Solution

APPLICATION OF DERIVATIVES WS 2

Class 12 - Mathematics

Section A

1. -2

   Explanation:

   $f(x) = x^2 + ax + 1$

   $\Rightarrow f'(x) = 2x + a$

   Since f(x) is strictly increasing on (1, 2), therefore $f'(x) = 2x + a > 0$ for all x in (1, 2)

   $\therefore$ On (1, 2) $1 < x < 2$

   $\Rightarrow 2 < 2x < 4$

   $\Rightarrow 2 + a < 2x + a < 4 + a$

   $\therefore$ Minimum value of f' (x) is 2 + a and maximum value is 4 + a.

   Since $f'(x) > 0$ for all x in (1, 2)

   $\therefore 2 + a > 0$ and $4 + a > 0$

   $\Rightarrow a > -2$ and $a > -4$

   Therefore least value of a is - 2.

2. Given:

   $f(x) = 4x^3 - 18x^2 + 27x - 27$

   $\therefore f'(x) = 12x^2 - 36x + 27$

   $= 12x^2 - 18x - 18x + 27$

   $= 3(2x - 3)^2$

   Now,

   $x \in R$

   $\Rightarrow (2x - 3)^2 > 0$

   $\Rightarrow 3(2x - 3)^2 > 0$

   $\Rightarrow f'(x) > 0$

   Hence the function is increasing.

3. Given: f(x) = sinx - ax + 4

   f (x) = cos x - a

   Given : F(x) is increasing on R

   $\Rightarrow f'(x) > 0$

   $\Rightarrow$ cosx - a > 0

   $\Rightarrow$ cos x > a

   We know

   Cos x > -1 , $\forall x \in R$

   $\therefore a < -1$

   $\Rightarrow a \in (-\infty, -1)$

4. Given: $f(x) = \frac{x^4}{4} + \frac{2}{3}x^3 - \frac{5}{2}x^2 - 6x + 7$

   $\Rightarrow f'(x) = x^3 + 2x^2 - 5x - 6$

   To find critical point for f(x), we must have

   $\Rightarrow f'(x) = 0$

   $\Rightarrow x^3 + 2x^2 - 5x - 6 = 0$

   $\Rightarrow (x + 1)(x - 2)(x + 3) = 0$

   $\Rightarrow x = -1, 2, -3$

   clearly, $f'(x) > 0$ if $-3 < x < -1$ and $x > 2$

   and $f'(x) < 0$ if $x < -3$ and $-3 < x < -1$

   Thus, f(x) increases on $(-3, -1) \cup (2, \infty)$

   and f(x) is decreasing on interval $(\infty, -3) \cup (-1, 2)$

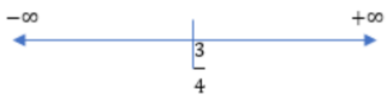
5. i. It is given that function $f(x) = 2x^2 - 3x$

$\Rightarrow$ $f'(x) = 4x - 3$

If $f'(x) = 0$, then we get,

$x = \frac{3}{4}$

So, the points $\frac{3}{4}$ divides the real line into two disjoint intervals, $\left(-\infty, \frac{3}{4}\right)$ and $\left(\frac{3}{4}, \infty\right)$



So, in interval $\left(\frac{3}{4}, \infty\right)$, $f'(x) = 4x - 3 > 0$

Therefore, the given function (f) is strictly increasing in interval $\left(\frac{3}{4}, \infty\right)$.

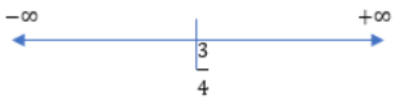
ii. It is given that function $f(x) = 2x^2 - 3x$

$\Rightarrow$ $f'(x) = 4x - 3$

If $f'(x) = 0$, then we get,

$x = \frac{3}{4}$

So, the point $\frac{3}{4}$ divides the real line into two disjoint intervals, $\left(-\infty, \frac{3}{4}\right)$ and $\left(\frac{3}{4}, \infty\right)$



So, in interval $\left(-\infty, \frac{3}{4}\right)$, $f'(x) = 4x - 3 < 0$

Therefore, the given function (f) is strictly decreasing in interval $\left(\frac{3}{4}, \infty\right)$.

6. Given: $f(x) = x^3 – 12x^2 + 36x + 17$

$\Rightarrow f(x) = \frac{d}{dx}\left(x^3 - 12x^2 + 36x + 17\right)$

$\Rightarrow$ $f'(x) = 3x^2 – 24x + 36$

For f(x) lets find critical point, we must have

$\Rightarrow$ $f'(x) = 0$

$\Rightarrow$ $3x^2 – 24x + 36 = 0$

$\Rightarrow$ $3(x^2 – 8x + 12) = 0$

$\Rightarrow$ $3(x^2 – 6x – 2x + 12) = 0$

$\Rightarrow$ $x^2 – 6x – 2x + 12 = 0$

$\Rightarrow$ $(x – 6)(x – 2) = 0$

$\Rightarrow$ $x = 2, 6$

clearly, $f'(x) > 0$ if $x < 2$ and $x > 6$

and $f'(x) < 0$ if $2 < x < 6$

Thus, the given function f(x) increases on $(–\infty, 2) \cup (6, \infty)$ and f(x) is decreasing on interval $x \in (2, 6)$.

7. Let $f(x)=\log(1+x)-x$

$f'(x) = \frac{1}{1+x} - 1 = -\frac{x}{1+x} f'(x) < 0, \textit{ for } x > 0 f(x) \textit{ is decreasing for } x > 0$

$\Rightarrow$ $f(x)<f(0)$, for $x>0$

$\Rightarrow$ $\log(1+x)-x<0$, for $x>0$

i.e, $\log(1+x)<x$, for $x>0$

8. Given:- $f(x) = (x - 1) e^x + 1$

$\Rightarrow f'(x) = \frac{d}{dx}\left((x-1)e^x + 1\right)$

$= f'(x) = e^x + (x - 1) e^x$

$= f'(x) = e^x(1 + x - 1)$

$= f'(x) = xe^x$

as given

$x > 0$

$= e^x > 0$

$= xe^x > 0$

$= f'(x) > 0$

Hence, the condition for f(x) to be increasing

Thus, f(x) is increasing for all x > 0

9. Given:

$f(x) = \frac{1}{1+x^2}$

Let $x_1 > x_2$

$\Rightarrow \quad x_1^2 > x_2^2$

$\Rightarrow \quad 1 + x_1^2 > 1 + x_2^2$

$\Rightarrow \quad \frac{1}{1+x_1^2} < \frac{1}{1+x_2^2}$

$\Rightarrow \quad f\left(x_1\right) < f\left(x_2\right)$

f(x) is decreasing on $[0, \infty)$

Case 2

$\Rightarrow \quad x_1^2 < x_2^2$

$\Rightarrow \quad 1 + x_1^2 < 1 + x_2^2$

$\Rightarrow \quad \frac{1}{1+x_1^2} > \frac{1}{1+x_2^2}$

$\Rightarrow \quad f\left(x_1\right) > f\left(x_2\right)$

So, f(x) is increasing on $[0, \infty)$

Thus, f(x) is neither increasing nor decreasing on R.

10. Given function is $f(x) = (x - 1)(x - 2)^2 = x^2 - 4x + 4\ (x - 1)$

$= x^3 - 4x^2 + 4x - x^2 + 4x - 4$

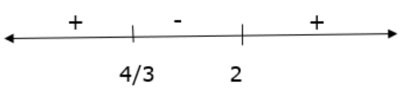
$f(x) = x^3 - 5x^2 + 8x - 4$

$f'(x) = 3x^2 - 10x + 8$

$f'(x) = 3x^2 - 6x - 4x + 8$

$= 3x(x - 2) -4(x - 2)$

$= (3x - 4)(x - 2)$



Function f(x) is decreasing for $x \in [4/3, 2]$ and increasing in $x \in (-\infty, 4/3) \cup (2, \infty)$.

11. $f(x) = x^2 + ax + 1$

$\Rightarrow f'(x) = 2x + a$

Since f(x) is strictly increasing on (1, 2), therefore $f'(x) = 2x + a > 0$ for all x in (1, 2)

$\therefore$ On (1, 2) $1 < x < 2$

$\Rightarrow 2 < 2x < 4$

$\Rightarrow 2 + a < 2x + a < 4 + a$

$\therefore$ Minimum value of f' (x) is 2 + a and maximum value is 4 + a.

Since $f'(x) > 0$ for all x in (1, 2)

$\therefore$ 2 + a > 0 and 4 + a > 0

$\Rightarrow$ a > -2 and a > -4

Therefore least value of a is - 2.

Which is the required solution.

12. Given: $f(x) = x^3 - 6x^2 + 9x + 15$

$\Rightarrow f(x) = \frac{d}{dx}\left(x^3 - 6x^2 + 9x + 15\right)$

$\Rightarrow f'(x) = 3x^2 - 12x + 9$

For f(x) lets find critical point, we must have

$\Rightarrow f'(x) = 0$

$\Rightarrow 3x^2 - 12x + 9 = 0$

$\Rightarrow 3(x^2 - 4x + 3) = 0$

$\Rightarrow 3(x^2 - 3x - x + 3) = 0$

$\Rightarrow x^2 - 3x - x + 3 = 0$

$\Rightarrow (x - 3)(x - 1) = 0$

$\Rightarrow x = 1, 3$

clearly, $f'(x) > 0$ if $x < 1$ and $x > 3$ and $f'(x) < 0$ if $1 < x < 3$

Thus, f(x) increases on the interval $(-\infty, 1) \cup (3, \infty)$

and f(x) is decreasing on interval $x \in (1, 3)$

13. Let $x_1, x_2 \in [0, \infty)$ such that $x_1 < x_2$. Then,

$x_1 < x_2 \Rightarrow x_1^2 < x_1x_2$.....(i)

$x_1 < x_2 \Rightarrow x_1x_2 < x_2$ ...(ii)

From (i) and (ii)

$x_1 < x_2 \Rightarrow x_1^2 < x_2^2 \Rightarrow f\left(x_1\right) < f\left(x_2\right)$

$x_1 < x_2 \Rightarrow f\left(x_1\right) < f\left(x_2\right)$ for all $x_1, x_2 \in [0, \infty)$

Hence f(x) is strictly increasing function in the interval $[0, \infty)$

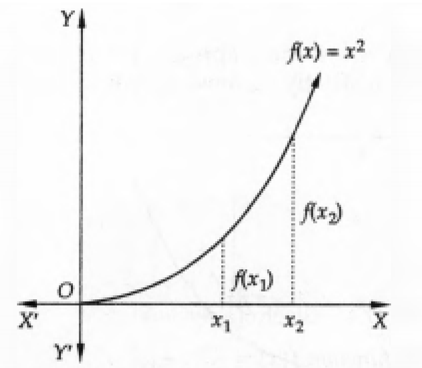


Fig. 17.9 Graph of $f(x) = x^2, x \geq 0$

14. Given: $f(x) = \log(1 + x) - \frac{2x}{2+x}$

$\Rightarrow \quad f'(x) = \frac{1}{1+x}\frac{d}{dx}(x + 1) - \frac{(2+x) \times 2 - 2x(0+1)}{(2+x)^2} = \frac{1}{1+x} - \frac{4}{(2+x)^2}$

$\Rightarrow \quad f'(x) = \frac{(2+x)^2 - 4(1+x)}{(2+x)^2(1+x)} = \frac{x^2}{(2+x)^2(1+x)} = \left(\frac{x}{2+x}\right)^2 \frac{1}{x+1}$

For f(x) to be increasing, we must have

$\Rightarrow \quad \left(\frac{x}{2+x}\right)^2 \frac{1}{x+1} > 0$

$\Rightarrow \quad \frac{1}{x+1} > 0$ and $x \neq 0$

$\Rightarrow \quad x + 1 > 0$ and $x \neq 0$

$\Rightarrow \quad x > -1$ and $x \neq 0$

so, f(x)is increasing on $(-1, 0) \cup (0, \infty)$

15. Given:

f(x) = sin x – bx +c

$f'(x) = \cos x - b$

Given that f(x) is a decreasing function on R

$\therefore f'(x) < 0$ for all $x \in R$

$\Rightarrow \cos x - b < 0$ for all $x \in R$

$\Rightarrow b > \cos x, \forall\, x \in R$

But the last values of cos x in 1

$\therefore b \geq 1$

16. f(x) = x^2 + ax + 1

$\therefore$ f'(x) = 2x + a

Now, function f will be increasing in (1, 2), if f'(x) > 0 in (1, 2).

$\Rightarrow$ 2x + a > 0

$\Rightarrow$ 2x > -a

$\Rightarrow$ $x > \frac{-a}{2}$

Therefore, we have to find the least value of a such that

$\Rightarrow$ $x > \frac{-a}{2}$, when $x \in (1, 2)$.

Thus, the least value of a for f to be increasing on (1, 2) is given by,

$\frac{-a}{2} = 1 \Rightarrow a = -2$

Hence, the required value of a is -2.

17. $\boldsymbol{f'(x) = \frac{1}{\sin x} . \cos x}$

$f'(x) = \cot x$

$\cot x > 0\ \forall x \in \left(0, \frac{\pi}{2}\right)$

and

$\cot x < 0\ \forall x \in \left(\frac{\pi}{2}, \pi\right)$

Hence f(x) = log sinx is strictly increasing on $\left(0, \frac{\pi}{2}\right)$ and decreasing on $\left(\frac{\pi}{2}, \pi\right)$

18. Note that the domain of f(x) is the set of all positive real numbers other than unity ie; $(0,1) \cup (1, \infty)$

Now $f(x) = \frac{x}{\log x}$

$\Rightarrow \quad f'(x) = \frac{\log x - 1}{(\log x)^2}$

For f(x) to be increasing function, we must have

$f'(x) > 0$

$\Rightarrow \quad \frac{\log x - 1}{(\log x)^2} > 0$

$\Rightarrow \quad \log x - 1 > 0$

$\log x > 1$

$x > e^1$

So, f(x) is increasing on (e, ∞)

For f(x) to be decreasing we must have

$\Rightarrow \quad \frac{\log x - 1}{(\log x)^2} < 0$

$\Rightarrow \quad \log x - 1 < 0$

$\Rightarrow \quad \log x < 1$

$\Rightarrow \quad x < e^1$

So f(x) is decreasing on $(0, e)$

19. Given function, $f(x) = \tan x - 4x \Rightarrow f'(x) = \sec^2 x - 4$

When $\frac{-\pi}{3} < x < \frac{\pi}{3}, 1 < \sec x < 2$

Therefore, $1 < \sec^2 x < 4 \Rightarrow -3 < \left(\sec^2 x - 4\right) < 0$

Thus for $\frac{-\pi}{3} < x < \frac{\pi}{3}, f'(x) < 0$

Hence, f is strictly decreasing on $\left(\frac{-\pi}{3}, \frac{\pi}{3}\right)$.

20. We have $f(x) = \sqrt{3}\sin x - \cos x - 2ax + b$

$\therefore f'(x) = \sqrt{3}\cos x + \sin x - 2a$

$= 2\left[\frac{\sqrt{3}}{2} \cdot \cos x + \frac{1}{2} \cdot \sin x\right] - 2a$

$= 2\left[\cos\frac{\pi}{6} \cdot \cos x + \sin\frac{\pi}{6} \cdot \sin x\right] - 2a = 2\cos\left(\frac{\pi}{6} - x\right) - 2a$

If f(x) is decreasing of R,

$\therefore f'(x) \le 0$ for all real x

$\Rightarrow 2\cos\left(\frac{\pi}{6} - x\right) - 2a \le 0$ for all real x

$\Rightarrow a \ge \cos\left(\frac{\pi}{6} - x\right)$ for all real x

$\Rightarrow a \ge$ max. value of $\cos\left(\frac{\pi}{6} - x\right)$

$\Rightarrow a \ge 1$

21. Given: $f(x) = x - \sin x$

$f'(x) = \frac{d}{dx}(x - \sin x)$

$\Rightarrow f'(x) = 1 - \cos x$

Now, for given $x \in R$, we have

$-1 < \cos x < 1$

$\Rightarrow -1 > \cos x > 0$

$\Rightarrow f'(x) > 0$

Hence, Condition for f(x) to be increasing is satisfied.

Thus, function f(x) is increasing on interval $x \in R$.

22. Let $f(x) = \cos 3x$

$\therefore f'(x) = -3\sin 3x$

Now, $f'(x) = 0$

$\Rightarrow \sin 3x = 0$

$\Rightarrow 3x = \pi$, as $x \in \left(0, \frac{\pi}{2}\right)$

$\Rightarrow x = \frac{\pi}{3}$

The point $x = \frac{\pi}{3}$ divides the interval $\left(0, \frac{\pi}{2}\right)$ into two distinct intervals.

i.e. $\left(0, \frac{\pi}{3}\right)$ and $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

Now, in the interval, $\left(0, \frac{\pi}{3}\right)$

$f'(x) = -3\sin 3x < 0$ as $\left(0 < x < \frac{\pi}{3} \Rightarrow 0 < 3x < \pi\right)$

Therefore, 'f ' is strictly decreasing in interval $\left(0, \frac{\pi}{3}\right)$

Now, in the interval $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

$f'(x) = -3\sin 3x > 0$ as $\frac{\pi}{3} < x < \frac{\pi}{2} \Rightarrow \pi < 3x < \frac{3\pi}{2}$

Therefore, 'f ' is strictly increasing in the interval $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$.

23. Given: f(x) = log sin x

$\Rightarrow f'(x) = \frac{1}{\sin x}\frac{d}{dx}(\sin x) = \frac{1}{\sin x}\cos x = \cot x$

On the interval $\left(0, \frac{\pi}{2}\right)$ i.e., in first quadrant,

f'(x) = cotx > 0

Therefore, f(x) is strictly increasing on $\left(0, \frac{\pi}{2}\right)$.

On the interval $\left(\frac{\pi}{2}, \pi\right)$ i.e., in second quadrant,

f'(x) = cot x < 0

Therefore, f(x) is strictly decreasing on $\left(\frac{\pi}{2}, \pi\right)$.

24. Given: $f(x) = x^3 - 15x^2 + 75x - 50$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

i. If f'(x) > 0 for all x ∈ (a, b), then f(x) is increasing on (a, b)

ii. If f'(x) < 0 for all x ∈ (a, b), then f(x) is decreasing on (a, b)

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

We have,

$f(x) = x^3 - 15x^2 + 75x - 50$

$\Rightarrow f(x) = \frac{d}{dx}\left(x^3 - 15x^2 + 75x - 50\right)$

$= f'(x) = 3x^2 - 30x + 75$

$= f'(x) = 3(x^2 - 10x + 25)$

$= f'(x) = 3(x - 5)^2$

Now, as given

$x \in R$

$= (x - 5)^2 > 0$

$= 3(x - 5)^2 > 0$

= f'(x) > 0

Hence, it is the condition for f(x) to be increasing

Thus, f(x) is increasing on the interval $x \in R$

25. Given interval: $x \in (\pi/2, \pi)$

$\Rightarrow \quad \pi/2 < x < \pi$

$x^{99} > 1$

$100x^{99} > 100$

Again, $x \in (\pi/2, \pi) \Rightarrow -1 < \cos x < 0 \Rightarrow 0 > \cos x > -1$

$100x^{99} > 100$ and $\cos x > -1$

$100x^{99} + \cos x > 100 - 1 = 99$

$100x^{99} + \cos x > 0$

$f'(x) > 0$

Thus f(x) is increasing on $(\pi/2, \pi)$

26. Given function: f(x) = tan x

$\Rightarrow f(x) = \frac{d}{dx}(\tan x)$

f'(x) = $\sec^2 x$

Now, as given

$x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

That is 4th quadrant, where

= $\sec^2 x > 0$

= f'(x) > 0

Hence, Condition for f(x) to be increasing

Thus, f(x) is increasing on the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

27. Given:- f(x) = $x^3 - 6x^2 + 12x - 18$

$\Rightarrow f'(x) = \frac{d}{dx}\left(x^3 - 6x^2 + 12x - 18\right)$

$\Rightarrow$ f'(x) = $3x^2 - 12x + 12$

$\Rightarrow$ f'(x) = $3(x^2 - 4x + 4)$

$\Rightarrow$ f'(x) = $3(x - 2)^2$

as given $x \in R$

$\Rightarrow (x - 2)^2 > 0$

$\Rightarrow 3(x - 2)^2 > 0$

$\Rightarrow$ f'(x) > 0

Hence, condition for f(x) to be increasing

Thus the given function f(x) is increasing on interval $x \in R$.

28. We have seen that the function, f'(x) > 0 for 0 < x < 1. But, f'(x) can be positive as well as negative when - 1 < x < 0. So, f'(x) can be positive as well as negative for $x \in (-1, 1)$. Hence, f(x) is neither increasing nor decreasing on (- 1, 1).

29. The given function is

f(x) = b (x + cosx) + C

$\Rightarrow$ f '(x) = b (1 - sin x)

Given f(x) is decreasing on R

$\Rightarrow$ f(x) <0

$\Rightarrow$ b (1 - sin x) <0...(1)

Also, We know that

Sin x < 1

$\Rightarrow$ 1 - sin x > 0

$\Rightarrow b < 0 \ \ldots$ [since $(1 - \sin x) \geq 0, b(1 - \sin x) < 0 \Rightarrow b$

$\Rightarrow b \in (-\infty, 0)$

30. we have, f(x)=$kx^3 - 9x^2 + 9x + 3$

$\Rightarrow f'(x) = 3kx^2 - 18x + 9$

Since f(x) is increasing on R, therefore , $f'(x) > 0 \ \forall x \in R$

$\Rightarrow 3kx^2 - 18x + 9 > 0, \forall x \in R \Rightarrow kx^2 - 6x + 3 > 0, \forall x \in R$

$\Rightarrow$ k>0 and 36-12k<0 [$\because ax^2 + bx + c > 0, \forall x \in R \Rightarrow a > 0$ *and discriminant* < 0]

$\Rightarrow k > 3$

Hence,f(x) is increasing on R, if k>3.

31. We have, f(x) = 2x + $\cot^{-1}x$ + log $(\sqrt{1 + x^2} - x)$

$\therefore f'(x) = 2 - \frac{1}{1+x^2} + \frac{1}{\sqrt{1+x^2} - 1}\left(\frac{x}{\sqrt{1+x^2}} - 1\right)$

$= \frac{1}{1+x^2} + \frac{1}{\sqrt{1+x^2}-x} \cdot \frac{x-\sqrt{1+x^2}}{\sqrt{1+x^2}} = 2 - \frac{1}{1+x^2} - \frac{1}{\sqrt{1+x^2}}$

Now, $1 + x^2$, $\sqrt{1+x^2} \geq 1$ for all real x,

$\therefore 2 - \frac{1}{1+x^2} - \frac{1}{\sqrt{1+x^2}} > 0$ for all real x,

$\therefore$ f'(x) > 0 for all real x

Thus f(x) is increasing of R.

Section B

32. According to the question, $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$

Therefore,on differentiating both sides w.r.t. x, we get

$f'(x) = 4x^3 - 24x^2 + 4x - 24$

$= 4(x^3 - 6x^2 + 11x - 6)$

$= 4(x - 1)(x^2 - 5x + 6)$

$= 4(x - 1)(x - 2)(x - 3)$

Put f'(x)= 0

$\Rightarrow 4(x - 1)(x - 2)(x - 3) = 0 \Rightarrow x = 1, 2, 3$

Therefore, the possible intervals are (-∞, 1), (1, 2), (2, 3) and (3, ∞).

For interval (-∞, 1), f'(x) < 0

For interval (1, 2), f'(x) > 0

For interval (2, 3), f'(x) < 0

For interval (3, ∞), f'(x) > 0.

Also, as f(x) is a polynomial function, so it is contiuous at x = 1, 2, 3.

Hence,

i. function increases in [1, 2] and [3, ∞).
ii. function decreases in (- ∞. 1] and [2, 3].

33. Here

$f(x) = (x+1)^3(x-3)^3$

$f'(x) = (x+1)^3.3(x-3)^2 + (x-3)^3.3(x+1)^2$

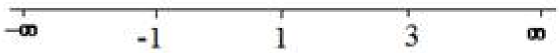
$= 3(x+1)^2(x-3)^2[x+1+x-3]$

$= 3(x+1)^2(x-3)^2[2x-2]$

$= 6(x+1)^2(x-3)^2(x-1)$

put f '(x) = 0

$x = -1, 3, 1$



int	Sign of f'(x)	Result
$(-\infty, -1)$	-tive	Decrease
$(-1, 1)$	-tive	Decrease
(1, 3)	+tive	Increase
(3, ∞)	+tive	Increase

34. We have,

$f(x) = \log(2+x) - \frac{2x}{2+x}$

$\Rightarrow f'(x) = \frac{d}{dx}\left(\log(2+x) - \frac{2x}{2+x}\right)$

$f'(x) = \frac{1}{2+x} - \frac{(2+x)2-2x\times 1}{(2+x)^2}$

$f'(x) = \frac{1}{2+x} - \frac{4}{(2+x)^2}$

$f'(x) = \frac{2+x-4}{(2+x)^2}$

$f'(x) = \frac{x-2}{(2+x)^2}$

For f(x) to be increasing, we must have

$\Rightarrow$ f'(x) > 0

$\frac{x-2}{(2+x)^2} > 0$

$\Rightarrow$ (x – 2) > 0

$\Rightarrow$ 2 < x < ∞

$\Rightarrow$ x ∈ (2, ∞)

Therefore, f(x) is increasing on interval (2, ∞)

Again, For f(x) to be decreasing, we must have

f'(x) < 0

$\frac{x-2}{(2+x)^2} > 0$

$\Rightarrow$ (x – 2) < 0

$\Rightarrow$ –∞ < x < 2

$\Rightarrow$ x ∈ (–∞, 2)

Therefore, f(x) is decreasing on interval (–∞, 2)

35. f(x) = tan x - 4x

f'(x) = $\sec^2 x - 4$

a. For f(x) to be strictly increasing

f'(x) > 0

$\Rightarrow \sec^2 x - 4 > 0$

$\Rightarrow \sec^2 x > 4$

$\Rightarrow \cos^2 x < \frac{1}{4}$

$\Rightarrow \cos^2 x < \left(\frac{1}{2}\right)^2$

$\Rightarrow -\frac{1}{2} < \cos x < \frac{1}{2}$

$\Rightarrow \frac{\pi}{3} < x < \frac{\pi}{2} \left[\because x \in \left(0, \frac{\pi}{2}\right)\right]$

b. For f(x) to be strictly decreasing

f'(x) < 0

$\Rightarrow \sec^2 x - 4 < 0$

$\Rightarrow \sec^2 x > 4$

$\Rightarrow \cos^2 x > \frac{1}{4}$

$\Rightarrow \cos^2 x > \left(\frac{1}{2}\right)^2$

$\Rightarrow \cos x > \frac{1}{2} \left[\because x \in \left(0, \frac{\pi}{2}\right)\right]$

$\Rightarrow 0 < x < \frac{\pi}{3}$

36. Here,it is given the function

f(x) = $x^3 + 2x^2 - 1$

$\Rightarrow$ f(x) = $3x^2 + 4x = 3x\left(x + \frac{4}{3}\right)$ ... (i)

1. (a) f(x) is increasing

$\Leftrightarrow$ f'(x) $\geq$ 0

$\Leftrightarrow\ 3x\left(x+\frac{4}{3}\right) \geq 0$

$\Leftrightarrow\ x\left(x+\frac{4}{3}\right) \geq 0$

$\Leftrightarrow$ [x $\geq$ 0 and $\left(x+\frac{4}{3}\right) \geq 0$] or [x $\leq$ 0 and $\left(x+\frac{4}{3}\right) \leq 0$]

$\Leftrightarrow$ [x $\geq$ 0 and $x \geq -\frac{4}{3}$] or [x $\leq$ 0 and $x \leq -\frac{4}{3}$]

$\Leftrightarrow$ (x $\geq$ 0) or $\left(x \geq -\frac{4}{3}\right)$

$\Leftrightarrow\ x \in [0, \infty]$ or $x \in \left[-\infty, -\frac{4}{3}\right]$

$\Leftrightarrow\ x \in \left[-\infty, -\frac{4}{3}\right] \cup [0, \infty]$

$\therefore$ f(x) is increasing on $\left[-\infty, -\frac{4}{3}\right] \cup [0, \infty]$

2. (b) f(x) is decreasing

f'(x) $\leq$ 0

$3x\left(x+\frac{4}{3}\right) \leq 0$

$x\left(x+\frac{4}{3}\right) \leq 0$

$-\frac{4}{3} \leq x \leq 0$

$x \in \left[-\frac{4}{3}, 0\right]$

$\therefore$ f(x) is decreasing on $\left[-\frac{4}{3}, 0\right]$

Hence, f(x) is increasing on $\left[-\infty, -\frac{4}{3}\right] \cup [0, \infty]$ and decreasing on $\left[-\frac{4}{3}, 0\right]$

37. We have

$f(x) = (a + 2)x^3 - 3ax^2 + 9ax - 1$ diff w.r.t.x,we obtain

$f'(x) = 3(a + 2)x^2 - 6ax + 9a$

Since f (x) is decreasing for all real values of x. Thus,

f'(x) < 0 for all x $\in$ R

$3(a + 2)x^2 - 6ax + 9a < 0$ for all x $\in$ R

$(a + 2)x^2 - 2ax + 3a < 0$ for all x $\in$ R

a + 2 < 0 and $4a^2 - 4 \times (a + 2) \times 3a < 0$ [since b^2-4ab<0]

a < -2 and $a^2 - 3a^2 - 6a < 0$

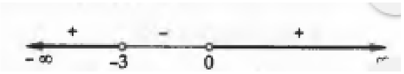
a < -2 and $-2a^2 - 6a < 0$

a < -2 and - 2a(a + 3) < 0

Now,

$-2a(a+3) < 0$

$\Rightarrow\ a(a+3) > 0$



$\Rightarrow\ a < -3$ or, $a > 0$

$\in (-\infty, -3) \cup (0, \infty)$

$a < -2$ and $-2a(a+3) < 0$

$a < -2$ and $a \in (-\infty, -3) \cup (0, \infty)$

$a \in (-\infty, -3)$

So, f(x) decreases for all $x \in R$, if $a \in (-\infty, -3)$

38. A continuous function y = f(x) is said to be increasing on R, if $\frac{dy}{dx} \geq 0, \forall x \in R$

Given, $y = x^3 - 3x^2 + 3x$

Differentiating both sides w.r.t. x,

$\frac{dy}{dx} = 3x^2 - 6x + 3$

$\Rightarrow \frac{dy}{dx} = 3(x^2 - 2x + 1)$

$\Rightarrow \frac{dy}{dx} = 3(x-1)^2$

Now, $3(x-1)^2 \geq 0$ for all real values of x, i.e $\forall x \in R$.

$\frac{dy}{dx} \geq 0 \quad \forall x \in R$

Hence, the given function is increasing on R.

39. $\boldsymbol{f(x) = \sin 3x}$

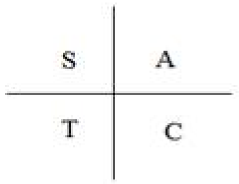
$f'(x) = 3\cos 3x$

$f'(x) = 0$

$\cos 3x = 0$

$3x = \frac{\pi}{2}$

$x = \frac{\pi}{6}$



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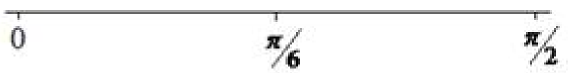


int.	**Sign of f'(x)**	**Result**
$\left[0, \frac{\pi}{6}\right)$	+tive	increase
$\left(\frac{\pi}{6}, \frac{\pi}{2}\right]$	-tive	Decrease

Hence, f(x) is increasing on $\left(0, \frac{\pi}{6}\right)$ and decreasing on $\left(\frac{\pi}{6}, \frac{\pi}{2}\right)$

40. Here,it is given that f(x)= $\{x(x-2)\}^2$

= f(x)= $\{(x^2-2x)\}^2$

$\Rightarrow f(x) = \frac{d}{dx}\left(\left[x^2 - 2x\right]^2\right)$

f'(x) = $2(x^2 - 2x)(2x - 2)$

f'(x) = $4x(x-2)(x-1)$

For f(x) lets find critical point, we must have

f'(x) = 0

$4x(x-2)(x-1) = 0$

$x(x - 2)(x - 1)= 0$

$x = 0, 1, 2$

Now, lets check values of f(x) between different ranges

Here points x = 0, 1, 2 divide the number line into disjoint intervals namely, $(-\infty, 0)$,(0, 1), (1, 2) and $(2, \infty)$

Lets consider interval $(-\infty, 0)$ and (1, 2)

In this case, we have $x(x - 2)(x - 1)< 0$

Thus, $f'(x) < 0$ when $x < 0$ and $1< x < 2$

Therefore, f(x) is strictly decreasing on interval $(-\infty, 0) \cup (1, 2)$

Now, consider interval (0, 1) and $(2, \infty)$

In this case, we have $x(x - 2)(x - 1) > 0$

Thus, $f'(x) > 0$ when $0 < x < 1$ and $x < 2$

Therefore, f(x) is strictly increases on interval $(0, 1) \cup (2, \infty)$

41. Here , it is given that the function $f(x) = 5 + 36x + 3x^2 - 2x^3$

We know that,

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

i. If $f'(x) > 0$ for all $X \in (a, b)$, then f(x) is increasing on (a, b)

ii. If $f'(x) < 0$ for all $X \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

i. Obtain the function and put it equal to f(x)

ii. Find f'(x)

iii. Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Now,

$f(x) = 5 + 36x + 3x^2 - 2x^3$

$\Rightarrow f'(x) = \frac{d}{dx}\left(5 + 36x + 3x^2 - 2x^3\right)$

$\Rightarrow f'(x) = 36 + 6x - 6x^2$

For f(x) lets find critical point, we must have

$\Rightarrow f'(x) = 0$

$\Rightarrow 36 + 6x - 6x^2 = 0$

$\Rightarrow 6(-x^2 + x + 6) = 0$

$\Rightarrow 6(-x^2 + 3x - 2x + 6) = 0$

$\Rightarrow -x^2 + 3x - 2x + 6 = 0$

$\Rightarrow x^2 - 3x + 2x - 6 = 0$

$\Rightarrow (x - 3)(x + 2) = 0$

$\Rightarrow x = 3 , - 2$

clearly, $f'(x) > 0$ if $-2< x < 3$

and $f'(x) < 0$ if $x < -2$ and $x > 3$

Therefore, f(x) increases on $x \in (-2,3)$

and f(x) is decreasing on interval $(-\infty,-2) \cup (3, \infty)$

42. Here,it is given the function

$f(x) = 2x^3 - 15x^2 + 36x + 6$

$f'(x) = 6x^2 - 30x + 36$

$= 6(x^2 - 5x + 6) = 6(x - 3)(x - 2)$ ... (i)

a. f(x) is increasing

$\Leftrightarrow f'(x) \geq 0$

$\Leftrightarrow 6(x - 3)(x - 2) \geq 0$ ... [(from (i)]

$\Leftrightarrow (x -3)(x -2) \geq 0$

$\Leftrightarrow [(x - 3) \geq 0 \text{ and } (x - 2) \geq 0]$

$\Leftrightarrow [(x - 3) \leq 0 \text{ and } (x - 2) \leq 0]$

$\Leftrightarrow [x \geq 3 \text{ and } x \geq 2] \text{ or } [x \leq 3 \text{ and } x \leq 2]$

$\Leftrightarrow$ [x $\geq$ 3] or [x $\leq$ 2]

$\Leftrightarrow$ $x \in [3, \infty]$ or $x \in [-\infty, 2]$

$\Leftrightarrow$ $x \in [-\infty, 2] \cup [3, \infty]$

$\therefore$ f(x) is increasing on $[-\infty, 2] \cup [3, \infty]$

b. f(x) is decreasing

$\Leftrightarrow$ f(x) $\leq$ 0

$\Leftrightarrow$ 6(x - 3)(x - 2) $\leq$ 0 ... [from (i)]

$\Leftrightarrow$ (x - 3)(x - 2) $\leq$ 0

$\Leftrightarrow$ 2 $\leq$ x $\leq$ 3

$\Leftrightarrow$ x $\in$ [2, 3].

$\therefore$ f(x) is decreasing on [2, 3].

Hence, f(x) is increasing on $[-\infty, 2] \cup [3, \infty]$ and decreasing on [2, 3].

43. Here,it is given the function $f(x) = (\sin^4 x + \cos^4 x)$

$\Rightarrow$ $f'(x) = 4\sin^3 x\cos x - 4\cos^3 x\sin x$

$= -4\sin x \cos x(\cos^2 x - \sin^2 x)$

$= -2\sin 2x \cos 2x = -\sin 4x$ ... (i)

Also, $0 \leq x \leq \frac{\pi}{2} \Leftrightarrow 0 \leq 4x \leq 2\pi$

1. f(x) is increasing

$\Leftrightarrow$ f'(x) $\geq$ 0

$\Leftrightarrow$ -sin 4x $\geq$ 0 ... [from (i)]

$\Leftrightarrow$ sin 4x $\leq$ 0

$\Leftrightarrow$ $\pi \leq 4x \leq 2\pi$

$\Leftrightarrow$ $\frac{\pi}{4} \leq x \leq \frac{\pi}{2}$

$\Leftrightarrow$ $x \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

f(x) is increasing on $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

2. f(x) is decreasing

$\Leftrightarrow$ f'(x) $\leq$ 0 ... [from (i)]

$\Leftrightarrow$ -sin 4x $\leq$ 0

$\Leftrightarrow$ sin 4x $\geq$ 0

$\Leftrightarrow$ $0 \leq 4x \leq \pi$

$\Leftrightarrow$ $0 \leq x \leq \frac{\pi}{4}$

$\Leftrightarrow$ $x \in \left[0, \frac{\pi}{4}\right]$

f(x) is decreasing on $\left[0, \frac{\pi}{4}\right]$

Therefore, f(x) is increasing on $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ and decreasing on $\left[0, \frac{\pi}{4}\right]$

44. The given function is: $f(x) = \sin x + \cos x$

$\Rightarrow$ $f'(x) = -\sin x + \cos x$

Now, let f'(x) = 0

$\Rightarrow$ $\cos x - \sin x = 0$

$\Rightarrow$ $\frac{\sin x}{\cos x} = \frac{\cos x}{\cos x}$

$\Rightarrow$ $\tan x = 1$

$\Rightarrow$ $x = \frac{\pi}{4}, \frac{5\pi}{4}$

$0 \leqslant x \leqslant 2\pi$

int	Sign of f'(x)	Result
$\left(0, \frac{\pi}{4}\right)$	+ve	Increase
$\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$	-ve	Decrease
$\left(\frac{5\pi}{4}, 2\pi\right)$	+ve	increase

45. Given,

$f(x) = \frac{x}{x^2+1}$

$\Rightarrow \quad f'(x) = \frac{\left(x^2+1\right) \times 1 - x(2x+0)}{\left(x^2+1\right)^2} = \frac{1-x^2}{\left(x^2+1\right)^2}$

For f(x) to be increasing, we must have

$\Rightarrow \quad \frac{1-x^2}{\left(x^2+1\right)^2} > 0$

$\Rightarrow \quad 1 - x^2 > 0$

$\Rightarrow \quad -\left(x^2 - 1\right) > 0$

$x^2 - 1 < 0$

$(x-1)(x+1) < 0$

$-1 < x < 1$

$x \in (-1, 1)$

Therefore, f(x) is increasing on (-1,1)

For f(x) to be decreasing, we must have

$\Rightarrow \quad \frac{1-x^2}{\left(x^2+1\right)^2} < 0$

$\Rightarrow \quad 1 - x^2 < 0$

$\Longrightarrow \quad -\left(x^2 - 1\right) < 0$

$x^2 - 1 > 0$

$\Longrightarrow \quad (x-1)(x+1) > 0$

$\Longrightarrow \quad x < -1$ or $x > 1$

Thus , f(x) is decreasing on $(-\infty, -1) \cup (1, \infty)$

46. Here,it is given the function

$f(x) = (x + 1)^3 (x - 3)^3$

$f'(x) = (x+1)^3 \cdot \frac{d}{dx}(x-3)^3 + (x-3)^3 \cdot \frac{d}{dx}(x+1)^3$

$= (x + 1)^3 3(x - 3)^2 + (x - 3)^3 3(x + 1)2$

$= 3(x + 1)^2(x - 3)^2[(x + 1) + (x - 3)]$

$= 6(x + 1)^2(x - 3)^2(x - 1)$ ... (i)

(a) f(x) is increasing

$\Leftrightarrow \; f'(x) \geq 0$

$\Leftrightarrow\ 6(x+1)^2(x-3)^2(x-1)\ \geq\ 0$ ... [from (i)]
$\Leftrightarrow\ (x-1)\ \geq\ 0$
$\Leftrightarrow\ x\ \geq\ 1$
$\Leftrightarrow\ x \in [1, \infty]$
$\therefore$ f(x) is increasing on $[1, \infty]$
(b) f(x) is decreasing
$\Leftrightarrow\ f'(x)\ \leq\ 0$
$\Leftrightarrow\ 6(x+1)^2(x-3)^2(x-1)\ \leq\ 0$ ... [from (i)]
$\Leftrightarrow\ (x-1)\ \leq\ 0\ \Leftrightarrow\ x\ \leq\ 1$
$\Leftrightarrow\ x \in [-\infty, 1]$
$\therefore$ f(x) is decreasing on $[-\infty, 1]$
Hence, f(x) is increasing on $[1, \infty]$ and decreasing on $[-\infty, 1]$

47. Here,it is given the function
$f(x) = (\sin x - \cos x),\ 0 < x < 2\pi$

$$\Rightarrow\ f'(x) = (\cos x + \sin x) = \sqrt{2}\left(\frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x\right)$$

$$\Rightarrow\ f'(x) = \sqrt{2}\left(\sin\frac{\pi}{4}\cos x + \cos\frac{\pi}{4}\sin x\right)$$

$$\Rightarrow\ f'(x) = \sqrt{2}\sin\left(\frac{\pi}{4} + x\right) \text{ ... (i)}$$

Now, $0 < x < 2\pi\ \Rightarrow\ \frac{\pi}{4} < \left(\frac{\pi}{4} + x\right) < \left(\frac{\pi}{4} + 2\pi\right)$

$$\Rightarrow\ \frac{\pi}{4} < \left(\frac{\pi}{4} + x\right) < \frac{9\pi}{4} \text{ ... (ii)}$$

a. f(x) is increasing

$\Leftrightarrow\ f'(x)\ \geq\ 0$

$$\Leftrightarrow\ \sqrt{2}\sin\left(\frac{\pi}{4} + x\right) \geq 0 \text{ ... [from (i)]}$$

$$\Leftrightarrow\ \sin\left(\frac{\pi}{4} + x\right) \geq 0$$

$$\Leftrightarrow\ \left\{\frac{\pi}{4} < \left(\frac{\pi}{4} + x\right) \leq \pi\right\} \text{ or } \left\{2\pi \leq \left(\frac{\pi}{4} + x\right) < \frac{9\pi}{4}\right\}$$

$$\Leftrightarrow\ \left\{0 < x \leq \frac{3\pi}{4}\right\} \text{ or } \left\{\frac{7\pi}{4} \leq x < 2\pi\right\}$$

$$\Leftrightarrow\ x \in [0, \frac{3\pi}{4}] \text{ or } x \in \left[\frac{7\pi}{4}, 2\pi\right]$$

$$\Leftrightarrow\ x \in \left[0, \frac{3\pi}{4}\right] \cup \left[\frac{7\pi}{4}, 2\pi\right]$$

$\therefore$ f(x) is increasing on $\left[0, \frac{3\pi}{4}\right] \cup \left[\frac{7\pi}{4}, 2\pi\right]$

b. f(x) is decreasing

$\Leftrightarrow\ f'(x)\ \leq\ 0$

$$\Leftrightarrow\ \sqrt{2}\sin\left(\frac{\pi}{4} + x\right) \leq 0 \text{ ... [from (i)]}$$

$$\Leftrightarrow\ \sin\left(\frac{\pi}{4} + x\right) \leq 0$$

$\Leftrightarrow\ \pi \le \left(\frac{\pi}{4} + x\right) \le 2\pi$

$\Leftrightarrow\ \frac{3\pi}{4} \le x \le \frac{7\pi}{4}$

$\Leftrightarrow\ x \in \left[\frac{3\pi}{4}, \frac{7\pi}{4}\right]$

$\therefore$ f(x) is decreasing on $\left[\frac{3\pi}{4}, \frac{7\pi}{4}\right]$.

48. Here,it is given the function

$f(x) = -2x^3 - 9x^2 - 12x + 1$

$\Rightarrow f'(x) = -6x^2 - 18x - 12 = -6(x^2 + 3x + 2)$

$= -6(x + 2)(x + 1)$ ... (i)

(a) f(x) is strictly increasing

$\Leftrightarrow f'(x) > 0$

$\Leftrightarrow -6(x + 2)(x + 1)$ [from (i)]

$\Leftrightarrow (x + 2)(x +1) < 0$

$\Leftrightarrow -2 < x < -1$

$\Leftrightarrow x \in [-2 ,-1]$.

(b) f(x) is strictly decreasing

$\Leftrightarrow f'(x) < 0$

$\Leftrightarrow -6\{x + 2)(x +1) < 0$ [from (i)]

$\Leftrightarrow (x + 2)(x +1) > 0$

$\Leftrightarrow [(x + 2) > 0$ and $(x + 1) > 0]$

or $[(x + 2) < 0$ and $(x +1) < 0]$

$\Leftrightarrow (x >-2$ and $x > -1)$ or $(x < -2$ and $x < -1)$

$\Leftrightarrow (x > -1)$ or.$(x < -2)$

$\Leftrightarrow x \in [-1, \infty]$ or $x \in [-\infty, -2]$

$\Leftrightarrow x \in [-\infty, -2] \cup [1 - 1, \infty]$

$\therefore$ f(x) is strictly decreasing on $[-\infty, -2] \cup [1 - 1, \infty]$

49. $f(x) = (x + 1)^3(x - 3)^3$

$f'(x) = (x + 1)^3.3(x - 3)^2 + (x - 3)^3.3(x + 1)^2$

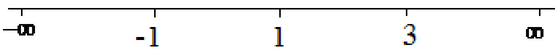
$= 3(x + 1)^2(x - 3)^2[x + 1 + x - 3]$

$= 3(x + 1)^2(x - 3)^2[2x - 2]$

$= 6(x + 1)^2(x - 3)^2(x - 1)$

Put $f'(x) = 0$

$x = -1, 3, 1$



int	Sign of f’(x)	Result
$(-\infty, -1)$	-ve	Decrease
(-1, 1)	-ve	Decrease
(1, 3)	+ve	Increase
$(3, \infty)$	+ve	Increase

50. Here,it is given the function

$f(x) = x^3 + 3x^2 - 105x + 25$

$\Rightarrow f'(x) = 3x^2 + 6x - 105 = 3(x^2 + 2x - 35) = 3(x + 7)(x-5)$ ... (i)

a. f(x) is increasing

$\Leftrightarrow f'(x) \ge 0$

$\Leftrightarrow 3(x + 7)\{x - 5) \ge 0$ [from (i)]

⇔ $(x + 7)(x - 5) \geq 0$

⇔ $[(x + 7) \geq 0$ and $(x - 5) \geq 0]$ or $[(x + 7) \leq 0$ and $(x - 5) \leq 0]$

⇔ $[x \geq -7$ and $x \geq 5]$ or $[x \leq -7$ and $x \leq 5]$

⇔ $(x \geq 5)$ or $(x \leq -7)$

⇔ $x \in [5, \infty]$ or $x \in [-\infty, -7]$

⇔ $x \in [-\infty, -7] \cup [5, \infty]$

∴ f(x) is increasing on $[-\infty, -7] \cup [5, \infty]$

b. f(x) is decreasing

⇔ $f'(x) \leq 0$

⇔ $3(x + 7)(x - 5) \leq 0$ ... [From (i)]

⇔ $(x + 7)(x - 5) \leq 0$

⇔ $-7 \leq x \leq 5$

⇔ $x \in [-7, 5]$.

∴ f(x) is decreasing on [-7, 5]

Hence, f(x) is increasing on $[-\infty, -7] \cup [5, \infty]$ and decreasing on [-7 , 5].

51. Given function is $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$

Differentiating both sides w.r.t. x,

$f'(x) = 12x^3 - 12x^2 - 24x$

For strictly increasing or strictly decreasing, put f' (x) = 0, we get

$12x^3 - 12x^2 - 24x = 0$

$\Rightarrow 12x(x^2 - x - 2) = 0$

$\Rightarrow 12x[x^2 - 2x + x - 2] = 0$

$\Rightarrow 12x(x + 1)(x - 2) = 0$

$\therefore x = 0, -1$ *or* 2

Interval	**f'(x) = 12x(x + 1)(x - 2)**	**Sign of f'(x)**
x<-1	(-)(-)(-)	-ve
-1<x<0	(-)(+)(-)	+ve
0< x < 2	(+)(+)(-)	-ve
x>2	(+)(+)(+)	+ve

A function f(x) is said to be strictly increasing, if f'(x) > 0 and it is said to be strictly decreasing, if f'(x) < 0. So, the given function f(x) is

i. strictly increasing on the intervals (-1, 0) and (2, ∞)
ii. strictly decreasing on the intervals (-∞, -1) and (0, 2)

52. Here,it is given function

$f(x) = 10 – 6x – 2x^2$

Theorem: Let f be a differentiable real function defined on an open interval (a, b).|

i. If f'(x) > 0 for all x ∈ (a, b), then f(x) is increasing on (a, b)
ii. If f'(x) < 0 for all **x** ∈ (**a**, **b**), then f(x) is decreasing on (a, b)

Algorithm:-

i. Obtain the function and put it equal to f(x)
ii. Find f'(x)
iii. Put f'(x) > 0 and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain, it is decreasing.

Here we have,

$f(x) = 10 – 6x – 2x^2$

$\Rightarrow f'(x) = \frac{d}{dx}\left(10 - 6x - 2x^2\right)$

$\Rightarrow f'(x) = –6 – 4x$

For f(x) to be increasing, we must have

$\Rightarrow f'(x) > 0$

$\Rightarrow$ –6 –4x > 0
$\Rightarrow$ –4x > 6
$\Rightarrow x < -\frac{6}{4}$
$\Rightarrow x < -\frac{3}{2}$
$\Rightarrow x \in \left(-\infty, -\frac{3}{2}\right)$

Therefore, f(x) is increasing on the interval $\left(-\infty, -\frac{3}{2}\right)$

Noow, f(x) to be decreasing, we must have
f'(x) < 0
$\Rightarrow$ –6 –4x < 0
$\Rightarrow$ –4x < 6
$\Rightarrow x > -\frac{6}{4}$
$\Rightarrow x > -\frac{3}{2}$
$\Rightarrow x \in \left(-\frac{3}{2}, \infty\right)$

Therefore, f(x) is decreasing on the interval $x \in \left(-\frac{3}{2}, \infty\right)$

53. Here,it is given function f(x) = cos x
We know that,
Theorem:- Let f be a differentiable real function defined on an open interval (a,b).
   i. If f'(x) > 0 for all x $\in$ (a, b), then f(x) is increasing on (a, b)
   ii. If f'(x) < 0 for all x $\in$ (a, b), then f(x) is decreasing on (a, b)

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.
Here we have,
f(x) = cos x
$\Rightarrow f'(x) = \frac{d}{dx}(\cos x)$
= f'(x) = –sinx
Taking different region from 0 to 2π
a) let x $\in$ (0, π)
= sin(x) > 0
= –sinx < 0
= f'(x) < 0
Theyesore, f(x) is decreasing in (0, π)
let x $\in$ (-π, 0)
= sin(x) < 0
= –sinx > 0
= f'(x) > 0
Theyesore, f(x) is increasing in (-π,0)
Therefore, from above condition we find that
f(x) is decreasing in (0, π) and increasing in (-π, 0)
Therefore ,the condition for f(x) neither increasing nor decreasing in (–π,π)

54. Here,it is given function f(x) = 8 + 36x + $3x^2$ – $2x^3$
We know that,
Theorem:- Let f be a differentiable real function defined on an open interval (a,b).
   i. If f'(x) > 0 for all x $\in$ (a, b), then f(x) is increasing on (a, b)
   ii. If f'(x) < 0 for all ,x $\in$ (a, b) then f(x) is decreasing on (a, b)

Algorithm:-

i. Obtain the function and put it equal to f(x)
ii. Find f'(x)
iii. Put f'(x) > 0 and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.
Given,

$f(x) = 8 + 36x + 3x^2 - 2x^3$

$\Rightarrow f'(x) = \frac{d}{dx}\left(8 + 36x + 3x^2 - 2x^3\right)$

$\Rightarrow f'(x) = 36 + 6x - 6x^2$

For f(x) lets find critical point, we must have

$\Rightarrow f'(x) = 0$

$\Rightarrow 36 + 6x - 6x^2 = 0$

$\Rightarrow 6(-x^2 + x + 6) = 0$

$\Rightarrow 6(-x^2 + 3x - 2x + 6) = 0$

$\Rightarrow -x^2 + 3x - 2x + 6 = 0$

$\Rightarrow x^2 - 3x + 2x - 6 = 0$

$\Rightarrow (x - 3)(x + 2) = 0$

$\Rightarrow x = 3, -2$

clearly, $f'(x) > 0$ if $-2 < x < 3$
and $f'(x) < 0$ if $x < -2$ and $x > 3$
Therefore, f(x) increases on $x \in (-2, 3)$
and f(x) is decreasing on interval $(-\infty, -2) \cup (3, \infty)$

55. i. The function is f(x) = sin x
Then, f'(x) = cos x

Since for each $x \in \left(0, \frac{\pi}{2}\right)$, cos x > 0, we have f'(x) > 0

Therefore, function f(x) is strictly increasing in $\left(0, \frac{\pi}{2}\right)$.

ii. The function is f(x) = sin x
Then, f'(x) = cos x

Since for each $x \in \left(\frac{\pi}{2}, \pi\right)$, cos x < 0, we have f'(x) < 0

Therefore, the function f(x) is strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$.

iii. The function is f(x) = sin x
Then, f'(x) = cos x

Since for each $x \in \left(0, \frac{\pi}{2}\right)$, cos x > 0, we have f'(x) > 0

Therefore, f(x) is strictly increasing in $\left(0, \frac{\pi}{2}\right)$ ...(i)

Now, the function is f(x) = sin x
Then, f'(x) = cos x

Since, for each $x \in \left(\frac{\pi}{2}, \pi\right)$, cos x < 0, we have f'(x) < 0

Therefore, f(x) is strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$ ...(ii)

From (i) and (ii)
It is clear that the function f(x) is neither increasing nor decreasing in $(0, \pi)$.

56. Here ,it is given function f(x) = $x^2 + 2x - 5$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

i. If f'(x) > 0 for all x ∈ (a, b), then f(x) is increasing on (a, b)
ii. If f'(x) < 0 for all x ∈ (a, b), then f(x) is decreasing on (a, b)

Algorithm:-

i. Obtain the function and put it equal to f(x)
ii. Find f'(x)
iii. Put f'(x) > 0 and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain, it is decreasing.

Here we have,

f(x) = $x^2 + 2x - 5$

$\Rightarrow f'(x) = \frac{d}{dx}\left(x^2 + 2x - 5\right)$

$\Rightarrow$ f'(x) = 2x + 2

For f(x) to be increasing, we must have

$\Rightarrow$ f'(x) > 0

$\Rightarrow$ 2x + 2 > 0

$\Rightarrow$ 2x < –2

$\Rightarrow x < -\frac{2}{2}$

$\Rightarrow$ x < –1

$\Rightarrow$ x ∈ (–∞,–1)

Therefore, f(x) is increasing on interval (–∞,–1)

Now, for f(x) to be decreasing, we must have

f'(x) < 0

$\Rightarrow$ 2x + 2 < 0

$\Rightarrow$ 2x > –2

$\Rightarrow x > -\frac{2}{2}$

$\Rightarrow$ x> –1

$\Rightarrow$ x ∈ (–1,∞)

Therefore, f(x) is decreasing on interval x ∈ (–1, ∞)

57. Here,it is given the function

f(x) = $\frac{4x^2+1}{x}, x \neq 0$

$\Rightarrow$ f(x) = $\left(4x + \frac{1}{x}\right), x \neq 0$

$\Rightarrow$ f(x) = $\left(4x - \frac{1}{x^2}\right)$

$\Rightarrow$ f'(x)

$= \frac{\left(4x^2-1\right)}{x^2}$ ... (i)

a. f(x) is increasing

$\Leftrightarrow$ f'(x) $\geq 0 \Leftrightarrow \frac{\left(4x^2-1\right)}{x^2} \geq 0$ ... [from (i)]

$\Leftrightarrow (4x^2 - 1) \geq 0$ ... [$\because x^2 > 0$]

$\Leftrightarrow (2x - 1)(2x + 1) \geq 0 \Leftrightarrow 2\left(x - \frac{1}{2}\right) \cdot 2\left(x + \frac{1}{2}\right) \geq 0$

$\Leftrightarrow \left(x - \frac{1}{2}\right)\left(x + \frac{1}{2}\right) \geq 0$

$\Leftrightarrow \left[\left(x-\frac{1}{2}\right) \geq 0 \text{ and } \left(x+\frac{1}{2}\right) \geq 0\right]$

or $\left[\left(x-\frac{1}{2}\right) \leq 0 \text{ and } \left(x+\frac{1}{2}\right) \leq 0\right]$

$\Leftrightarrow \left[x \geq \frac{1}{2} \text{ and } x \geq -\frac{1}{2}\right] \text{ or } \left[x \leq \frac{1}{2} \text{ and } x \leq -\frac{1}{2}\right]$

$\Leftrightarrow \left(x \geq \frac{1}{2}\right) \text{ or } \left(x \leq -\frac{1}{2}\right)$

$\Leftrightarrow x \in \left[\frac{1}{2}, \infty\right] \text{ or } x \in \left[-\infty, -\frac{1}{2}\right]$

$\Leftrightarrow x \in \left[-\infty, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, \infty\right]$

$\therefore$ f(x) is increasing on $\left[-\infty, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, \infty\right]$

b. f(x) is decreasing

$\Leftrightarrow f'(x) \leq \frac{\left(4x^2-1\right)}{x^2} \leq 0$ ... [from (i)]

$\Leftrightarrow (4x^2 - 1) < 0$ $[x^2 > 0]$

$\Leftrightarrow (2x-1)(2x+1) < 0 \; 2\left(x-\frac{1}{2}\right) \cdot 2\left(x+\frac{1}{2}\right) \leq 0$

$\Leftrightarrow \left(x-\frac{1}{2}\right)\left(x+\frac{1}{2}\right) \leq 0$

$\Leftrightarrow -\frac{1}{2} \leq x \leq \frac{1}{2}$

$\Leftrightarrow x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

f(x) is decreasing on $\left[-\frac{1}{2}, \frac{1}{2}\right]$

Hence, f(x) is increasing on $\left[-\infty, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, \infty\right]$ and decreasing on $\left[-\frac{1}{2}, \frac{1}{2}\right]$

58. Given, f(x) = sin 3x - cos 3x, $0 < x < \pi$

Therefore, on differentiating both sides w.r.t. x, we get,

f'(x) = 3cos 3x + 3sin 3x

On putting f'(x) = 0, we get,

sin 3x = - cos 3x

$\Rightarrow$ tan 3x = -1

$\Rightarrow \quad 3x = \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{4}$

[$\because$ tanθ is negative in Ilnd and IVth quadrants]

$\Rightarrow \quad x = \frac{\pi}{4}, \frac{7\pi}{12}, \frac{11\pi}{12}$

Now, we find intervals and check in which intervals f(x) is strictly increasing or strictly decreasing.

Interval	**Test value**	**f'(x) = 3(cos x + sin 3x)**	**Sign of f'(x)**
$0 < x < \frac{\pi}{4}$	At x = $\frac{\pi}{6}$	$3\left(\cos\frac{\pi}{2} + \sin\frac{\pi}{2}\right) = 3(0 + 1) = 3$	+ve
$\frac{\pi}{4} < x < \frac{7\pi}{12}$	At x = $\frac{\pi}{3}$	$3(\cos \pi + \sin \pi) = 3(-1 + 0) = -3$	-ve

$\frac{7\pi}{12} < x < \frac{11\pi}{12}$	At x = $\frac{3\pi}{4}$	$3\left(\cos\frac{9\pi}{4} + \sin\frac{9\pi}{4}\right) = 3\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = 3\sqrt{2}$	+ve
$\frac{11\pi}{12} < x < \pi$	At x = $\frac{23\pi}{24}$	$3\left(\cos\frac{23\pi}{8} + \sin\frac{23}{8}\pi\right)$ $= 3\left[\cos\left(3\pi - \frac{\pi}{8}\right) + \sin\left(3\pi - \frac{\pi}{8}\right)\right] = 3\left(-\cos\frac{\pi}{8} + \sin\frac{\pi}{8}\right)$ $= 3\left(\sin\frac{\pi}{8} - \cos\frac{\pi}{8}\right) < 0$	-ve

Here, we see that f'(x)>0, for 0< x< $\frac{\pi}{4}$ and $\frac{7\pi}{12} < x < \frac{11\pi}{12}$, therefore, f(x) is strictly increasing in the intervals $\left(0, \frac{\pi}{4}\right)$ and $\left(\frac{7\pi}{12}, \frac{11\pi}{12}\right)$

while f'(x)< 0 in $\frac{\pi}{4} < x < \frac{7\pi}{12}$ and $\frac{11\pi}{12} < x < \pi$

therefore, f(x) is strictly decreasing in the intervals $\left(\frac{\pi}{4}, \frac{7\pi}{12}\right)$ and $\left(\frac{11\pi}{12}, \pi\right)$

59. $f(x) = \log(1 + x) - \frac{x}{1+x}$

Domain of f(x) is $(-1, \infty)$

$f'(x) = \frac{1}{1+x} - \left\{\frac{1+x-x}{(1+x)^2}\right\}$

$= \frac{1}{1+x} - \frac{1}{(1+x)^2}$

$= \frac{x}{(1+x)^2}$

For f(x) to be increasing, we must have

f'(x) > 0

$\Rightarrow \frac{x}{(1+x)^2} > 0$

$\Rightarrow$ x > 0 [$\because$ $(1 + x)^2 > 0$, Domain: $(-1, \infty)$]

$\Rightarrow x \in (0, \infty)$

So, f(x) is increasing on $(0, \infty)$

For f(x) to be decreasing, we must have

f'(x) < 0

$\Rightarrow \frac{x}{(1+x)^2} < 0$

$\Rightarrow$ x < 0 [$\because$ (1 + x)2 > 0, Domain: $(-1, \infty)$]

$\Rightarrow x \in (-1, 0)$

So, f(x) is decreasing on (-1, 0).

60. $\boldsymbol{y = \frac{4\sin\theta}{2+\cos\theta} - \theta}$

$f'(\theta) = \frac{(2+\cos\theta)(4\cos\theta) + 4\sin^2\theta}{(2+\cos\theta)^2} - 1$

$= \frac{8\cos\theta + 4}{(2+\cos\theta)^2} - 1$

$= \frac{8\cos\theta + 4 - \left(4 + \cos^2\theta + 4\cos\theta\right)}{(2+\cos\theta)^2}$

$f'(\theta) = \frac{\cos\theta(4-\cos\theta)}{(2+\cos\theta)^2}$

Since $-1 \le \cos\theta \le 1, 4 - \cos\theta > 0\ in\ [0, \frac{\pi}{2}]$

and $(2 + \cos\theta)^2 > 0$ *in* $[0, \frac{\pi}{2}]$ [Being a perfect square]

$$\Rightarrow f'(\theta) = \frac{\cos\theta(4-\cos\theta)}{2+\cos\theta} > 0 \ \forall\ \theta \in \left[0, \frac{\pi}{2}\right]$$

Hence, y is strictly increasing function in $[0, \frac{\pi}{2}]$.

61. Given: $y = \frac{4\sin\theta}{(2+\cos\theta)} - \theta$

$$\Rightarrow \frac{dy}{d\theta} = \frac{(2+\cos\theta).4\cos\theta - 4\sin\theta(-\sin\theta)}{(2+\cos\theta)^2} - 1$$

$$= \frac{8\cos\theta + 4\cos^2\theta + 4\sin^2\theta}{(2+\cos\theta)^2} - 1$$

$$\Rightarrow \frac{dy}{d\theta} = \frac{8\cos\theta + 4\left(\cos^2\theta + \sin^2\theta\right) - (2+\cos\theta)^2}{(2+\cos\theta)^2}$$

$$= \frac{8\cos\theta + 4 - (2+\cos\theta)^2}{(2+\cos\theta)^2}$$

$$\Rightarrow \frac{dy}{d\theta} = \frac{(8\cos\theta + 4) - \left(4 + 4\cos\theta + \cos^2\theta\right)}{(2+\cos\theta)^2}$$

$$= \frac{4\cos\theta - \cos^2\theta}{(2+\cos\theta)^2}$$

$$= \frac{\cos\theta(4-\cos\theta)}{(2+\cos\theta)^2}$$

Since $0 \leqslant \theta \leqslant \frac{\pi}{2}$ and we have $0 \leqslant \cos\theta \leqslant 1$, therefore $4 - \cos\theta > 0$.

$$\therefore \frac{dy}{dx} \geqslant 0 \text{ for } 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

Hence, y is an increasing function of θ in $\left[0, \frac{\pi}{2}\right]$

62. $y = \frac{4\sin\theta}{2+\cos\theta} - \theta$

$$f'(\theta) = \frac{(2+\cos\theta)(4\cos\theta) + 4\sin^2\theta}{(2+\cos\theta)^2} - 1$$

$$= \frac{8\cos\theta + 4}{(2+\cos\theta)^2} - 1$$

$$= \frac{8\cos\theta + 4 - (4 + \cos^2\theta + 4\cos\theta)}{(2+\cos\theta)^2}$$

$$f'(\theta) = \frac{\cos\theta(4-\cos\theta)}{(2+\cos\theta)^2}$$

$$\Rightarrow f'(\theta) = \frac{\cos\theta(4-\cos\theta)}{(2+\cos\theta)^2} > 0 \ \forall\ \theta \notin \left(0, \frac{\pi}{2}\right)$$

This shows that $y = \frac{4\sin\theta}{2+\cos\theta} - \theta$ is increasing on $\left(0, \frac{\pi}{4}\right)$

63. $f(x) = \frac{4\sin x - 2x - x\cos x}{2+\cos x}$

$$= \frac{4\sin x - x(2+\cos x)}{2+\cos x}$$

$$= \frac{4\sin x}{2+\cos x} - \frac{x(2+\cos x)}{(2+\cos x)}$$

$$f(x) = \frac{4\sin x}{2+\cos x} - x$$

$$f'(x) = \frac{\cos x(4-\cos x)}{(2+\cos x)^2}$$

$[\because\ -1 \leqslant \cos x \leqslant 1]$

Hence,

$$\frac{\cos x(4-\cos x)}{(2+\cos x)^2} > 0 \ \forall\ x \in \left(0, \frac{\pi}{2}\right) \text{ and } \left(\frac{3\pi}{2}, 2\pi\right)$$

$\frac{\cos x(4-\cos x)}{(2+\cos x)^2} < 0 \ \forall x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

f(x) is increasing in $\left(0, \frac{\pi}{2}\right)$ and $\left(\frac{3\pi}{2}, 2\pi\right)$

f(x) is decreasing in $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

64. Here, it is given the function, $f(x) = \tan^{-1}(\sin x + \cos x)$

$\Rightarrow f'(x) = \frac{d}{dx}\left(\tan^{-1}(\sin x + \cos x)\right)$

$\Rightarrow f'(x) = \frac{1}{1+(\sin x+\cos x)^2} \times (\cos x - \sin x)$

$\Rightarrow f'(x) = \frac{(\cos x - \sin x)}{1+\sin^2 x+\cos^2 x+2\sin x\cos x}$

$\Rightarrow f'(x) = \frac{\cos x - \sin x}{2(1+\sin x\cos x)}$

Now, as given

$x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

= Cosx – sinx< 0; as here cosine values are smaller than sine values for same angle

$= \frac{\cos x - \sin x}{2(1+\sin x\cos x)} < 0$

= f'(x) < 0

So, Condition for f(x) to be decreasing

Therefore, f(x) is decreasing on interval $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

65. Given, $f(x) = (x - 1)(x - 2)^2$

On differentiating both sides w.r.t. x, we get

$f'(x) = (x - 1)\frac{d}{dx}(x-2)^2 + (x-2)^2\frac{d}{dx}(x-1)$[by using product rule of derivative]

$\Rightarrow$ $f'(x) = 2(x - 1)(x - 2) + (x - 2)^2 \cdot 1$

$\Rightarrow$ $f'(x) = 2(x - 1)(x - 2) + (x - 2)^2$

$\Rightarrow$ f'(x) = (x - 2) [2x - 2 + x - 2]

$\Rightarrow$ f'(x) = (x - 2) (3x - 4)

On putting f'(x) = 0, we get

(x - 2) (3x - 4) = 0

$\Rightarrow$ x - 2 = 0 or 3x - 4 = 0

Now, we find the intervals in which f(x) is strictly increasing or strictly decreasing.

Interval	**f'(x) =(x - 2)(3x - 4)**	**Sign of f'(x)**
$x<\frac{4}{3}$	(-)(-)	+ve
$\frac{4}{3}<x<2$	(-)(+)	-ve
x > 2	(+)(+)	+ve

We know that, a fuction f(x) is sad to be a strictly increasing fuction, if f'(x) > 0 and strictly decreasing function when f'(x) < 0.

So, f(x) is increasing on $\left(-\infty, \frac{4}{3}\right)$ and (2, ∞) and decreasing on $\left(\frac{4}{3}, 2\right)$.

Since, f(x) is a polynomial function, so it is continuous at x = 4/3 and 2.

Hence, given function is increasing on interval $\left(-\infty, \frac{4}{3}\right]$ and [2, ∞) and decreasing on interval $\left[\frac{4}{3}, 2\right]$.

66. Given: $y = \log(1+x) - \frac{2x}{2+x}$

$$\therefore \frac{dy}{dx} = \frac{1}{1+x}\frac{d}{dx}(1+x) - \left[\frac{(2+x)\frac{d}{dx}(2x) - 2x\frac{d}{dx}(2+x)}{(2+x)^2}\right]$$

$$= \frac{1}{1+x} - \frac{(4+2x-2x)}{(2+x)^2}$$

$$= \frac{1}{1+x} - \frac{4}{(2+x)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(2+x)^2 - 4(1+x)}{(1+x)(2+x)^2}$$

$$= \frac{x^2}{(1+x)(2+x)^2} \quad ...(i)$$

Domain of the given function is given to be $x > -1$

$\Rightarrow x + 1 > 0$

Also $(2+x)^2 > 0$ and $x^2 \geqslant 0$

$\therefore$ From eq. (i), $\frac{dy}{dx} \geqslant 0$ for all x in domain $x > -1$ and f is an increasing function.

Section C

67. $C = 40000h^2 + 5000x^2$

as $x^2h = 250$

$\Rightarrow C = \frac{40000(250)^2}{x^4} + 5000x^2$

68. $\frac{dC}{dx} = \frac{-160000(250)^2}{x^5} + 10000x$

69. For minimum cost $\frac{dc}{dx} = 0$

$\Rightarrow 10000x^6 = 250 \times 250 \times 160000$

$\Rightarrow x = 10$

showing $\frac{d^2C}{dx^2} > 0$ at x = 10

$\therefore$ cost is minimum when x = 10

70. $\frac{dC}{dx} = \frac{-160000(250)^2}{x^4} + 10000x$

$\frac{dC}{dx} = 0$ gives x = 10

$\frac{dC}{dx} > 0$ in $(10, \infty)$ and $\frac{dC}{dx} < 0$ in (0, 10).

Hence, cost function is neither increasing nor decreasing for x > 0

Section D

71. i. Let $f_1(x) = \cos x$

$\therefore f_1'(x) = -\sin x$

In interval $\left(0, \frac{\pi}{2}\right)$, $f_1'(x) = -\sin x < 0$.

Therefore, $f_1(x) = \cos x$ is strictly decreasing in interval $\left(0, \frac{\pi}{2}\right)$.

ii. Let $f_2(x) = \cos 2x$

$\therefore f_2'(x) = -2\sin 2x$

Now, $0 < x < \frac{\pi}{2}$

$\Rightarrow 0 < 2x < \pi$

$\Rightarrow \sin 2x > 0$

$\Rightarrow -2\sin 2x < 0$

$\therefore\ f_2'(x) = -2 \sin 2x < 0$ on $\left(0, \frac{\pi}{2}\right)$

Therefore, $f_2(x) = \cos 2x$ is strictly decreasing in interval $\left(0, \frac{\pi}{2}\right)$.

iii. Let $f_3(x) = \cos 3x$

$\therefore\ f_3'(x) = -3 \sin 3x$

Now, $f_3' = 0$

$\Rightarrow\ \sin 3x = 0$

$\Rightarrow\ 3x = \pi$, as $x \in \left(0, \frac{\pi}{2}\right)$

$\Rightarrow x = \frac{\pi}{3}$

The point $x = \frac{\pi}{3}$ divides the interval $\left(0, \frac{\pi}{2}\right)$ into two distinct intervals.

i.e. $\left(0, \frac{\pi}{3}\right)$ and $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

Now, in interval, $\left(0, \frac{\pi}{3}\right)$

$f_3'(x) = -3\sin 3x < 0$ as $\left(0 < x < \frac{\pi}{2} \Rightarrow 0 < 3x < \pi\right)$

Therefore, f_3 is strictly decreasing in interval $\left(0, \frac{\pi}{3}\right)$

Now, in interval $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

$f_3'(x) = -3\sin 3x >$ as $\frac{\pi}{3} < x < \frac{\pi}{2} \Rightarrow \pi < 3x < \frac{3\pi}{2}$

Therefore, f_3 is strictly increasing in interval $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$.

iv. Let $f_4 = \tan x$

$\therefore f_3'(x) = \sec^2 x$

In interval $\left(0, \frac{\pi}{2}\right)$

$f_3'(x) = \sec^2 x > 0$

Therefore, f_4 is strictly increasing in interval $\left(0, \frac{\pi}{2}\right)$

72. We have,

$f(x) = \cot^{-1}(\sin x + \cos x)$

$\Rightarrow f'(x) = \frac{-1}{1 + (\sin x + \cos x)^2} \times (\cos x - \sin x)$

$= \frac{\sin x - \cos x}{1 + \sin^2 x + \cos^2 x + 2\sin x\cos x}$

$= \frac{\sin x - \cos x}{1 + 1 + 2\sin x\cos x}$

$= \frac{\sin x - \cos x}{2 + 2\sin x\cos x}$

$= \frac{1}{2} \times \frac{\sin x - \cos x}{1 + \sin x\cos x}$

For f(x) to be decreasing, we must have

$f'(x) < 0$

$\Rightarrow \frac{1}{2} \times \frac{\sin x - \cos x}{1 + \sin x \cos x} < 0$

$\Rightarrow \frac{\sin x - \cos x}{1 + \sin x \cos x} < 0$

$\Rightarrow$ sin x - cos x < 0 (In first quadrant)

$\Rightarrow$ sin x < cos x

$\Rightarrow$ tan x < 1

$\Rightarrow 0 < x < \frac{\pi}{4}$

So, f (x) is decreasing on $\left(0, \frac{\pi}{4}\right)$

For f(x) to be increasing, we must have

f'(x) > 0

$\Rightarrow \frac{1}{2} \times \frac{\sin x - \cos x}{1 + \sin x \cos x} > 0$

$\Rightarrow \frac{\sin x - \cos x}{1 + \sin x \cos x} > 0$

$\Rightarrow$ sin x - cos x > 0 (In first quadrant)

$\Rightarrow$ sin x > cos x

$\Rightarrow$ tan x > 1

$\Rightarrow \frac{\pi}{4} < x < \frac{\pi}{2}$

So, f (x) is increasing on $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$.

73. Given, f(x) = 20 - 9x + $6x^2$ - x^3.

On differentiating both sides w.r.t. x, we get

f'(x) = - 9 + 12x - $3x^2$

On putting f'(x) = 0, we get

-9 + 12x - $3x^2$ = 0 $\Rightarrow$ -3(x^2 - 4x + 3) = 0

$\Rightarrow$ -3(x - 1) (x - 3) = 0 $\Rightarrow$ (x - 1)(x - 3) = 0

$\Rightarrow$ x - 1 = 0 or x - 3 = 0 $\Rightarrow$ x = 1 or 3

Now, we find intervals in which f(x) is strictly increasing or strictly decreasing.

Interval	**f'(x) =-3(x - 1) (x - 3)**	**Sign of f'(x)**
x<1	(-) (-) (-)	-ve
1 < x < 3	(-) (+) (-)	+ve
x > 3	(-) (+) (+)	-ve

Therefore, by definition, the given function f(x) is

i. strictly increasing on the interval (1, 3) and
ii. strictly decreasing on the intervals (-∞, 1) and (3, ∞)

74. $f(x) = \tan^{-1}(\sin x + \cos x)$

$f'(x) = \frac{1}{1 + (\sin x + \cos x)^2}.(\cos x - \sin x)$

$= \frac{\cos x(1 - \tan x)}{1 + (\sin x + \cos x)^2}$

For critical points let f '(x)=0

$\frac{\cos x(1 - \tan x)}{1 + (\sin x + \cos x)^2} = 0$

cosx (1 - tanx) = 0

$\Rightarrow$ cosx = 0, or 1 - tanx = 0

since $\tan x < 1 \ \forall x \in \left(0, \frac{\pi}{4}\right)$

$f'(x) < 0 \ \forall x \in \left(0, \frac{\pi}{4}\right)$

Hence f(x) is strictly increasing on $\left(0, \frac{\pi}{4}\right)$

75. We have,

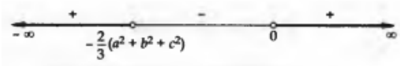
$$f(x) = \begin{vmatrix} x + a^2 & ab & ac \\ ab & x + b^2 & bc \\ ac & bc & x + c^2 \end{vmatrix}$$

$$\Rightarrow \quad f'(x) = \begin{vmatrix} 1 & 0 & 0 \\ ab & x + b^2 & bc \\ ac & bc & x + c^2 \end{vmatrix} + \begin{vmatrix} x + a^2 & ab & ac \\ 0 & 1 & 0 \\ ac & bc & x + c^2 \end{vmatrix} + \begin{vmatrix} x + a^2 & ab & ac \\ ab & x + b^2 & bc \\ 0 & 0 & 1 \end{vmatrix}$$

$$\Rightarrow \quad f'(x) = \left(x + b^2\right)\left(x + c^2\right) - b^2c^2 + \left(x + a^2\right)\left(x + c^2\right) - a^2c^2 + (x + a)^2(x + b)^2 - a^2b^2$$

$$\Rightarrow \quad f'(x) = 3x^2 + 2x\left(a^2 + b^2 + c^2\right)$$

For (x) to be increasing, we must have



$$\Rightarrow \quad 3x^2 + 2x\left(a^2 + b^2 + c^2\right) > 0$$

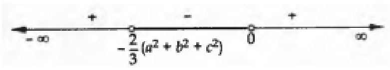
$$\Rightarrow \quad x\left\{3x + 2\left(a^2 + b^2 + c^2\right)\right\} > 0$$

$$\Rightarrow \quad x < -\frac{2}{3}\left(a^2 + b^2 + c^2\right) \text{ or, } x > 0$$

$$\Rightarrow \quad x \in \left(-\infty, -\frac{2}{3}\left(a^2 + b^2 + c^2\right)\right) \cup (0, \infty)$$

So, f(x) is increasing on $\left(-\infty, -\frac{2}{3}\left(a^2 + b^2 + c^2\right)\right) \cup (0, \infty)$

For f(x) to be decreasing we, must have



$$\Rightarrow \quad 3x^2 + 2x\left(a^2 + b^2 + c^2\right) < 0$$

$$\Rightarrow \quad x\left\{3x + 2\left(a^2 + b^2 + c^2\right)\right\} < 0$$

$$\Rightarrow \quad -\frac{2}{3}\left(a^2 + b^2 + c^2\right) < x < 0$$

$$\Rightarrow \quad x \in \left(-\frac{2}{3}\left(a^2 + b^2 + c^2\right), 0\right)$$

So, f(x) is decreasing on $\left(-\frac{2}{3}\left(a^2 + b^2 + c^2\right), 0\right)$

hence, f (x) is increasing on $\left(-\infty, -\frac{2}{3}\left(a^2 + b^2 + c^2\right)\right) \cup (0, \infty)$ and decreasing on $\left(-\frac{2}{3}\left(a^2 + b^2 + c^2\right), 0\right)$