}{8}$$

$$\Rightarrow \frac{dV}{dt} = \frac{3\pi l^2}{8} \frac{dl}{dt}$$

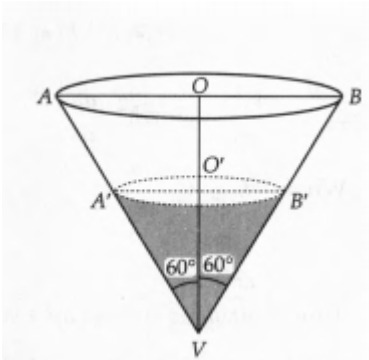
Given that  $\frac{dV}{dt} = -4 \text{ cm}^3 \text{ sec}$  (negative sign due to decrease)

$$\therefore -4 = \frac{3\pi}{8} l^2 \frac{dl}{dt}$$

$$\Rightarrow \frac{dl}{dt} = -\frac{32}{3\pi l^2}$$

$$\frac{dl}{dt} = -\frac{32}{3\pi(3)^2} = -\frac{32}{27\pi} \text{ cm/sec}$$

Thus, the slant height of the water - cone is decreasing at the rate of  $\frac{32}{27\pi} \text{ cm/sec}$



37. Marginal cost =  $\frac{dC}{dx}$

$$= \frac{d}{dx} (0.007x^3 - 0.003x^2 + 15x + 4000)$$

$$= 0.021x^2 - 0.006x + 15$$

Now, when  $x = 17$ , MC

$$= 0.021(17)^2 - 0.006 \times 17 + 15$$

$$= 0.069 - 0.102 + 15 = 20.967$$

Therefore, required Marginal cost is Rs 20.97

38. We have, rate of decrease of the volume of spherical ball of salt at any instant is proportional to surface area. Let the radius of the spherical ball of the salt be  $r$ .

$$\therefore \text{Volume of the ball (V)} = \frac{4}{3}\pi r^3$$

and surface area (s) =  $4\pi r^2$

$$\therefore \frac{dV}{dt} \propto s$$

$$\Rightarrow \frac{d}{dt} \left( \frac{4}{3}\pi r^3 \right) \propto 4\pi r^2$$

$$\Rightarrow \frac{4}{3}\pi 3r^2 \frac{dr}{dt} \propto 4\pi r^2$$

$$\Rightarrow \frac{dr}{dt} \propto \frac{4\pi r^2}{4\pi r^2}$$

$$\Rightarrow \frac{dr}{dt} = k.1 \text{ [where, k is the proportionality constant]}$$

$$\Rightarrow \frac{dr}{dt} = k$$

Hence, the radius of ball is decreasing at a constant rate.

39. Let  $x$  cm be the radius and  $y$  be the enclosed area of the circular wave at any time  $t$ .

Rate of increase of radius of circular wave = 5 cm/sec

$$\Rightarrow \frac{dx}{dt} \text{ is positive and } = 5 \text{ cm/sec}$$

$$\Rightarrow \frac{dx}{dt} = 5 \text{ cm/sec ... (i)}$$

$$y = \pi x^2$$

$$\therefore \text{Rate of change of area} = \frac{dy}{dt} = \pi \frac{d}{dt} x^2$$

$$= \pi.2x \frac{dx}{dt} = 2\pi x(5) \text{ (from (i))}$$

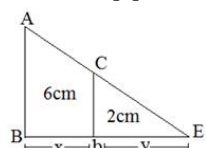
$$= 10\pi x \text{ cm}^2/\text{sec}$$

Putting  $x = 8 \text{ cm}$  (given),

$$\frac{dy}{dt} = 10\pi(8) = 80\pi \text{ cm}^2/\text{sec}$$

Since  $\frac{dy}{dt}$  is positive, therefore area of circular wave is increasing at the rate of  $80\pi \text{ cm}^2/\text{sec}$ .

40. AB is lamp post DC is man



$$\frac{dx}{dt} = 5 \text{ km/h}, \frac{dy}{dt} = ?$$

$$\triangle DEC \sim \triangle BEA$$

$$\frac{2}{6} = \frac{y}{x+y}$$

$$x + y = 3y$$

$$x = 2y$$

$$\frac{dx}{dt} = 2 \frac{dy}{dt}$$

$$5 = 2 \frac{dy}{dt}$$

$$\frac{5}{2} \text{ km/h} = \frac{dy}{dt}$$

41. Given,  $P(x) = 0.005x^3 + 0.02x^2 + 30x$

On differentiating both sides w.r.t.  $x$ , we get

$$P'(x) = 3 \times 0.005x^2 + 2 \times 0.02x + 30$$

On putting  $x = 3$  we get

$$\begin{aligned} P'(3) &= 3 \times 0.005 \times 9 + 2 \times 0.02 \times 3 + 30 \\ &= 0.135 + 0.12 + 30 \\ &= 30.255 \end{aligned}$$

Therefore, the marginal increase in pollution content when 3 diesel vehicles are added is 30.255.

Pollution content in the city increases with the increase in number of diesel vehicles.

42. Given: The rate of change of volume of a sphere is equal to the rate of change of its radius

$$\Rightarrow \frac{dV}{dt} = \frac{dr}{dt}$$

We know that,

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$4\pi r^2 = 1$$

$$r = \frac{1}{\sqrt{4\pi}}$$

$$\text{Radius of sphere} = \frac{1}{2\sqrt{\pi}} \text{ units}$$

43. Here,

$$y = \frac{2}{3}x^3 + 1$$

Differentiating both sides with respect to  $t$ ,

$$\Rightarrow \frac{dy}{dt} = 2x^2 \frac{dx}{dt}$$

$$\Rightarrow 2 \frac{dx}{dt} = 2x^2 \frac{dx}{dt} \left[ \because \frac{dy}{dt} = 2 \frac{dx}{dt} \right]$$

$$\Rightarrow x = \pm 1$$

Substituting the value of  $x = 1$  and  $x = -1$  in  $y = \frac{2}{3}x^3 + 1$ , we get

$$\Rightarrow y = \frac{5}{3} \text{ and } y = \frac{1}{3}$$

So, the points are  $\left(1, \frac{5}{3}\right)$  and  $\left(-1, \frac{1}{3}\right)$

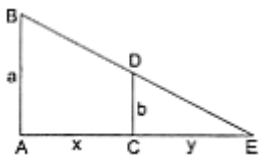
### Section B

44. Suppose AB be the lamp post, the lamp being at B. Then,  $AB = a$  metres. At any instant  $t$ , suppose CD be the position of the boy and CE be his shadow.

Then,  $CD = b$  metres.

Let  $AC = x$  metres and  $CE = y$  metres.

Given that  $\frac{dx}{dt} = c$  metres /min



Clearly,  $\triangle BAE \sim \triangle DCE$

$$\therefore \frac{AB}{CD} = \frac{AE}{CE}$$

$$\Rightarrow \frac{a}{b} = \frac{x+y}{y}$$

$$\Rightarrow (a - b)y = bx$$

$$\Rightarrow (a - b) \frac{dy}{dt} = b \cdot \frac{dx}{dt} = bc \left[ \because \frac{dx}{dt} = c \right]$$

$$\Rightarrow \frac{dy}{dt} = \frac{bc}{(a-b)} \text{metres/min}$$

Therefore, the shadow is increasing at the rate of  $\frac{bc}{(a-b)}$  metres/min

45. Given curve is  $6y = x^3 + 2 \dots(i)$

$$\text{so, } 6 \frac{dy}{dt} = 3x^2 \frac{dx}{dt}$$

$$\Rightarrow 6 \times 8 \frac{dx}{dt} = 3x^2 \frac{dx}{dt} \left[ \because \frac{dy}{dt} = 8 \frac{dx}{dt} \right]$$

$$\Rightarrow 16 = x^2$$

$$\Rightarrow x = \pm 4$$

Put the value of x in equation (1)

When  $x = 4$

$$6y = (4)^3 + 2$$

$$\Rightarrow 6y = 64 + 2$$

$$\Rightarrow 6y = 66$$

$$\therefore y = \frac{66}{6} = 11$$

So, point is (4, 11)

Now, When  $x = -4$

$$6y = (-4)^3 + 2$$

$$= -64 + 2$$

$$\therefore y = \frac{-62}{6} = \frac{-31}{3}$$

So the point is  $\left(-4, \frac{-31}{3}\right)$

46. Let the required point be (x, y). Then,

$$\frac{dy}{dt} = -\frac{dx}{dt} \dots (i)$$

Given curve is  $16x^2 + 9y^2 = 400 \dots (ii)$

On differentiating both sides of (ii) w.r.t. t, we get

$$32x \cdot \frac{dx}{dt} + 18y \cdot \frac{dy}{dt} = 0$$

$$\Rightarrow 32x \cdot \frac{dx}{dt} + 18y \cdot \left(-\frac{dx}{dt}\right) = 0 \text{ [using (i)]}$$

$$\Rightarrow (32x - 18y) = 0 \Rightarrow y = \frac{16}{9}x$$

Substituting  $y = \frac{16}{9}x$  in (ii), we obtain

$$16x^2 + 9 \times \left(\frac{16x}{9}\right)^2 = 400 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3$$

$$\text{Now, } x = 3 \Rightarrow y = \left(\frac{16}{9} \times 3\right) = \frac{16}{3},$$

$$x = -3 \Rightarrow y = \left[\frac{16}{9} \times (-3)\right] = \frac{-16}{3}$$

We get, the required points are  $\left(3, \frac{16}{3}\right)$  and  $\left(-3, \frac{-16}{3}\right)$

47. Given: Rate of decrease of length x of rectangle is 5 cm/minute.

$$\Rightarrow \frac{dx}{dt} \text{ is negative} = -5 \text{ cm/minute}$$

$$\Rightarrow \frac{dx}{dt} = -5 \text{ cm/minute} \dots (i)$$

Also, Rate of increase of width y of rectangle is 4 cm/minute

$$\Rightarrow \frac{dy}{dt} \text{ is positive} = 4 \text{ cm/minute}$$

$$\Rightarrow \frac{dy}{dt} = 4 \text{ cm/minute} \dots \text{(ii)}$$

let z denotes the perimeter of rectangle at any time t

$$\therefore z = 2x + 2y$$

$$\Rightarrow \frac{dz}{dt} = 2 \frac{dx}{dt} + 2 \frac{dy}{dt} \text{ (from (i) and (ii))}$$

$$= 2(-5) + 2(4) = -2 \text{ which is negative.}$$

$\therefore$  Perimeter of the rectangle is decreasing at the rate of 2 cm/min.

**(b)** Let z denotes the area of rectangle at any time t

$$\Rightarrow z = xy$$

$$\Rightarrow \frac{dz}{dt} = x \frac{dy}{dt} + y \frac{dx}{dt}$$

$$= 8(4) + 6(-5) = 2 \text{ which is positive. (from (i) and (ii))}$$

$\therefore$  Area of the rectangle is increasing at the rate of 2 cm<sup>2</sup>/min

48. Let 'a' be the side of an equilateral triangle and 'A' be the area of an equilateral triangle. Then,  $\frac{dA}{dt} = 2 \text{ cm}^2/\text{s}$

We know that, area of an equilateral triangle,  $A = \frac{\sqrt{3}}{4} a^2$

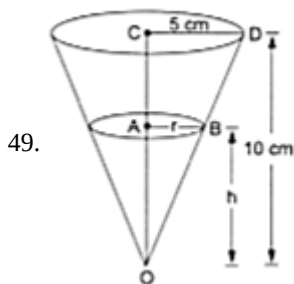
On differentiating both sides w.r.t. t, we get

$$\frac{dA}{dt} = \frac{\sqrt{3}}{4} \times 2a \times \frac{da}{dt}$$

$$\Rightarrow \frac{dA}{dt} = \frac{\sqrt{3}}{4} \times 2 \times 20 \times 2 \text{ [given, } a = 20]$$

$$\therefore \frac{dA}{dt} = 20\sqrt{3} \text{ cm}^2/\text{s}$$

Thus, the rate of area increasing is  $20\sqrt{3} \text{ cm}^2/\text{s}$



Now, at any instant t, suppose r be the radius of the water level, h the height of the water level and V the volume of the water in the conical funnel. Then,

$$\frac{dV}{dt} = -5 \dots \text{(given)} \dots \text{(i)}$$

From similar  $\triangle$ s OAB and OCD, we have

$$\frac{AB}{OA} = \frac{CD}{OC} \Rightarrow \frac{r}{h} = \frac{5}{10} = \frac{1}{2} \Rightarrow r = \frac{1}{2}h$$

$$\text{Now, } V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times \left(\frac{1}{2}h\right)^2 \times h = \frac{1}{12}\pi h^3$$

$$\therefore \frac{dV}{dt} = \left(\frac{dV}{dh} \times \frac{dh}{dt}\right) = \frac{d}{dh} \left(\frac{1}{12}\pi h^3\right) \cdot \frac{dh}{dt} = \frac{\pi r^2}{4} \cdot \frac{dh}{dt}$$

$$\Rightarrow -5 = \frac{\pi h^2}{4} \cdot \frac{dh}{dt}$$

$$\Rightarrow \frac{dh}{dt} = \frac{-20}{\pi h^2}$$

$$\Rightarrow \left(\frac{dh}{dt}\right)_{h=7.5\text{cm}} = \frac{-20}{\pi \times (7.5)^2} \dots \text{ [ } \because h = (10 - 2.5)\text{cm} = 7.5 \text{ cm]}$$

$$= \frac{-16}{45\pi} \text{ cm/sec}$$

= Therefore, the rate of change of water level at h = 7.5 cm is  $\frac{-16}{45\pi} \text{ cm/sec}$

50. Given: Equation of the curve  $6y = x^3 + 2 \dots \text{(i)}$

Let (x, y) be the required point on curve (i)

since y-coordinate is changing 8 times as fast as the x-coordinate

$$\frac{dy}{dt} = 3x^2 \frac{dx}{dt} \dots \text{(ii)}$$

$$\therefore \text{From eq. (i), } 6y = x^3 + 2$$

$$6 \frac{dy}{dt} = 3x^2 \frac{dx}{dt}$$

now from (ii) we substitute the value of  $\frac{dy}{dt}$

$$\Rightarrow 6 \left( 8 \frac{dx}{dt} \right) = 3x^2 \cdot \frac{dx}{dt}$$

$$\Rightarrow 6 \times 8 = 3x^2 \text{ (we cancel } \frac{dx}{dt} \text{ from both side)}$$

$$\Rightarrow x^2 = \frac{6 \times 8}{3}$$

$$\Rightarrow x = \pm 4$$

Taking  $x = 4$ ,  $6y = 64 + 2$

$$\Rightarrow y = 11$$

$\therefore$  Required point is (4, 11).

Taking  $x = -4$ ,  $6y = -64 + 2$

$$\Rightarrow y = \frac{-31}{3}$$

$\therefore$  Required point is  $\left( -4, \frac{-31}{3} \right)$ .

so the required points are  $\left( -4, \frac{-31}{3} \right)$  and (4, 11).

51.  $\frac{dx}{dt} = 2 \text{ cm/s}$

$$\frac{dx}{dt} = 0.02 \text{ m/sec}$$

$$x^2 + y^2 = 5^2$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

When  $x = 4$

$$y = \sqrt{5^2 - 4^2} = 3$$

$$2 \times 4(0.02) + 2 \times 3 \frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = \frac{2 \times 4 \times 0.02}{2 \times 3}$$

$$= -\frac{0.08}{3} \times 100$$

$$= -\frac{8}{3} \text{ cm/sec}$$

52.



Let radius of the cone =  $r$

Height of the cone =  $h$

Volume of the cone =  $\frac{1}{3}\pi r^2 h$

$$\frac{dv}{dt} = 12 \text{ cm}^3/\text{s} \text{ (Given)}$$

$$h = \frac{1}{6}r \text{ (Given)}$$

$$v = \frac{1}{3}\pi r^2 h$$

$$v = \frac{1}{3} \cdot \pi \cdot (6h)^2 \cdot h \left[ \because h = \frac{1}{6}r \right]$$

$$\Rightarrow v = \frac{1}{3} \cdot \pi \cdot 36 \cdot h^3$$

$$\Rightarrow v = 12\pi h^3$$

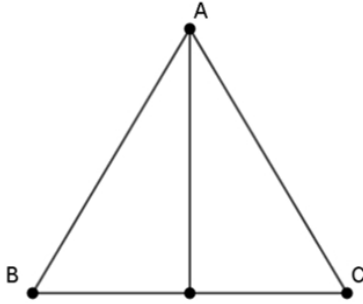
$$\text{Now } \frac{dv}{dt} = 12\pi \cdot 3h^2 \cdot \frac{dh}{dt}$$

$$12 = 12\pi \cdot 3(4)^2 \cdot \frac{dh}{dt} \quad [h = 4\text{cm}]$$

$$\Rightarrow \frac{12}{12\pi \cdot 3 \cdot 16} = \frac{dh}{dt}$$

$$\therefore \frac{dh}{dt} = \frac{1}{48\pi} \text{cm/s}$$

53. In an equilateral triangle, median is same as altitude. Let 'h' denote the length of the median (or altitude) and 'x' be the side of  $\triangle ABC$ .



$$\text{Then, } h = \frac{\sqrt{3}}{2}x \text{ or } x = \frac{2h}{\sqrt{3}} \dots(i)$$

It is given that  $\frac{dh}{dt} = 2\sqrt{3}$  So, by (i) we have

$$\frac{dx}{dt} = \frac{2}{\sqrt{3}} \frac{dh}{dt} \Rightarrow \frac{dx}{dt} = 4$$

Thus, the side of  $\triangle ABC$  is increasing at the rate of 4 cm/sec.

54. Let x be the side, V be the volume and S be the surface area of cube

$$\frac{dv}{dt} = 9\text{cm}^3/\text{s}$$

$$v = x^3$$

$$\Rightarrow \frac{dv}{dt} = 3x^2 \frac{dx}{dt}$$

$$\Rightarrow 9 = 3x^2 \frac{dx}{dt}$$

$$\Rightarrow \frac{3}{x^2} = \frac{dx}{dt}$$

$$\text{Now, } s = 6x^2$$

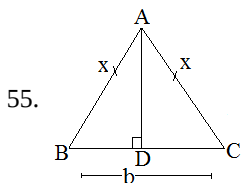
$$\frac{ds}{dt} = 12x \frac{dx}{dt}$$

$$= 12x \times \frac{3}{x^2}$$

$$\frac{ds}{dt} = \frac{36}{x}$$

$$\left. \frac{ds}{dt} \right|_{x=10} = \frac{36}{10}$$

$$= 3.6 \text{ cm}^2/\text{sec}$$



$$\frac{dx}{dt} = -3\text{cm/s}$$

Let A be area of  $\Delta$

$$A = \frac{1}{2} \times b \times AD$$

$$\left[ AD = \sqrt{x^2 - \frac{b^2}{4}} = \frac{\sqrt{4x^2 - b^2}}{2} \right]$$

$$= \frac{1}{2} \times b \times \frac{\sqrt{4x^2 - b^2}}{2}$$

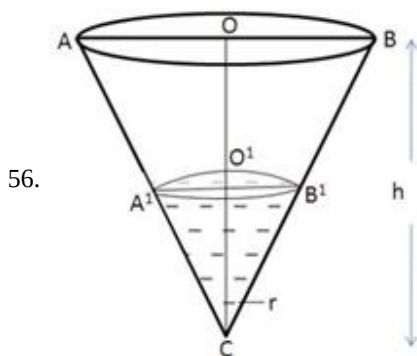
$$\frac{dA}{dt} = \frac{1}{4} b \cdot \frac{2}{2\sqrt{4x^2 - b^2}} \cdot 8x \cdot \frac{dx}{dt}$$

$$\left( \frac{dA}{dt} \right)_{x=b} = \frac{b}{\sqrt{4 \times b^2 - b^2}} \times b(-3) [x=b]$$

$$= -\frac{3b^2}{\sqrt{3b}}$$

$$= -\frac{\sqrt{3} \cdot \sqrt{3}}{\sqrt{3}} \cdot b$$

$$\frac{dA}{dt} = -\sqrt{3}bcm^2/s$$



Suppose be the semi vertical angle of the cone CAB whose height CO is half of r

$$\tan \alpha = \frac{OB}{CO}$$

$$= \frac{OB}{2OB}$$

$$\tan \alpha = \frac{1}{2}$$

Suppose V be the volume of the sand in the cone

$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi \left(\frac{h}{2}\right)^2 h$$

$$= \frac{\pi}{12}h^3$$

$$\frac{dV}{dt} = \frac{3\pi}{12}h^2 \frac{dh}{dt}$$

$$50 = \frac{3\pi}{12}h^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{200}{\pi h^2}$$

$$= \frac{200}{\pi(5)^2}$$

$$\frac{dh}{dt} = \frac{8}{3.14} \text{cm/min}$$

Therefore, rate of increasing of height is  $= \frac{8}{3.14} \text{cm/min}$

57.  $\frac{dx}{dt} = 3 \text{cm/min}$ ,  $\frac{dy}{dt} = 2 \text{cm/min}$

a. Let P be the perimeter

$$P = 2(x + y)$$

$$\frac{dP}{dt} = 2 \left( \frac{dx}{dt} + \frac{dy}{dt} \right)$$

$$= 2(-3 + 2)$$

$$= -2 \text{ cm/min}$$

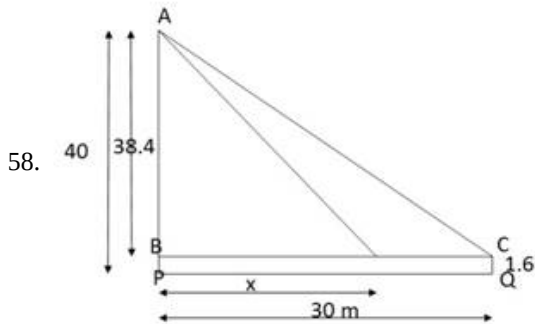
b.  $A = xy$

$$\frac{dA}{dt} = x \frac{dy}{dt} + y \frac{dx}{dt}$$

$$= 10(2) + 6(-3)$$

$$= 20 - 18$$

$$= 2 \text{ cm}^2/\text{min}$$



$$\frac{dx}{dt} = -2 \text{ cm/s}$$

$$\tan Q = \frac{38.4}{x}$$

$$Q = \tan^{-1} \frac{38.4}{x}$$

$$\frac{dQ}{dt} = \frac{1}{1 + \frac{38.4^2}{x^2}} \left( -\frac{1}{x^2} \right) \cdot 38.4$$

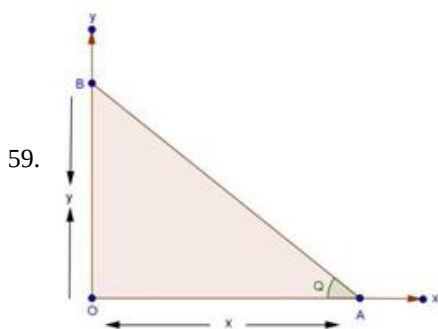
$$\frac{dQ}{dt} = \frac{x^2}{x^2 + 1474.56} \left( -\frac{1}{x^2} \right) \cdot 38.4$$

$$\frac{dQ}{dt} = -\frac{1}{30^2 + 1474.56} \cdot 38.4 \cdot \frac{dx}{dt}$$

$$\frac{dQ}{dt} = -\frac{1}{30^2 + 1474.56} \cdot 38.4 \times 2$$

$$\frac{dQ}{dt} = -0.032 \text{ radian/sec therefore}$$

Angle of elevation is decreasing at the rate 0.032 degree /sec.



Suppose AB be the position of the ladder, at time  $t$  such that  $OA = x$  and  $OB = y$

By triangle law,

$$OA^2 + OB^2 = AB^2$$

$$x^2 + y^2 = (13)^2$$

$$x^2 + y^2 = 169 \dots(i)$$

And  $\frac{dx}{dt} = 1.5 \text{ m/sec}$

From figure,  $\tan \theta = \frac{y}{x} \dots(ii)$

Differentiating equation (i) with respect to t,

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

$$2(1.5)x + 2y \frac{dy}{dt} = 0$$

$$3x + 2y \frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{3x}{2y} \dots$$

differentiating equation (ii) with respect to t

$$\sec^2 \theta \frac{d\theta}{dt} = \frac{x \frac{dy}{dt} - y \frac{dx}{dt}}{x^2}$$

$$= \frac{x \times \left(-\frac{3x}{2y}\right) - y(1.5)}{x^2}$$

$$= \frac{-1.5x^2 - 1.5y^2}{yx^2}$$

$$\frac{d\theta}{dt} = \frac{-1.5(x^2 + y^2)}{x^2 y \sec^2 \theta}$$

$$= \frac{-1.5(x^2 + y^2)}{x^2 y (1 + \tan^2 \theta)}$$

$$\frac{d\theta}{dt} = \frac{-1.5(x^2 + y^2)}{x^2 y \left(1 + \frac{y^2}{x^2}\right)}$$

$$= \frac{-1.5(x^2 + y^2) \times x^2}{x^2 y (x^2 + y^2)}$$

$$= \frac{-1.5}{y}$$

$$= \frac{-1.5}{\sqrt{169 - x^2}}$$

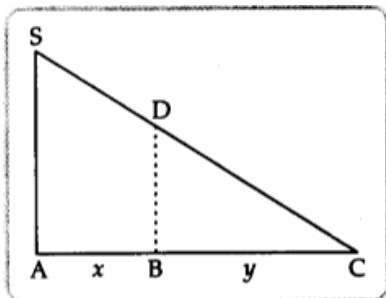
$$= \frac{-1.5}{\sqrt{169 - 144}}$$

$$= \frac{-1.5}{5}$$

$$= 0.3 \text{ radian/sec}$$

Therefore, angle between ladder and ground is decreasing at the rate of 0.3 radian/sec

60. Let S be the position of source of light.



Let BD be the position of the man at time t.

Let AB = x and BC = length of the shadow = y

$$\text{Now, } \frac{dx}{dt} = 140$$

Here,  $\triangle ASC \sim \triangle BDC$

$$\therefore \frac{AS}{BD} = \frac{AC}{BC}$$

$$\Rightarrow \frac{40}{5} = \frac{x+y}{y}$$

$$\Rightarrow 8y = x + y$$

$$\Rightarrow 7y = x$$

Differentiating both sides w.r.t. 't', we get

$$7 \frac{dy}{dt} = \frac{dx}{dt}$$

$$\therefore \frac{dy}{dt} = \frac{1}{7}(140) = 20 \text{ ft/min}$$

Therefore, the shadow of the man is lengthening at the rate of 20 ft/min.

The tip of the shadow is at C. Let AC = Z

$$\text{Also, } \frac{AS}{BD} = \frac{AC}{BC} \quad [\because \triangle ASC \sim \triangle BDC]$$

$$\therefore \frac{40}{5} = \frac{z}{z-x}$$

$$\Rightarrow 8(z-x) = z$$

$$\Rightarrow 7z = 8x$$

Differentiating both sides w.r.t. 't', we get

$$7 \frac{dz}{dt} = 8 \frac{dx}{dt}$$

$$\Rightarrow \frac{dz}{dt} = \frac{8}{7} \times 140$$

$$= 160 \text{ ft/min}$$

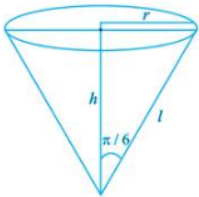
Therefore, the tip of the shadow is moving at the rate 160 ft/min.

61. Given that  $\frac{dv}{dt} = 1 \text{ cm}^3/\text{s}$  where v is the volume of water in the conical vessel.

From the Fig,  $l = 4 \text{ cm}$ ,  $h = l \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2} l$  and  $r = l \sin \frac{\pi}{6} = \frac{l}{2}$ .

$$\text{Therefore, } v = \frac{1}{3} \pi r^2 h = \frac{\pi l^2 \sqrt{3}}{3 \cdot 4 \cdot 2} l = \frac{\sqrt{3}}{24} l^3$$

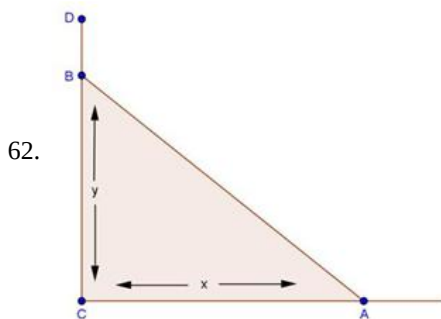
$$\frac{dv}{dt} = \frac{\sqrt{3}\pi}{8} l^2 \frac{dl}{dt}$$



Therefore, Rate of decrease of slant height, when slant height is 4 units.  $= \frac{\sqrt{3}\pi}{8} 16 \cdot \frac{dl}{dt}$

$$\Rightarrow \frac{dl}{dt} = \frac{1}{2\sqrt{3}\pi} \text{ cm/s}$$

Therefore, the rate of decrease of slant height  $= \frac{1}{2\sqrt{3}\pi} \text{ cm/s}$ .



Suppose, CD be the wall and AB is the ladder

here, AB = 6 meter and  $\left(\frac{dx}{dt}\right)_x = 0.5 \text{ m/sec}$ .

$$AB^2 = x^2 + y^2$$

$$(6)^2 = x^2 + y^2$$

$$36 = x^2 + y^2$$

Differentiating it with respect to t,

$$0 = 2x \frac{dx}{dt} + 2y \frac{dy}{dt}$$

$$\frac{dy}{dt} = -\frac{x}{y} \frac{dx}{dt}$$

$$\left(\frac{dy}{dt}\right)_{x=4} = \frac{4(0.5)}{\sqrt{36-x^2}}$$

$$= -\frac{2}{\sqrt{36-16}}$$

$$= -\frac{2}{2\sqrt{5}}$$

$$= -\frac{1}{\sqrt{5}} \text{m/sec}$$

Therefore, ladder top is sliding at the rate of  $\frac{1}{\sqrt{5}}$  m/sec

Now, to find x when  $\frac{dx}{dt} = -\frac{dy}{dt}$

$$\frac{dy}{dt} = -\frac{x}{y} \frac{dx}{dt}$$

$$-\frac{dx}{dt} = -\frac{x}{y} \frac{dx}{dt}$$

$$x = y$$

Now,

$$36 = x^2 + y^2$$

$$36 = x^2 + x^2$$

$$2x^2 = 36$$

$$x^2 = 18$$

$$x = 3\sqrt{2} \text{m}$$

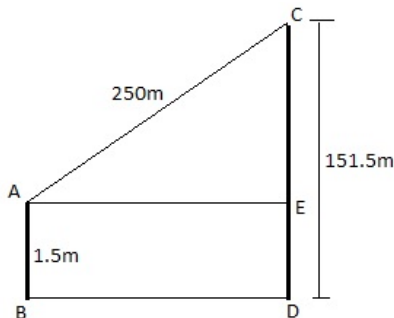
When foot and top are moving at the same rate, foot of wall is  $3\sqrt{2}$  meters away from the wall

63. We have, height ( $h$ ) = 151.5m/s, speed of kite ( $v$ ) = 10m/s

Let CD be the height of kite and AB be the height of boy.

Let  $DB = xm = EA$  and  $AC = 250m$

$$\therefore \frac{dx}{dt} = 10m/s$$



From the figure, we see that

$$EC = 151.5 - 1.5 = 150m$$

And  $AE = x$

Also,  $AC = 250m$

In right angles  $\triangle CEA$ ,

$$AE^2 + EC^2 = AC^2$$

$$\Rightarrow x^2 + (150)^2 = y^2 \dots(i)$$

$$\Rightarrow x^2 + (150)^2 = (250)^2$$

$$\Rightarrow x^2 = (250)^2 - (150)^2$$

$$= (250 + 150)(250 - 150)$$

$$= 400 \times 100$$

$$\therefore x = 20 \times 10 = 200$$

From Eq. (i), on differentiating w.r.t t, we get

$$2x \frac{dx}{dt} + 0 = 2y \frac{dy}{dt}$$

$$\Rightarrow 2y \frac{dy}{dt} = 2x \frac{dx}{dt}$$

$$\therefore \frac{dy}{dt} = \frac{x}{y} \cdot \frac{dx}{dt}$$

$$= \frac{200}{250} \cdot 10 = 8 \text{ m/s} \left[ \because \frac{dx}{dt} = 10 \text{ m/s} \right]$$

So, the required rate at which the string is being let out is 8 m/s.

64. Given, increasing rate of volume of cube,

$$\frac{dV}{dt} = 9 \text{ cubic inches/s}$$

Since, Volume of a cube,  $V = a^3$

Differentiating both sides w.r.t 't', we get

$$\frac{dV}{dt} = 3a^2 \frac{da}{dt}$$

$$\therefore 9 = 3a^2 \frac{da}{dt}$$

$$\Rightarrow \frac{da}{dt} = \frac{3}{a^2} \dots \text{(i)}$$

Now, surface area of cube,  $S = 6a^2$

$$\therefore \text{Rate of change of surface area, } \frac{dS}{dt} = 12a \frac{da}{dt}$$

$$\therefore \frac{dS}{dt} = 12a \frac{3}{a^2} \text{ [from (i)]}$$

$$\Rightarrow \frac{dS}{dt} = \frac{36}{12} = 3 \text{ inches}^2/\text{s} \text{ [Given } a = 12 \text{ inches]}$$

### Section C

65.  $v = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^3$  [as  $\theta = 45^\circ$  gives  $r = h$ ]

$$66. \frac{dv}{dt} = \pi r^2 \frac{dr}{dt}$$

$$\Rightarrow \left( \frac{dr}{dt} \right)_{r=2\sqrt{2}} = -\frac{1}{4\pi} \text{ cm/sec}$$

67.  $C = \pi r l = \pi r \sqrt{2} r = \sqrt{2} \pi r^2$

$$\frac{dC}{dt} = \sqrt{2} \pi 2r \frac{dr}{dt}$$

$$\left( \frac{dC}{dt} \right)_{r=2\sqrt{2}} = -2 \text{ cm}^2/\text{sec}$$

68.  $l^2 = h^2 + r^2$

$$l = 4 \Rightarrow r = h = 2\sqrt{2}$$

$$h = r \Rightarrow \frac{dh}{dt} = \frac{dr}{dt} = -\frac{1}{4\pi} \text{ cm/sec}$$

### Section D

69. (b)  $S = \pi r^2 + \left( \frac{600 - 2\pi r}{4} \right)^2$

**Explanation:** Let the radius of circular garden be  $r$  (in meters) and the side of the square garden be  $x$  (in meters). Then,  $600 = 2$

$$\pi r + 4x \Rightarrow x = \frac{600 - 2\pi r}{4} \dots \text{(i)}$$

The sum of the areas,  $S = \pi r^2 + x^2$

$$\Rightarrow S = \pi r^2 + \left( \frac{600 - 2\pi r}{4} \right)^2$$

70. (a)  $r = \frac{300}{\pi + 4}$

$$\text{Explanation: } \Rightarrow \frac{dS}{dr} = 2\pi r - \frac{4\pi}{16}(600 - 2\pi r) = \frac{\pi}{2}(4r - 300 + \pi r)$$

$$\text{For local points of maxima and/or minima, } \frac{dS}{dr} = \frac{\pi}{2}(4r - 300 + \pi r) = 0$$

$$\Rightarrow \pi r = 300 - 4r$$

$$\Rightarrow r = \left( \frac{300}{\pi+4} \right) \text{metres.}$$

71. (d)  $\frac{d^2s}{dr^2} > 0$

**Explanation:** Also  $\frac{d^2s}{dr^2} = \frac{\pi}{2}(4 + \pi)$

$$\begin{aligned} \therefore \left[ \frac{d^2s}{dr^2} \right]_{\text{at } r = \frac{300}{\pi+4}} \\ = \frac{\pi}{2}(4 + \pi) > 0 \end{aligned}$$

72. (a)  $x = 2r$

**Explanation:**  $x = 2r$

73. (b)  $\frac{1}{2}$

**Explanation:**  $\frac{1}{2}$

74. Let the rate at which the water is running out be given by  $-\frac{dL}{dt}$ .

Given  $L = 200(10 - t)^2$

Now differentiating the above equation with respect to t, we get

$$\Rightarrow -\frac{dL}{dt} = -\frac{d(200(10-t)^2)}{dt}$$

Taking out the constant term, we get

$$\Rightarrow -\frac{dL}{dt} = -200 \cdot \frac{d((10-t)^2)}{dt}$$

Applying the power rule of differentiating, we get

$$\Rightarrow -\frac{dL}{dt} = -200 \cdot 2(10-t) \cdot \frac{d(10-t)}{dt}$$

$$\Rightarrow -\frac{dL}{dt} = -400(10-t) \cdot (-1)$$

$$\Rightarrow -\frac{dL}{dt} = 400(10-t) \dots(i)$$

Now we need to find how fast is the water running out at the end of 5 sec, so finding the value of equation (i) at  $t = 5$ , we get the answer

$$\Rightarrow \left( -\frac{dL}{dt} \right)_{t=5} = (400(10-t))_{t=5}$$

$$\Rightarrow \left( -\frac{dL}{dt} \right)_{t=5} = 400(10-5)$$

$$\Rightarrow \left( -\frac{dL}{dt} \right)_{t=5} = 400(5)$$

$$\Rightarrow \left( -\frac{dL}{dt} \right)_{t=5} = 2000 \frac{L}{s} \dots(ii)$$

Hence the rate at which the water is running out of the pool at the end of 5s is 2000 L/s

Now to find the initial rate we will substitute  $t = 0$  in equation (i), we get

$$\Rightarrow \left( -\frac{dL}{dt} \right)_{t=0} = (400(10-t))_{t=0}$$

$$\Rightarrow \left( -\frac{dL}{dt} \right)_{t=0} = 400(10-0)$$

$$\Rightarrow \left( -\frac{dL}{dt} \right)_{t=0} = 4000 \text{ L/s} \dots(iii)$$

So equation (ii) is the final rate and equation (iii) is the initial rate

Hence the average rate during 5s is

$$= \frac{\text{initial rate} + \text{final rate}}{2}$$

Substituting the corresponding values, we get

$$= \frac{4000 + 2000}{2} = 3000 \text{ L/s}$$

Hence, the average rate at which the water flows out during the first 5s is 3000 L/s .

75. Let two men start from the point C with velocity  $v$  each at the same time.

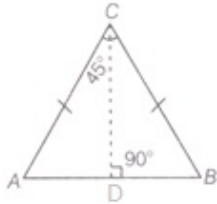
Also,  $\angle BCA = 45^\circ$

Since, A and B are moving with same velocity  $v$ , so they will cover same distance in same time.

Therefore,  $\triangle ABC$  is an isosceles triangle with  $AC = BC$ .

Now, draw  $CD \perp AB$

Let at any instant  $t$ , the distance between them is  $AB$



Let  $AC = BC = x$  and  $AB = y$

In  $\triangle ACD$  and  $\triangle DCB$ ,

$\angle CAD = \angle CBD$  [  $\because AC = BC$  ]

$\angle CDA = \angle CDB = 90^\circ$

$\therefore \angle ACD = \angle DCB$

or  $\angle ACD = \frac{1}{2} \times \angle ACB \Rightarrow \angle ACD = \frac{1}{2} \times 45^\circ$

$$\Rightarrow \angle ACD = \frac{\pi}{8}$$

$$\Rightarrow \sin \frac{\pi}{8} = \frac{AD}{AC}$$

$$\Rightarrow \sin \frac{\pi}{8} = \frac{y/2}{x} \left[ \because AD = \frac{y}{2} \right]$$

$$\Rightarrow \frac{y}{2} = x \sin \frac{\pi}{8}$$

$$\Rightarrow y = 2x \sin \frac{\pi}{8}$$

Now, differentiating both sides w.r.t  $t$ , we get

$$\frac{dy}{dx} = 2 \cdot \sin \frac{\pi}{8} \cdot \frac{dx}{dt}$$

$$= 2 \cdot \sin \frac{\pi}{8} \cdot v \left[ \because v = \frac{dx}{dt} \right]$$

$$= 2v \cdot \frac{\sqrt{2-\sqrt{2}}}{2} \left[ \because \sin \frac{\pi}{8} = \frac{\sqrt{2-\sqrt{2}}}{2} \right]$$

$$= \sqrt{2-\sqrt{2}} v \text{ unit /s}$$

which is the rate at which A and B are being separated.