CBSE

APPLICATION OF DERIVATIVES WS 2

Class 12 - Mathematics

Section A

1. Find the least value of a such that the function f given by f(x) = x^2 + ax + 1 is strictly increasing on (1, 2). **[2]**
2. Prove that the function f(x) = $4x^3$ - $18x^2$ + 27x - 27 is increasing on R. **[2]**
3. Find the values of a for which the functionf(x) = sin x - ax + 4 is increasing function on R. **[2]**
4. Find the interval in function $f(x) = \frac{x^4}{4} + \frac{2}{3}x^3 - \frac{5}{2}x^2 - 6x + 7$ is increasing or decreasing. **[2]**
5. Find the intervals in which the function f given by f(x) = $2x^2$ - 3x is **[2]**

   i. increasing

   ii. decreasing
6. Find the interval of the function f(x) = x^3 -$12x^2$ + 36x + 17 is increasing or decreasing. **[2]**
7. Find the set of values of x for which log(1+x)<x **[2]**
8. Show that f(x) = (x – 1) e^x + 1 is an increasing function for all x > 0. **[2]**
9. Show that $f(x) = \frac{1}{1+x^2}$ is neither increasing nor decreasing on R. **[2]**
10. Find the intervals of function f(x) = (x - 1)$(x - 2)^2$ is **[2]**

    a. increasing

    b. decreasing.
11. Find the least value of a such that the function f given by f(x) = x^2 + ax + 1 is strictly increasing on (1, 2). **[2]**
12. Find the interval in which the function $f(x) = x^3 - 6x^2 + 9x + 15$ is increasing or decreasing. **[2]**
13. Show that the function f(x) = x^2 is strictly increasing function on [0, ∞) **[2]**
14. Find the intervals in which the function f(x) = log $(1 + x) - \frac{2x}{2+x}$ is increasing or decreasing. **[2]**
15. Find the values of b for which the function f(x) = sin x -bx + c is a decreasing function on R. **[2]**
16. For what values of a the function f given by f(x) = x^2 + ax + 1 is increasing on [1, 2]? **[2]**
17. Prove that the function given by f(x) = log sinx is strictly increasing on $\left(0, \frac{\pi}{2}\right)$ and strictly decreasing on $\left(\frac{\pi}{2}, \pi\right)$ **[2]**
18. Find the intervals in which $f(x) = \frac{x}{\log x}$ is increasing or decreasing. **[2]**
19. Prove that the function $f(x) = \tan x - 4x$ is strictly decreasing on $\left(\frac{-\pi}{3}, \frac{\pi}{3}\right)$. **[2]**
20. Show that for a ≥ 1, f(x) = $\sqrt{3}$ sinx - cosx – 2ax + b is decreasing in **R**. **[2]**
21. Show that f(x) = x - sinx is increasing for all $x \in R$ **[2]**
22. Is the function $cos3x$ decreasing on $(0, \frac{\pi}{2})$? **[2]**
23. Prove that the function f given by f(x) = log sinx is increasing on $\left(0, \frac{\pi}{2}\right)$ and decreasing on $\left(\frac{\pi}{2}, \pi\right)$. **[2]**
24. Show that f(x) = x^3 - $15x^2$ + 75x - 50 is an increasing function for all x ∈ R **[2]**
25. Show that the function f(x) = x^{100} + sin x - 1 is increasing on the interval $(\frac{\pi}{2}, \pi)$ **[2]**
26. Show that f(x) = tan x is an increasing function on $(\frac{-\pi}{2}, \frac{\pi}{2})$ **[2]**
27. Prove that the function f(x) = x^3 - $6x^2$ +12x -18 is increasing on R. **[2]**
28. On which of the following intervals is the function f(x) = x^{100} + sin x - 1 increasing **[2]**

29. Find the set of values of b for which f (x) = b (x + cos x) + C is decreasing on R. **[2]**

30. Find the values of x for which the function , **[2]**

$f(x) = kx^3 - 9x^2 + 9x + 3$ is increasing in R

31. Show that f(x) = 2x + $\cot^{-1}$x + log $(\sqrt{1+x^2} - x)$ is increasing in **R**. **[2]**

Section B

32. Find the intervals in which the function given by f(x) = x^4 - $8x^3$ + $22x^2$ - 24x + 21 is (i) increasing. (ii) decreasing. **[3]**

33. Find the intervals in which the function $(x+1)^3(x-3)^3$ is strictly increasing or strictly decreasing. **[3]**

34. Find the interval in function f(x) = $\frac{x^4}{4}$ - x^3 - $5x^2$ + 24x + 12 is increasing or decreasing. **[3]**

35. Find the intervals in which the function f given by f(x) = tan x - 4x, $x \in \left(0, \frac{\pi}{2}\right)$ is **[3]**

a. strictly increasing

b. strictly decreasing

36. Find the intervals on which the function f(x) = x^3 + $2x^2$ - 1 is **[3]**

a. increasing

b. decreasing.

37. Find the values 'a' for which the function f (x) = (a + 2)x^3 - $3ax^2$ + 9ax - 1 decreases for all real values of x. **[3]**

38. Show that the function f(x) = x^3 - $3x^2$ + 3x, x ∈ R is increasing on R. **[3]**

39. Find intervals in which the function given by f (x) = sin 3x, x ∈ $\left[0, \frac{\pi}{2}\right]$ is (a) increasing (b) decreasing. **[3]**

40. Find the interval in function f(x) = $\{x(x-2)\}^2$ is increasing or decreasing. **[3]**

41. Find the interval in function f(x) = 5 + 36x + $3x^2$ - $2x^3$ is increasing or decreasing. **[3]**

42. Find the intervals on which the function f(x) = $2x^3$ - $15x^2$ + 36x + 6 is **[3]**

a. increasing

b. decreasing.

43. Separate the interval $\left[0, \frac{\pi}{2}\right]$ into sub-intervals in which f(x) = ($\sin^4$x + $\cos^4$x) is **[3]**

a. increasing

b. decreasing.

44. Find the intervals in which the function f given by $f(x) = \sin x + \cos x, 0 \leq x \leq 2\pi$ is increasing or decreasing. **[3]**

45. Find for which values of x, the function $f(x) = \frac{x}{x^2+1}$ is increasing and for which values of x it is decreasing. **[3]**

46. Find the intervals on which the function f(x) = $(x + 1)^3$ $(x - 3)^3$ is (a) increasing (b) decreasing. **[3]**

47. Find the intervals on which the function f(x) = (sin x - cos x), 0 < x < 2π is **[3]**

a. increasing

b. decreasing.

48. Find the intervals on which the function f(x) = -$2x^3$ - $9x^2$ - 12x + 1 is (a) strictly increasing (b) strictly decreasing. **[3]**

49. Find the interval of the function that is strictly increasing or decreasing: $(x + 1)^3$ $(x - 3)^3$ **[3]**

50. Find the intervals on which the function f(x) = x^3 + $3x^2$ - 105x + 25 is **[3]**

a. increasing

b. decreasing.

51. Find the intervals in which the function f(x) = $3x^4 - 4x^3 - 12x^2 + 5$ is **[3]**

i. strictly increasing.

ii. strictly decreasing.

52. Find the interval in function f(x) = $10 - 6x - 2x^2$ is increasing or decreasing. **[3]**

53. Show that f(x) = cos x is a decreasing function on (0, π), increasing in (- π , 0) and neither increasing nor decreasing in (- π , π). **[3]**

54. Find the interval in function f(x) = $8 + 36x + 3x^2 - 2x^3$ is increasing or decreasing. **[3]**

55. Show that the function given by f(x) = sin x is **[3]**

i. strictly increasing in $\left(0, \frac{\pi}{2}\right)$

ii. strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$

iii. neither increasing nor decreasing in $(0, \pi)$

56. Find the interval in function f(x) = $x^2 + 2x - 5$ is increasing or decreasing. **[3]**

57. Find the intervals on which the function f(x) = $\frac{4x^2+1}{x}$, $(x \neq 0)$ is **[3]**

a. increasing

b. decreasing.

58. Find the intervals in which the function f(x) = sin 3x - cos 3x, 0 < x < π, is strictly increasing or strictly decreasing. **[3]**

59. Find the intervals in which f(x) = $\log(1 + x) - \frac{x}{1+x}$ is increasing or decreasing. **[3]**

60. Prove that $y = \frac{4\sin\theta}{2+\cos\theta} - \theta$ is an increasing function for θ in $\left[\theta, \frac{\pi}{2}\right]$ **[3]**

61. Prove that $y = \frac{4\sin\theta}{(2+\cos\theta)} - \theta$ is an increasing function of θ in $\left[0, \frac{\pi}{2}\right]$ **[3]**

62. Prove that $y = \frac{4\sin\theta}{2+\cos\theta} - \theta$ is an increasing function for θ in $\left[\theta, \frac{\pi}{2}\right]$ **[3]**

63. Find the intervals in which the function f given by $f(x) = \frac{4\sin x - 2x - x\cos x}{2+\cos x}$ is (i) increasing (ii) decreasing **[3]**

64. Show that f(x) = tan -1 (sin x + cos x) is a decreasing function on the interval ($\frac{\pi}{4}$, $\frac{\pi}{2}$). **[3]**

65. Find the interval in function f(x) = (x - 1)$(x - 2)^2$ is increasing or decreasing. **[3]**

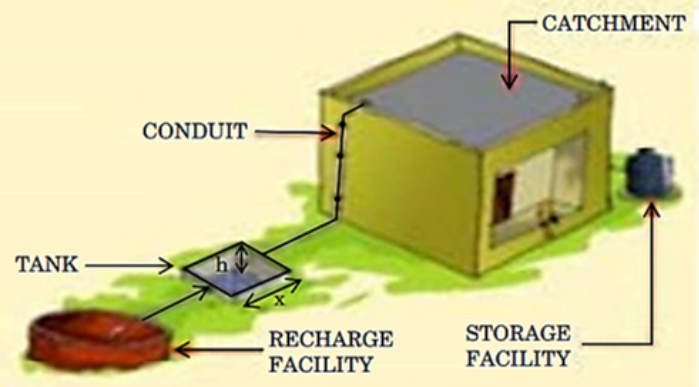
66. Show that $y = \log(1 + x) - \frac{2x}{2+x}, x > -1$ is an increasing function of x throughout its domain. **[3]**

Section C

Question No. 67 to 70 are based on the given text. Read the text carefully and answer the questions: **[4]**

In order to set up a rain water harvesting system, a tank to collect rain water is to be dug. The tank should have a square base and a capacity of 250 m^3. The cost of land is ₹ 5,000 per square metre and cost of digging increases with depth and for the whole tank, it is ₹ 40,000 h^2, where h is the depth of the tank in metres. x is the side of the square base of the tank in metres.

ELEMENTS OF A TYPICAL RAIN WATER HARVESTING SYSTEM



67. Find the total cost C of digging the tank in terms of x.

68. Find $\frac{dC}{dx}$.

69. Find the value of x for which cost C is minimum.

70. Check whether the cost function C(x) expressed in terms of x is increasing or not, where x > 0.

Section D

71. Which of the following functions are decreasing on $(0, \frac{\pi}{2})$? **[5]**

    i. cos x

    ii. cos 2x

    iii. cos 3x

    iv. tan x

72. Show that the function f(x) = $\cot^{-1}$(sin x + cos x) is decreasing on (0, $\frac{\pi}{4}$) and increasing on $(\frac{\pi}{4}, \frac{\pi}{2})$ **[5]**

73. Find the intervals in which the function f(x) = 20- 9x + $6x^2$ - x^3 is (i) strictly increasing. (ii) strictly decreasing. **[5]**

74. Show that $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in $(0, \frac{\pi}{4})$ **[5]**

75. if a, b, c are real numbers, then find the intervals in which **[5]**

$$f(x) = \begin{vmatrix} x+a^2 & ab & ac \\ ab & x+b^2 & bc \\ ac & bc & x+c^2 \end{vmatrix}$$ is increasing or decreasing